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PAPER

Coefficients—Delay Simultaneous Adaptation Scheme for Linear Equalization of Nonminimum Phase Channels

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SUMMARY An efficient adaptation technique of the delay is introduced for accomplishing more accurate adaptive linear equalization of nonminimum phase channels. It is focused that the filter structure and adaptation procedure of the adaptive Butler-Cantoni (ABC) equalizer is very suitable to deal with a variable delay for each iteration, compared with a classical adaptive linear transversal equalizer (LTE). We derive a cost function by comparing the system mismatch of an optimum equalizer coefficient vector with an equalizer coefficient vector with several delay settings. The cost function is square of difference of absolute values of the first element and the last element for the equalizer coefficient vector. The delay adaptation method based on the cost function is developed, which is involved with the ABC equalizer. The delay is adapted by checking the first and last elements of the equalizer coefficient vector and this results in an LTE providing a lower mean square error level than the other LTEs with the same order. We confirm the performance of the ABC equalizer with the delay adaptation method through computer simulations.

key words: *adaptive channel equalizer, linear transversal filter, optimum delay, nonminimum phase channel, channel estimation, Levinson-Trench algorithm*

1. Introduction

Modern telephone network systems have required efficient and effective elimination of the intersymbol interference for accomplishing a fast data transmission [1]. Therefore, fast convergence called "fast start-up equalization" is desired for adaptive equalizers in the training mode. From this point of view, two commonly used linear transversal equalizers (LTEs), the least mean square (LMS) and recursive least square (RLS) transversal equalizers, have been investigated and modified by many researchers [2]–[8].

Recently, Shimamura et al. have proposed an adaptive LTE involving a channel estimator, which is called the adaptive Butler-Cantoni (ABC) equalizer [9], [10]. The ABC equalizer converges faster than the LMS and RLS transversal equalizers, because it is indirectly adapted from the results of the channel estimator by use of the Butler-Cantoni non-iterative method [11] for each iteration. The convergence property of the adaptive Butler-Cantoni equalizer depends only on the input signal for the channel estimator, which is in many cases a pseudo-random sequence. This fact of the ABC equalizer leads to a fastest converging LTE.

The LTEs usually require a fixed delay for a nonminimum phase channel, which is often set to half of the equalizer length. This is because we have no priori information about the channel. However, this is not ideal for channel equalization, because the delay setting affects the performance of the adaptive LTEs [12]. The delay is desired to be adjusted at the receiver side, but this is not easy.

In noniterative channel equalization, some techniques have been introduced for the delay adjustment [11], [13]. Halpern et al. [13] have derived a determination method of the delay for the LTEs. They have focused that a nonminimum phase channel can be constructed by a cascade connection of a minimum phase channel and a maximum phase channel. This fact means that, when the minimum phase part of the nonminimum phase channel is canceled by embedding its inverse in the LTE, the remaining part becomes the maximum phase one. Then, the delay adjustment problem for the nonminimum phase channel reduces that for the maximum phase channel. Thus, the delay setting becomes easy. The determination method, however, has two problems for its application to adaptive equalization. The first problem is that the determination method is too complicated. This is because the calculation of the zeros of the channel is required for suppressing the minimum phase part at each iteration. The second problem is that we cannot deploy the determination method in noisy environments. The signal-to-noise ratio (SNR) is a factor to decide the delay. The determination method, however, utilizes only the channel impulse response and does not utilize the information of the SNR. From the above reasons, it is difficult to apply the determination method to adaptive equalization.

On the other hand, Butler and Cantoni [11] have suggested a delay selection method to minimize the mean square error (MSE). The delay selection method compares error signals with several delay settings. Therefore, the selection method requires to solve the normal equation and to calculate the equalizer output and the error signal at each delay setting. The additional computational complexity is proportional to M^2, where M is the equalizer length, which makes the delay selection method complicated. However, we can utilize an efficient recursive algorithm [11] in the delay selection method. Then, the additional computational complexity is proportional to M. However, the selection method is still complicated for its application to adaptive equalization. This is because the computation of the ABC

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equalizer itself even is proportional to M^2.

In adaptive channel equalization, some methods [14]–[16] have been developed against the delay adjustment problem. Qureshi [14] has developed a gradient-direct method to search the optimum delay, which allows the LTE to adapt the delay for 200 iterations. This method, however, may require a lot of number of iterations to converge. The other methods [15], [16] are suitable on only multichannels. Hence, we cannot treat them as traditional methods.

We investigate the delay adjustment problem for the LTE on nonminimum phase channels in noiseless and white noise environments. First, we discuss two LTEs, the classical LTE and the ABC equalizer. The coefficients of the classical LTE is directly adapted with feedback of error signals. The coefficients of the ABC equalizer is, however, indirectly adapted by using information of the channel estimator. From this reason, we mention that the filter structure and adaptation procedure of the ABC equalizer is very suitable to change the delay at each sampling time rather than the classical LTE. Next, we derive a new cost function [17]. The cost function is derived by comparing the system mismatch between an optimum equalizer coefficient vector and an equalizer coefficient vector with several delay settings. The cost function is square of difference of absolute values of the first and last elements of the coefficient vector for the LTE, which is minimized when the optimum delay is set. We evaluate the cost function by comparing numerical results with simulation results. Following the derivation and the evaluation, we develop a delay adaptation scheme based on the cost function, which is involved in the ABC equalizer. The delay adaptation method minimizes the cost function when the adaptation is enough. The computation required by the delay adaptation method is proportional to neither M nor the length of the channel. We consider the performance of the delay adaptation scheme in white noise environments. We evaluate the performance of the ABC equalizer with the delay adaptation method in noiseless and white noise environments through computer simulations.

The rest of this paper is organized as follows. Sect. 2 gives preliminaries where the classical LTE and the ABC equalizer are briefly discussed. Sect. 3 proposes the delay adaptation method and explains its actual implementation in the ABC equalizer. Sect. 4 demonstrates the performance of the ABC equalizer with the delay adaptation method through simulation results. Sect. 5 draws concluding remarks.

2. Preliminaries

2.1 Discrete-Time Channel Model

It is assumed that a digital communication channel is given by the following equation:

$$x_n = s_n + v_n \tag{1}$$

$$= \sum_{k=0}^{L-1} h_k u_{n-k} + v_n \tag{2}$$

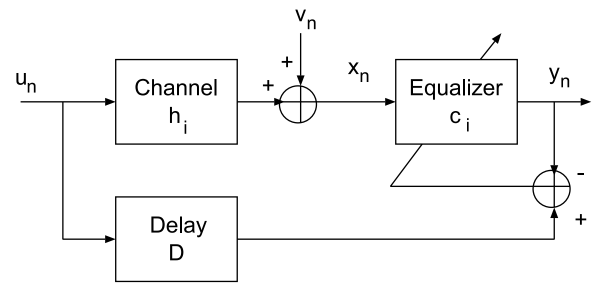

Fig. 1 System configuration of a classical linear transversal equalizer in the training mode.

where $h_0, h_1, \ldots, h_{L-1}$ are the channel impulse response, u_n is the transmitted signal with values of $+1$ or -1, and v_n is an additive white Gaussian noise uncorrelated with u_n. The channel output x_n corresponds to the received signal, which is used as the input signal for an adaptive equalizer.

2.2 Classical Adaptive LTEs

Figure 1 illustrates the system configuration of a classical LTE in the training mode where a D sample delayed transmitted signal, u_{n-D}, is assumed to be generated at the receiver side (D is a constant number). When the LMS adaptation is deployed for the LTE, the coefficients adaptation procedure is described as follows:

$$y_n = X(n)^T C(n) \tag{3}$$

$$e_n = u_{n-D} - y_n \tag{4}$$

$$C(n+1) = C(n) + \nabla_{lms}(n) \tag{5}$$

$$= C(n) + \mu e_n X(n) \tag{6}$$

where T denotes transpose and $X(n)$ and $C(n)$ are the input and coefficient vectors for the equalization filter at n-th iteration, which are given by

$$X(n) = [x_n, x_{n-1}, \ldots, x_{n-M+1}]^T \tag{7}$$

and

$$C(n) = [c_0(n), c_1(n), \ldots, c_{M-1}(n)]^T, \tag{8}$$

respectively. The μ is the step size parameter for the LMS adaptation, which controls the convergence.

As shown by the above descriptions, the LMS adaptation scheme for the classical LTE are performed by adding the gradient vector, $\nabla_{lms}(n)$, to the current coefficient vector $C(n)$. Thus, the previous coefficient vector $C(n-1)$ directly affects the current coefficient vector $C(n)$. This fact means that the delay used at previous iteration also directly affects the current coefficient vector. This is because the gradient vector contains the error signal being the difference between the delayed transmitted signal u_{n-D} and the LTE output y_n. Hence, it seems to be difficult for the classical adaptive LTEs to change the delay D at each sampling time.

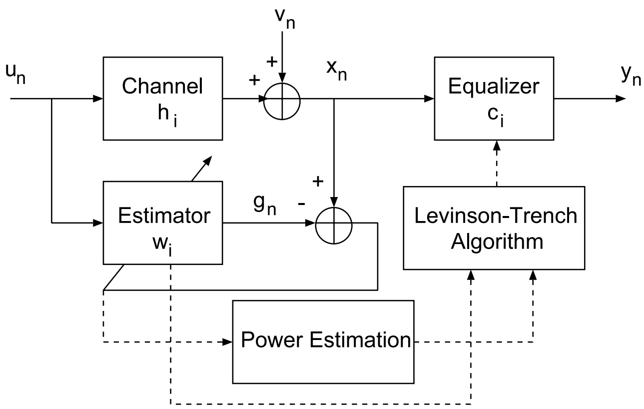

Fig. 2 System configuration of the adaptive Butler-Cantoni equalizer in the training mode.

2.3 ABC Equalizer

The system configuration of the ABC equalizer is illustrated in Fig. 2. The ABC equalizer has two linear transversal filters, the channel estimator and the equalization filter. The equalization filter has M coefficients, the output signal of which is also calculated by (3). The channel estimator has L coefficients, the output signal of which is obtained by

$$g_n = U(n)^T W(n) \tag{9}$$

where $U(n)$ and $W(n)$ are the input and coefficient vectors for the channel estimator, which are given by

$$U(n) = [u_n, u_{n-1}, \ldots, u_{n-L+1}]^T \tag{10}$$

and

$$W(n) = [w_0(n), w_1(n), \ldots, w_{L-1}(n)]^T, \tag{11}$$

respectively. Here, we deploy the LMS adaptation scheme for updating the coefficients of the channel estimator given by

$$f_n = x_n - g_n, \tag{12}$$
$$W(n+1) = W(n) + \mu f_n U(n). \tag{13}$$

By utilizing the coefficients of the channel estimator, the coefficients of the equalization filter are calculated by solving the following normal equation:

$$C(n) = A(n)^{-1} B(n) \tag{14}$$

for each iteration, where $A(n)$ and $B(n)$ denote the auto-correlation matrix and the cross-correlation vector at the n-th iteration. Equation (14) can be efficiently solved with the Levinson-Trench algorithm [11]. In the ideal case, $A(n)$ and $B(n)$ reduce to the true auto-correlation matrix A and the true cross-correlation vector B, respectively. The elements of A and B are given by

$$a_{ij} = \sum_{m=0}^{L-1} h_m h_{m+|i-j|} + \sigma^2 \delta(i-j),$$
$$i, j = 0, 1, \ldots, M-1 \tag{15}$$
$$b_i = h_{D-i},$$
$$i = 0, 1, \ldots, M-1 \tag{16}$$

where σ^2 is the variance of additive noise and $|\cdot|$ and $\delta(\cdot)$ denote the absolute value operation and the Kronecker delta function, respectively.

Actually, instead of A and B, we use $A(n)$ and $B(n)$, the elements of which are given by

$$a_{ij}(n) = \sum_{m=0}^{L-1} w_m(n) w_{m+|i-j|}(n) + \hat{\sigma}^2(n)\delta(i-j),$$
$$i, j = 0, 1, \ldots, M-1 \tag{17}$$
$$b_i(n) = w_{D-i}(n),$$
$$i = 0, 1, \ldots, M-1 \tag{18}$$

from the results of the channel estimator. The $\hat{\sigma}^2(n)$ is a noise variance estimate obtained by the averaging operation shown as

$$\hat{\sigma}^2(n) = \frac{1}{P} \sum_{k=0}^{P-1} f_{n-k}^2 \tag{19}$$

where P is a sample number.

As described above, in the ABC equalization scheme, the previous coefficient vector $C(n-1)$ does not directly affect the current vector $C(n)$ as shown by (14). This fact means that the delay used at previous iteration does not affect the current coefficient vector. Therefore, it is very easy for the ABC equalizer to change the delay D at each sampling time. This property of the ABC equalizer is utilized in the delay adaptation scheme, which will be described in Sect. 3.3.

3. Delay Adaptation for LTEs on Nonminimum Phase Channels

In this section, we derive the delay adaptation for the LTE on nonminimum phase channels. This section consists of five subsections. Sect. 3.1 gives a motivation to this subject and derives the cost function for the delay adaptation in noiseless cases. In Sect. 3.2, the cost function is evaluated. Sect. 3.3 develops an actual technique of delay adaptation. Performance of the derived cost function in noisy environments is investigated in Sect. 3.4. Sect. 3.5 discusses the complexity of the delay adaptation technique.

3.1 Motivation and Derivation of the Cost Function

The transfer function of the nonminimum phase channel given by (2) is reexpressed by

$$H(z) = \sum_{k=0}^{L-1} h_k z^{-k} \tag{20}$$

$$= h_0 \prod_{k=1}^{L-1} (1 - r_k z^{-1}) \tag{21}$$

where r_k and z denote the zeros of $H(z)$ and the forward shift operator, respectively($h_0 \neq 0$ is assumed here). In noiseless environments, an inverse of $H(z)$, $C(z)$, is expressed with a delay D_c in the form of partial fraction expansion [12] as

$$C(z) = \frac{z^{-D_c}}{H(z)} \tag{22}$$

$$= \frac{1}{h_0}\Bigg(\sum_{k=1}^{L-1} \frac{q_k}{1 - r_k z^{-1}}\Bigg) z^{-D_c} \tag{23}$$

$$= \frac{1}{h_0}\Bigg(\sum_{s} \frac{q_s}{1 - r_s z^{-1}} + \sum_{t} \frac{q_t}{1 - r_t z^{-1}}\Bigg) z^{-D_c} \tag{24}$$

where $|r_s| > 1$ and $|r_t| < 1$. From the expression of (24), the impulse response of $C(z)$, c_i, is given by

$$c_i = \begin{cases} \frac{1}{h_0} \sum_s -q_s (r_s)^{i-D_c} & 0 \leq i \leq D_c - 1 \\ \frac{1}{h_0} \sum_t q_t (r_t)^{i-D_c} & D_c \leq i \leq M - 1. \end{cases} \tag{25}$$

Equation (25) denotes the impulse response of $C(z)$ with D_c. The impulse response of $C(z)$ with $D_c + \kappa$ (κ is an integer) is therefore expressed by

$$c_i = \begin{cases} \frac{1}{h_0} \sum_s -q_s (r_s)^{i-D_c} & -\kappa \leq i \leq D_c - 1 \\ \frac{1}{h_0} \sum_t q_t (r_t)^{i-D_c} & D_c \leq i \leq M - 1 - \kappa. \end{cases} \tag{26}$$

The delay D required for the LTE in Sect. 2 is related with D_c as

$$D = D_w + D_c \tag{27}$$

where D_w is the delay by which $|h_{D_w}|$ becomes greater than any other $|h_k|$ for $k = 0, \ldots, L-1$. Equation (27) means that the main peak of the transmitted pulse is delayed by D_w in the channel and it is further delayed by D_c in the equalization filter. The decision of the delay D does not affect D_w, because D_w is determined by $H(z)$. However, D_c is affected. Thus, we investigate how to decide D_c to minimize the MSE of the equalizer output.

From (25), we can see that c_i for $i = 0, \ldots, D_c - 1$ is used for equalization of the maximum phase part of the non-minimum phase channel, because c_i for $i = 0, \ldots, D_c - 1$ includes r_s. Similarly, c_i for $i = D_c, \ldots, M - 1$ is used for equalization of the minimum phase part, because c_i for $i = D_c, \ldots, M - 1$ includes r_t. Thus, in a situation where $|r_s|$ is close to 1, equalization of maximum phase part is an ill-condition for adaptive channel equalization. Then, the residual MSE becomes larger one and $|c_i|/|c_{D_c}|$ for $i = 0, \ldots, D_c - 1$ also becomes larger value. Also, in the case of $|r_t|$, such a property is satisfied. Therefore, we deduce that equalization accuracy of the maximum phase part depends on $|c_i|/|c_{D_c}|$ for $i = 0, \ldots, D_c - 1$ and that of the minimum phase part depends on $|c_i|/|c_{D_c}|$ for $i = D_c + 1, \ldots, M - 1$.

We will further investigate the effects of decision of D_c on the behavior of c_i. In particular, we study a relation between M and c_i because we consider that M is also an important factor to decide D_c. If $M \approx \infty$ is satisfied in noiseless environments, $D_c \approx \infty$ is satisfied in the case where $D_c = M/2$. This satisfaction and (25) result in $|c_0| \approx 0$ and $|c_{M-1}| \approx 0$.

On the other hand, if M is set to a smaller number in noiseless environments, both $|c_0|$ and $|c_{M-1}|$ become larger. From these relationships between M and c_i in noiseless cases, we consider that equalization accuracy of the maximum phase part depends on $|c_0|/|c_{D_c}|$ and that of the minimum phase part depends on $|c_{M-1}|/|c_{D_c}|$. From this point of view, if D_c is set so that

$$\frac{|c_0|}{|c_{D_c}|} \approx \frac{|c_{M-1}|}{|c_{D_c}|}, \tag{28}$$

equalization accuracy of the maximum phase part becomes close to that of the minimum phase part. In stationary cases, $|c_{D_c}|$ is considered to be constant. Hence, we reduce (28) to

$$|c_0| \approx |c_{M-1}|. \tag{29}$$

In this paper, it is presented that (29) is satisfied when the optimum delay is set. To derive (29), we utilize the following knowledge [12]:

$$\begin{cases} MMSE_{D_{opt}} < MMSE_{D_{opt}-1} < MMSE_{D_{opt}-2} < \cdots \\ MMSE_{D_{opt}} < MMSE_{D_{opt}+1} < MMSE_{D_{opt}+2} < \cdots \end{cases} \tag{30}$$

where $MMSE_{D_{opt}}$ denotes the minimum MSE (MMSE) of the equalizer output with an optimum delay D_{opt} on a non-minimum phase channel. Hence, we can compare and consider the characteristics of the LTE with only D_{opt}, $D_{opt} - 1$, and $D_{opt} + 1$. We first prepare three coefficient vectors with D_c, $D_c - 1$, and $D_c + 1$ given by

$$C_{D_c} = [c_0, c_1, \ldots, c_{M-2}, c_{M-1}]^T, \tag{31}$$

$$C_{D_c-1} = [c_1, c_2, \ldots, c_{M-1}, c_M]^T, \tag{32}$$

$$C_{D_c+1} = [c_{-1}, c_0, \ldots, c_{M-3}, c_{M-2}]^T, \tag{33}$$

where all c_i are calculated with (25). To simplify the comparison of C_{D_c}, C_{D_c-1}, and C_{D_c+1}, $C'_{D_c} = C_{D_c} z^{-1}$ and $C'_{D_c-1} = C_{D_c-1} z^{-2}$ are made and zero padding in C'_{D_c} and C'_{D_c+1} is assumed, which yield

$$C'_{D_c} = [0, c_0, c_1, \ldots, c_{M-2}, c_{M-1}, 0]^T, \tag{34}$$

$$C'_{D_c-1} = [0, 0, c_1, c_2, \ldots, c_{M-1}, c_M]^T, \tag{35}$$

$$C'_{D_c+1} = [c_{-1}, c_0, \ldots, c_{M-3}, c_{M-2}, 0, 0]^T. \tag{36}$$

Then, for the comparison, we will obtain system mismatch of the three coefficient vectors for the LTE. As an optimum coefficient vector, we deploy an infinite impulse response (IIR). This is because performance of the LTE with the IIR

Table 1 Coefficient vectors for the LTE where C_{IIR}, C''_{D_c}, C''_{D_c-1}, and C''_{D_c+1} denote the IIR and FIR with a delay, D_c, D_c-1, D_c+1, respectively.

C_{IIR}	$c_{-\infty}$	$\cdots$	c_{-2}	c_{-1}	c_0	c_1	$\cdots$	c_{M-2}	c_{M-1}	c_M	c_{M+1}	$\cdots$	$c_{+\infty}$
C''_{D_c}	0	$\cdots$	0	0	c_0	c_1	$\cdots$	c_{M-2}	c_{M-1}	0	0	$\cdots$	0
C''_{D_c-1}	0	$\cdots$	0	0	0	c_1	$\cdots$	c_{M-2}	c_{M-1}	c_M	0	$\cdots$	0
C''_{D_c+1}	0	$\cdots$	0	c_{-1}	c_0	c_1	$\cdots$	c_{M-2}	0	0	0	$\cdots$	0

is generally better than that with a finite impulse response (FIR) when we tackle an inverse modeling problem like channel equalization. Table 1 gives the IIR and the FIRs where C_{IIR} denotes the IIR, the elements of which are also calculated with (25). For the comparison, in the FIRs, C'_{D_c}, C'_{D_c-1}, and C'_{D_c+1}, zero padding is made as C''_{D_c}, C''_{D_c-1}, and C''_{D_c+1}, which are also shown in Table 1. As described above, the comparison is made with system mismatch defined by

$$V = E[(C_{opt} - C)^T (C_{opt} - C)] \tag{37}$$

where C_{opt} denotes the optimum coefficient vector, $E[\cdot]$ is an expectation operation, and C_{IIR} is used as C_{opt}. We here focus on $E[C_{IIR}] = C_{IIR}$ and $E[C] = C$, which we can see in (25). Instead of (37), therefore, we actually use

$$V = (C_{IIR} - C)^T (C_{IIR} - C). \tag{38}$$

Then, the system mismatches are obtained as

$$\begin{aligned} V_{D_c} &= (C_{IIR} - C''_{D_c})^T (C_{IIR} - C''_{D_c}) \\ &= \sum_{i=-\infty}^{-2} c_i^2 + c_{-1}^2 + c_M^2 + \sum_{i=M+1}^{+\infty} c_i^2, \end{aligned} \tag{39}$$

$$\begin{aligned} V_{D_c-1} &= (C_{IIR} - C''_{D_c-1})^T (C_{IIR} - C''_{D_c-1}) \\ &= \sum_{i=-\infty}^{-2} c_i^2 + c_{-1}^2 + c_0^2 + \sum_{i=M+1}^{+\infty} c_i^2, \end{aligned} \tag{40}$$

$$\begin{aligned} V_{D_c+1} &= (C_{IIR} - C''_{D_c+1})^T (C_{IIR} - C''_{D_c+1}) \\ &= \sum_{i=-\infty}^{-2} c_i^2 + c_{M-1}^2 + c_M^2 + \sum_{i=M+1}^{+\infty} c_i^2. \end{aligned} \tag{41}$$

where V_{D_c}, V_{D_c-1}, and V_{D_c+1} are the system mismatches of C''_{D_c}, C''_{D_c-1}, and C''_{D_c+1}, respectively.

Along the strategy of (30), we obtain $V_{D_c-1} - V_{D_c}$ and $V_{D_c+1} - V_{D_c}$ as

$$V_{D_c-1} - V_{D_c} = c_0^2 - c_M^2, \tag{42}$$

and

$$V_{D_c+1} - V_{D_c} = c_{M-1}^2 - c_{-1}^2, \tag{43}$$

respectively. Equations (42) and (43) are represented as

$$V_{D_c-1} - V_{D_c} = c_0^2 - \alpha_1^2 c_{M-1}^2 \tag{44}$$

and

$$V_{D_c+1} - V_{D_c} = c_{M-1}^2 - \alpha_2^2 c_0^2, \tag{45}$$

respectively, where $|c_M| = \alpha_1 |c_{M-1}|$ and $|c_{-1}| = \alpha_2 |c_0|$ (α_1 and α_2 are positive real numbers). If (29) is satisfied, both

$$V_{D_c-1} - V_{D_c} \approx c_0^2 (1 - \alpha_1^2) > 0 \tag{46}$$

and

$$V_{D_c+1} - V_{D_c} \approx c_0^2 (1 - \alpha_2^2) > 0 \tag{47}$$

are satisfied because both

$$0 < \alpha_1 < 1 \tag{48}$$

and

$$0 < \alpha_2 < 1 \tag{49}$$

are roughly satisfied under the following constraint:

$$\begin{cases} |c_{D_c}| > |c_{D_c-1}| > \cdots > |c_1| > |c_0| \\ |c_{D_c}| > |c_{D_c+1}| > \cdots > |c_{M-2}| > |c_{M-1}|. \end{cases} \tag{50}$$

Thus, when (50) is satisfied, both (46) and (47) are satisfied. Therefore, we insist that when the optimum delay is set, the difference between $|c_0|$ and $|c_{M-1}|$ is minimized, the property of which may be used as the cost function for delay adaptation.

In the above comparison of system mismatches with (46) and (47), we consider only (29) as the cost function. Thus, we will further investigate a general form of (29) expressed as

$$|c_0|^\gamma \approx |c_{M-1}|^\gamma \tag{51}$$

where γ is a positive integer and is not equal to 1. We will compare, in particular, (29) and

$$|c_0|^2 \approx |c_{M-1}|^2, \tag{52}$$

for use as the cost function. This is because (46) and (47) consist of square of the equalizer coefficients.

Equations (29) and (52) mean minimization of

$$\big||c_0| - |c_{M-1}|\big| \tag{53}$$

and

$$\left||c_0|^2 - |c_{M-1}|^2\right| = \big||c_0| + |c_{M-1}|\big| \times \big||c_0| - |c_{M-1}|\big|, \tag{54}$$

respectively. Thus, the distinction between (53) and (54) is

$$|c_0| + |c_{M-1}|. \tag{55}$$

Here, let us consider a situation as

$$|c_{M-1}| - |c_M| = |c_1| - |c_0|. \tag{56}$$

Equation (56) may be satisfied because of (50). Then, (56) is represented as

$$|c_0| + |c_{M-1}| = |c_1| + |c_M|. \tag{57}$$

In this case, an addition of absolute values of first and last coefficients for C_{D_c} (i.e. $|c_0| + |c_{M-1}|$) is equivalent to that for C_{D_c-1} (i.e. $|c_1| + |c_M|$). Equation (54) is, in the case of $D_c - 1$, presented as

$$\left||c_1|^2 - |c_M|^2\right| = ||c_1| + |c_M|| \times ||c_1| - |c_M||\,. \tag{58}$$

It means that the first term of the right-hand side in (54) is equivalent to that of (58). Hence, (55) is not always desired to be minimized. Thus, we insist that minimization of (53) is suitable for delay adaptation. Also, from the point of computational complexity, minimization of (53) is favored. We therefore consider the use of (29), that is (53), as the cost function. Its evaluation by comparing numerical and simulation examples will be given in the following subsection.

By the way, to derive the cost function, we have utilized (20) under the constraint of $h_0 \neq 0$. In the case of dispersive channels like the raised cosine channel [18], however, $h_0 = 0$. In this case, we focus that, with $H(z)$, the transfer function of a dispersive channel can be expressed as

$$H_{dis}(z) = H(z)z^{-1}. \tag{59}$$

The essential difference between $H(z)$ and $H_{dis}(z)$ is the number of the zeros. The zeros of $H(z)$ are $r_k (k = 1, \ldots, L-1)$. On the other hand, the zeros of $H_{dis}(z)$ are $r_k (k = 1, \ldots, L-1)$ and 0. From this point of view, $1/H_{dis}(z)$ with D_c is equivalent to $1/H(z)$ with $D_c + 1$ because of (25). Hence, we can also deploy the cost function for the delay adaptation method on dispersive channels.

3.2 Evaluation of the Cost Function

In this subsection, we evaluate the derived cost function by comparing numerical and simulation results. Figure 3 depicts the dependency on the delay for the ABC equalizer on a nonminimum phase channel:

$$H_1(z) = 1 + 2.2z^{-1} + 0.4z^{-2} \tag{60}$$

where $M = 11$ and the SNR is 100 dB. It is confirmed that $D = 8$ provides the lowest MSE level. From this point of view, in this situation, $D = 8$ is the optimum delay for the LTE.

Table 2 gives equalizer coefficient vectors with three delay settings ($D = 7$, $D = 8$, and $D = 9$) on $H_1(z)$ in the case of SNR of 100 dB. The equalizer coefficients are calculated by the normal equation(ideal case of (14)):

$$C = A^{-1}B. \tag{61}$$

The elements of A and B are calculated by (15) and (16), respectively. Then, to derive A, $\sigma^2 = 6\times10^{-10}$ is considered, because of $E[s_n^2] = 6$. With the derived cost function:

$$\xi = ||c_0| - |c_{M-1}||\,, \tag{62}$$

the following cost functions are also drawn.

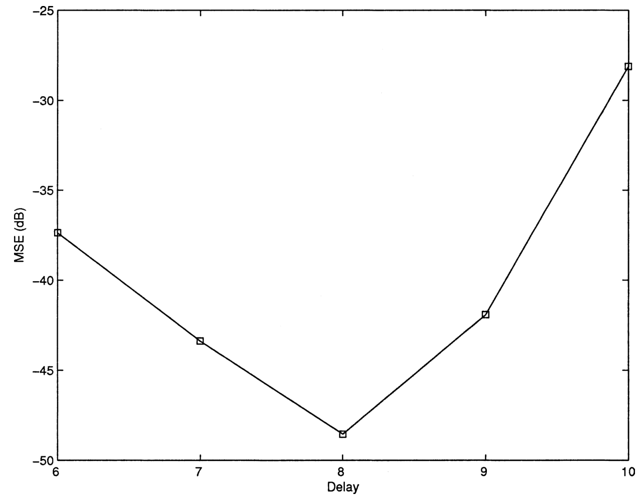

Fig. 3 Dependency on the delay for the ABC equalizer on $H_1(z)$.

Table 2 Relation between coefficients and delay for the ABC equalizer where $D = 7$, $D = 8$, and $D = 9$ mean that the fixed delay, D, is 7, 8, and 9, respectively. All the coefficients are calculated with (61).

	$D = 7$	$D = 8$	$D = 9$
c_0	0.0059	−0.0029	0.0015
c_1	−0.0158	0.0079	−0.0039
c_2	0.0339	−0.0170	0.0085
c_3	−0.0690	0.0345	−0.0172
c_4	0.1387	−0.0693	0.0346
c_5	−0.2777	0.1388	−0.0693
c_6	0.5555	−0.2777	0.1387
c_7	−0.1111	0.5555	−0.2773
c_8	0.0222	−0.1109	0.5547
c_9	−0.0044	0.0219	−0.1094
c_{10}	0.0008	−0.0038	0.0192
$\|\|c_0\| - \|c_{10}\|\|$	5.1×10^{-3}	0.9×10^{-3}	1.77×10^{-2}
$\|c_0\| + \|c_{10}\|$	6.7×10^{-3}	6.7×10^{-3}	2.07×10^{-2}
$(\|c_0\| - \|c_{10}\|)^2$	2.60×10^{-5}	8.1×10^{-7}	3.133×10^{-4}
$\left\|\|c_0\|^2 - \|c_{10}\|^2\right\|$	3.42×10^{-5}	6.03×10^{-6}	3.664×10^{-4}

$$\xi^2 = (|c_0| - |c_{M-1}|)^2 \tag{63}$$

$$\xi' = \left||c_0|^2 - |c_{M-1}|^2\right| \tag{64}$$

$$\xi'' = |c_0| + |c_{M-1}| \tag{65}$$

From Table 2, we can see that the important relation between the equalizer coefficients given by (50) is satisfied. Also, we can confirm that ξ, ξ^2, and ξ' with $D = 8$ are smaller values than those with $D = 7$ or $D = 9$, respectively. On the contrary, ξ'' with $D = 8$ is equivalent to that with $D = 7$. As mentioned above, these examples demonstrate that ξ'' is not always desired to be minimized and it suggests that ξ' is not also favored for use as the cost function, because of

$$\xi' = \xi \times \xi''. \tag{66}$$

We will further evaluate the derived cost function. To derive the cost function, we utilize the relationships that c_i with D_c is equivalent to c_{i-1} with $D_c - 1$ and that c_i with D_c is equivalent to c_{i+1} with $D_c + 1$. However, the relationships are not strictly satisfied as shown in Table 2. We can see that c_0 with $D = 7$ is 0.0059 and c_1 with $D = 8$ is 0.0079. Hence, we can insist that satisfaction of the relationships is not necessarily strict. As a conclusion, the derived cost function ξ in (62) is suitable under the following constraints.

1. Equation (50) is satisfied as strictly as possible.
2. Difference between c_i with D_c and c_{i-1} with $D_c - 1$ is small.
3. Difference between c_i with D_c and c_{i+1} with $D_c + 1$ is small.

The detail of the proposed adaptation scheme based on the cost function will be described in the following subsection.

3.3 Adaptation Procedure

Based on the derived cost function, involved with the ABC equalization scheme, the proposed delay adaptation is implemented at the n-th iteration as follows.

1. We calculate $|c_0(n)|$ and $|c_{M-1}(n)|$ by solving (14) with the Levinson-Trench algorithm [11].
2. The average of the delays is calculated as
$$D_{av}(n) = \frac{1}{Q}\sum_{q=0}^{Q-1} D(n-q) \tag{67}$$
where $D(n)$ denotes the estimated delay at the n-th iteration and Q is a positive even number.
3. The absolute value of the difference between $D(n)$ and $D_{av}(n)$ is calculated as
$$\zeta(n) = |D(n) - D_{av}(n)|. \tag{68}$$
4. If all the elements of $\zeta(n-q)$ for $q = 0, \ldots, Q-1$ are not equal to 0.5, go to Step 7. Otherwise, go to the next step.
5. Instead of ξ,
$$\hat{\xi}(n) = ||c_0(n)| - |c_{M-1}(n)|| \tag{69}$$
is calculated.
6. If $\hat{\xi}(n) > \hat{\xi}(n-1)$,
$$D(n+F) = D(n-1). \tag{70}$$
Otherwise,
$$D(n+F) = D(n) \tag{71}$$
where $F = 1, 2, \ldots, \infty$. Go to Step 8.
7. If $|c_0(n)| > |c_{M-1}(n)|$,
$$D(n+1) = D(n) + 1. \tag{72}$$
Otherwise,
$$D(n+1) = D(n) - 1. \tag{73}$$
8. The iteration number n is set to $n+1$ and return to Step 1.

Note that $W(n)$, $C(n)$, and $D(n)$ are initialized as $W(0) = \mathbf{0}$, $C(0) = \mathbf{0}$, and $D(0) = M/2$, respectively, before the adaptation.

Let us discuss the behavior of the delay adaptation scheme. First, let us consider the use of only Steps 1, 7, and 8 and their behavior. Step 1 calculates $|c_0(n)|$ and $|c_{M-1}(n)|$ to be used in the cost function. The purpose of Step 7 is given as follows. If $|c_0(n)| > |c_{M-1}(n)|$, equalization performance of the minimum phase part is better than that of the maximum phase part. Then, we must add $D(n)$ and a positive integer so that equalization performance of the minimum phase part become close to that of the maximum phase part. Here, we use 1 as the positive integer. Hence, the operation given by (72) is implemented. Similarly, if $|c_0(n)| \le |c_{M-1}(n)|$, we must add $D(n)$ and a negative integer (-1 is used) as shown by (73). Therefore, with only the three steps, it is expected that the convergence of the ABC equalizer leads to either

$$\begin{cases} D(n+F) = D_{opt} \\ D(n+F+1) = D_{opt} + 1 \end{cases} \tag{74}$$

or

$$\begin{cases} D(n+F) = D_{opt} \\ D(n+F+1) = D_{opt} - 1. \end{cases} \tag{75}$$

Thus, D_{opt} is either $D(n+F)$ or $D(n+F+1)$. With only the three steps, however, we do not know which is D_{opt}. Thus, the remaining steps are necessary.

Next, Steps 2, 3, and 4 are discussed. These three steps are implemented to check that either (74) or (75) is satisfied. When (74) is satisfied, $D_{av}(n)$ becomes

$$\begin{aligned} D_{av}(n) &= \frac{1}{Q}\left(\frac{Q}{2}(D_{opt}+1) + \frac{Q}{2}D_{opt}\right) \\ &= D_{opt} + \frac{1}{2}. \end{aligned} \tag{76}$$

Similarly, when (75) is satisfied, $D_{av}(n)$ becomes

$$D_{av}(n) = \frac{1}{Q}\left(\frac{Q}{2}(D_{opt}-1) + \frac{Q}{2}D_{opt}\right) = D_{opt} - \frac{1}{2}. \quad (77)$$

From the results, when either (74) or (75) is satisfied, we have $\zeta(n) = 0.5$. Hence, when all the elements of $\zeta(n-q)$ for $q = 0, \ldots, Q-1$ are equal to 0.5, either (74) or (75) is satisfied. To accomplish the above operation, Q must be a large even number. This is because $Q/2$ must be an integer and neither (74) nor (75) is satisfied with small Q.

Steps 5 and 6 are finally discussed. The ξ in Step 2 is the cost function to select either $D(n+F)$ or $D(n+F+1)$ as D_{opt}. If $\hat{\xi}(n) > \hat{\xi}(n-1)$, $D(n-1)$ is selected as D_{opt}. Otherwise, $D(n)$ is selected. Equations (70) and (71) mean that the delay is fixed at the $n+F$-th iteration. Therefore, Step 2 calculates the cost function and Step 5 fixes the delay, which is also used in the tracking mode.

3.4 Discussions

In Sect. 3.1, we have investigated the delay adjustment problem in noiseless cases. In this subsection, we discuss the use of the derived cost function and that based delay adaptation technique for linear equalization of nonminimum phase channels in white noise environments.

We consider that the delay adjustment problem in a noiseless case is essentially equivalent to that in a white noise case. In this case, (30) is satisfied even in white noise case. Hence, we discuss to take the same approach to the delay adjustment problem for the LTE on nonminimum phase channels in white noise as that in noiseless. In particular, we focus on the satisfaction of (50) in white noise.

As shown by (15) and (16), the elements of B are given by

$$b_i = \sum_{j=0}^{M-1} a_{ij}c_j \qquad i = 0, \ldots, M-1. \quad (78)$$

From this equation, magnitude of $|c_i|$ for $i = 0, \ldots, M-1$ becomes smaller as the SNR decreases. Here, we consider an effect with decrease of the SNR on $|c_{D_c}|$ is larger than that on $|c_i|$ for $i = 0, \ldots, D_c-1, D_c+1, \ldots, M-1$. This means that magnitude of $|c_{D_c-k}|$ for $k = D_c, \ldots, 1, -1, \ldots, D_c-M+1$ becomes smaller as $|k|$ increases in low SNR cases. This is because decrease of the SNR effects only the diagonal elements of A. Therefore, difference between $|c_i|$ and $|c_{i-1}|$ becomes smaller as the SNR decreases. However, we consider that the relationship of (50) is still satisfied in noisy cases, because $|c_{D_c-k}|$ with large $|k|$ becomes close to 0 in low SNR environments.

They are confirmed with numerical results. Table 3 shows the coefficient vectors with $D = 8$ for the LTE on $H_1(z)$ where the SNRs are 100 dB, 20 dB, 10 dB, and 0 dB, respectively. The coefficient vectors are calculated by (61), which is the same as in Table 2. From this table, we can observe that the important relation between the equalizer coefficients given by (50) is satisfied in both noiseless and noisy cases. Thus, we can insist that the derived adaptation scheme is suitable in both the cases. Performance of the ABC equalizer with the derived adaptation scheme is evaluated in Sect. 4.

Table 3 Effect of the SNR on the equalizer coefficients for the conventional ABC equalizer on $H_1(z)$ where $D = 8$. All the coefficients are calculated with (61).

SNR(dB)	100	20	10	0
c_0	−0.0029	−0.0023	−0.0002	0
c_1	0.0079	0.0062	0.0008	0
c_2	−0.0170	−0.0138	−0.0025	0
c_3	0.0345	0.0290	0.0072	−0.0002
c_4	−0.0693	−0.0603	−0.0205	0.0012
c_5	0.1388	0.1250	0.0575	−0.0029
c_6	−0.2777	−0.2589	−0.1578	−0.0126
c_7	0.5555	0.5347	0.4165	0.1762
c_8	−0.1109	−0.0969	−0.0255	0.0430
c_9	0.0219	0.0145	−0.0180	−0.0177
c_{10}	−0.0038	−0.0009	0.0099	0.0031

3.5 Computational Complexity

As shown in Sect. 3.3, the proposed delay adaptation method can be implemented with the simple procedure, which is combined with the conventional ABC equalizer [9]. In Step 2, the proposed scheme requires the average operation of (67), which is reexpressed as

$$D_{av}(n) = D_{av}(n-1) + \frac{D(n) - D(n-Q)}{Q}. \quad (79)$$

With the above technique, the proposed delay adaptation scheme approximately requires five additions, one division, and four absolute value operations for each iteration. A point to be focused is that the additional complexity depends on neither the length of the channel estimator, L, nor that of the equalization filter, M.

4. Simulations

4.1 Conditions

Simulation experiments are carried out to evaluate the performance of the proposed adaptive scheme. We use two nonminimum phase channels whose transfer functions are given by (60) and

$$H_2(z) = 0.06 - 0.07z^{-1} + 0.1z^{-2} - 0.5z^{-3} - 0.9z^{-4} + 1.0z^{-5} + 0.3z^{-6} + 0.2z^{-8} + 0.05z^{-9} + 0.1z^{-10} \quad (80)$$

where the conventional and proposed ABC equalizers are compared. The two ABC equalizers commonly use $\mu = 0.01$ and $P = 20$. The length of channel estimator for the ABC equalizers is $L = 3$ for $H_1(z)$ and $L = 11$ for $H_2(z)$, which are equivalent to the channel length, respectively. The proposed ABC equalizer uses $Q = 100$ for use in Step 2 described in Sect. 3.3. The MSE performance is evaluated by averaging 100 individual trials and the BER performance is evaluated by averaging 10 individual trials.

4.2 Performance in High SNR Environments

Figure 4 illustrates the convergence of the conventional and proposed ABC equalizers on $H_1(z)$ where $M = 11$ and the SNR is 100 dB (this is considered as a noiseless environment). From this figure, it is confirmed that the proposed ABC equalizer provides an improvement of about 18 dB, compared with the conventional one with $D = 5$ being half of equalizer length. We can also see that the convergence of the proposed ABC equalizer is very similar with that of the conventional one with $D = 8$ being optimum in the sense of providing the lowest MSE level. The delay behavior of the proposed scheme is depicted in Fig. 5. The convergence of the proposed ABC equalizer leads to (75) as shown around the range of 350–450 iterations. Furthermore, it is observed that the final decision of the delay using (69) is performed around 450 iterations.

Figure 6 depicts the convergence of the ABC equalizers on $H_2(z)$ where $M = 16$ and the SNR is 100 dB. We can see that the convergence of the proposed ABC equalizer is similar with that of the conventional one with $D_{opt} = 16$. Figure 7 illustrates the delay behavior of the proposed ABC equalizer in the case of Fig. 6. We can confirm that the proposed adaptation scheme succeeds in estimating the optimum delay on $H_2(z)$ which has multiple zeros whose absolute values are greater than 1 and multiple zeros whose absolute values are less than 1. Also, we can observe that the

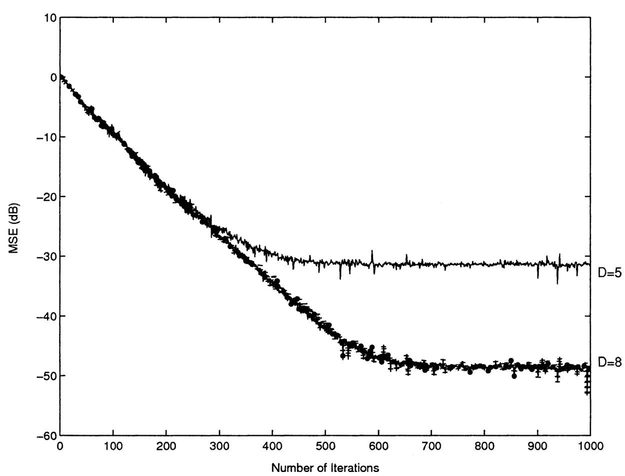

Fig. 4 Convergence of the two ABC equalizers on $H_1(z)$ where $M = 11$ and the SNR is 100 dB. The solid and dotted lines correspond to the conventional and proposed ABC equalizers, respectively.

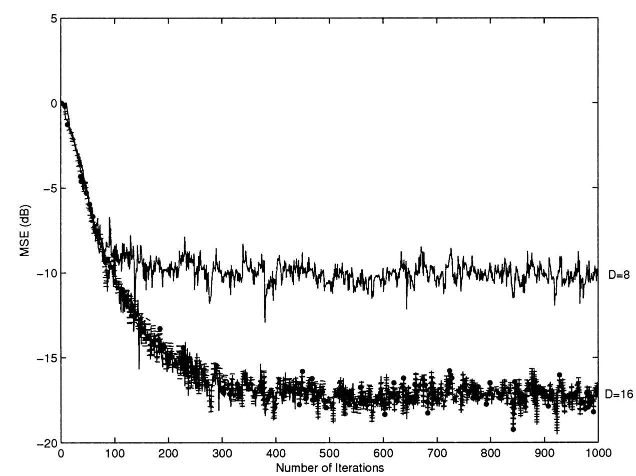

Fig. 6 Convergence of the two ABC equalizers on $H_2(z)$ where $M = 16$ and the SNR is 100 dB.

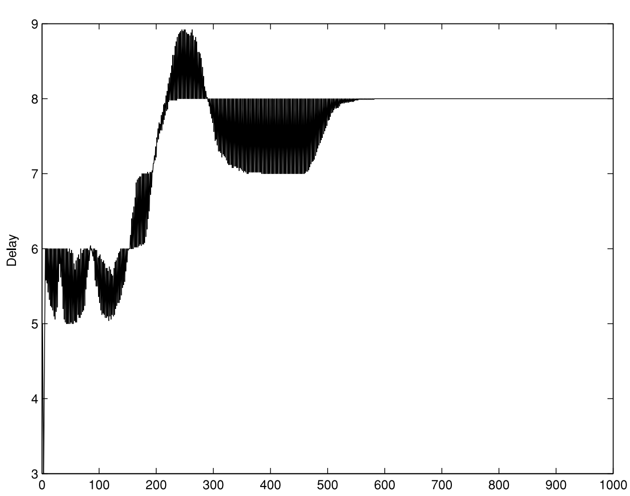

Fig. 5 Delay behavior of the proposed ABC equalizer in the case of Fig. 4.

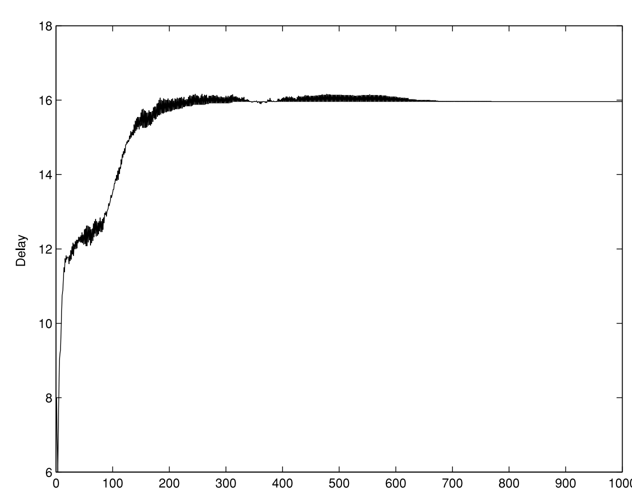

Fig. 7 Delay behavior of the proposed ABC equalizer in the case of Fig. 6.

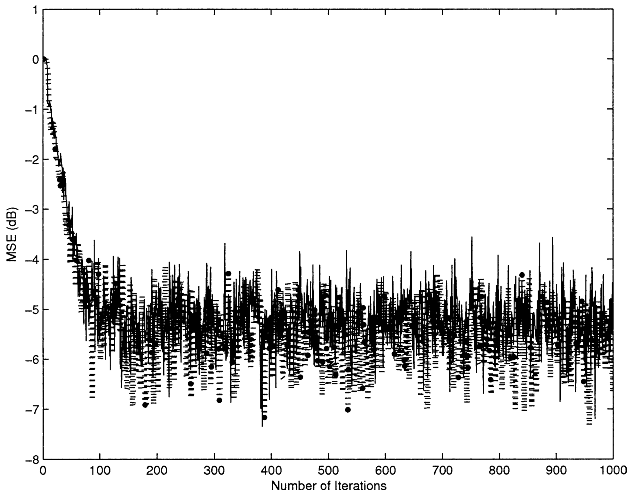

Fig. 8 Convergence of the two ABC equalizers on $H_2(z)$ where $M = 16$ and the SNR is 5 dB. The fixed delay for the conventional ABC equalizer is set to $D = 8$, which is $M/2$.

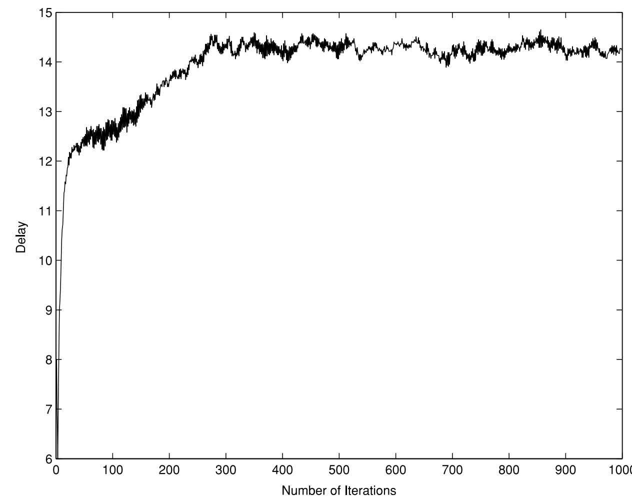

Fig. 9 Delay behavior of the proposed ABC equalizer in the case of Fig. 8.

behavior of the delay becomes close to (74) as the proposed ABC equalizer converges and (69) is implemented around 400 iterations. Furthermore, through Figs. 4–7, we can see that the speed of convergence of the delay is approximately equivalent to that of the MSE. Therefore, the proposed ABC equalizer accomplishes both fast start-up and excellent accuracy with a small increase of computation.

4.3 Performance in Low SNR Environments

In the previous subsection, the performance of the proposed ABC equalizer in noiseless environments has been investigated. In this subsection, further simulations are carried out to investigate the performance of the proposed ABC equalizer in white noise environments.

Figure 8 compares the MSE performance of the two ABC equalizers on $H_2(z)$ where $M = 16$, the SNR is 5 dB, and the conventional ABC equalizer uses $D = M/2 = 8$ being half of the equalizer length. From this result, we can see that the proposed ABC equalizer provides an improvement, compared with the conventional one with $D = M/2 = 8$ Behavior of adapted delay of the proposed ABC equalizer in the case of Fig. 10 is shown in Fig. 9. Similarly in noiseless cases, the convergence speed of the adapted delay of the proposed ABC equalizer is roughly the same with that of the MSE. Comparison of the conventional ABC equalizer with the optimum delay and the proposed ABC equalizer is depicted in Fig. 10. We can observe that the convergence of the proposed ABC equalizer is similar with that of the conventional one with $D_{opt} = 14$.

As shown in Fig. 9, however, the average of the adapted delays is not always an integer in noisy cases. This means that the additive noise corrupts both the equalizer coefficients and the delay for the proposed ABC equalizer. For the tracking mode of the proposed adaptation scheme in noisy cases, the delay is recommended to be fixed. To solve this problem, we use an integer close to the average of the

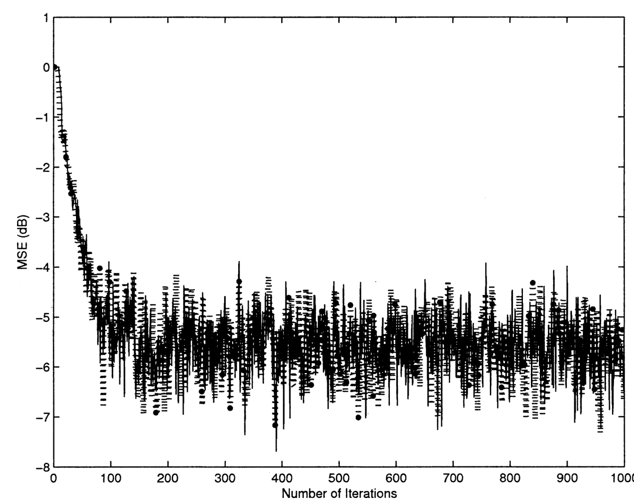

Fig. 10 Convergence of the two ABC equalizers on $H_2(z)$ where $M = 16$ and the SNR is 5 dB. The optimum delay, $D_{opt} = 14$, is used for the conventional ABC equalizer.

adapted delays $D_{av}(n)$. In the case of Fig. 9, $D = 14$ is a closest integer, which we deploy in the tracking mode. In noisy environments, we use $D_{av}(n)$ for the proposed ABC equalizer in the tracking mode.

It is desired to investigate a bit-error-rate (BER) performance in addition to the MSE performance. Figure 11 illustrates the BER performance on $H_2(z)$ where both the ABC equalizers commonly use $M = 16$. Here, $D = M/2 = 8$ is used for the conventional ABC equalizer. From this figure, it is shown that the proposed ABC equalizer provides better performance than the conventional one with $D = 8$. We can also observe that a degree of performance improvement becomes larger as the SNR becomes higher. This is because the effect of decision of the delay for the LTE on nonminimum phase channels becomes larger as the SNR increases.

Figure 12 shows the BER performance of the two ABC equalizers against the equalizer length on $H_2(z)$ where the

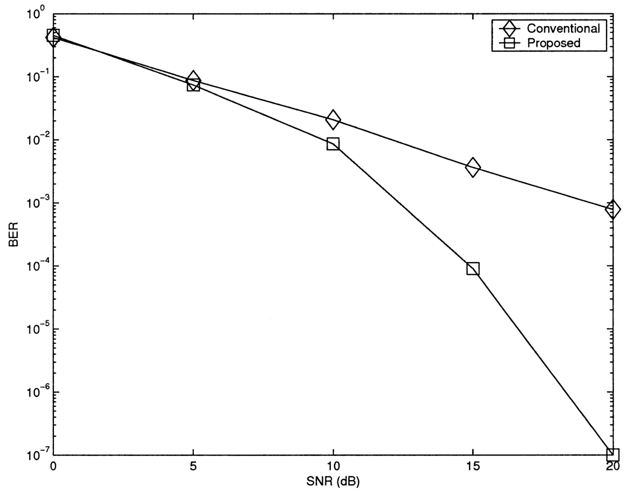

Fig. 11 BER performance of the two ABC equalizers on $H_2(z)$ against the SNR where $M = 16$.

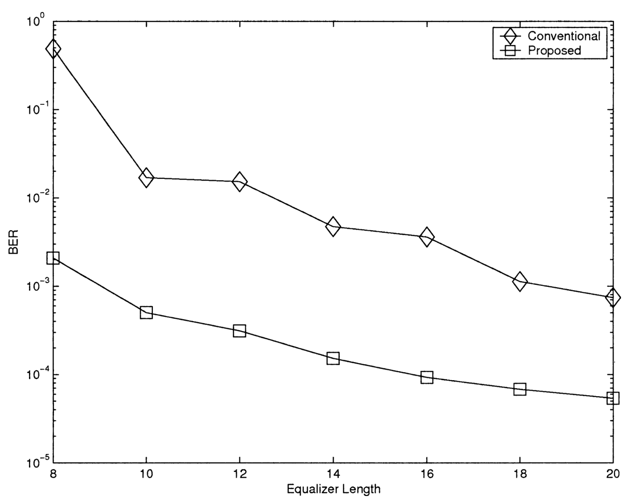

Fig. 12 BER performance of the two ABC equalizers on $H_2(z)$ against the equalizer length where the SNR is 15 dB.

SNR is 15 dB. We can see that a degree of performance improvement provided by the proposed delay adaptation scheme becomes larger as the equalizer length M becomes smaller. This is because the absolute values of first and last coefficients for the equalization filter become larger as M becomes a smaller positive integer. This fact results in worse error performance of the conventional ABC equalizer as discussed in Sect. 3.1. The proposed scheme is, however, more insensitive to the equalizer length than the conventional one as shown in this figure. Hence, we can insist that the proposed delay adaptation scheme works well in both noiseless and noisy environments.

5. Concluding Remarks

The problem of delay adaptation for the LTE on nonminimum phase channels has been investigated. We have focused that the filter structure and adaptation procedure of the ABC equalizer are very suitable to change the delay at each sampling time. Also, we have derived the novel cost function, which is square of difference of absolute values of the first and last elements of the equalizer coefficient vector. We have evaluated the cost function by comparing numerical and simulation examples. As a result, it has been shown that the cost function is suitable for delay adaptation under the constraints described in Sect. 3.2. Based on the cost function, we have developed the delay adaptation method, which is involved with the ABC equalization scheme. Computer simulations have demonstrated that the proposed ABC equalizer succeeds in estimating the optimum delay and provides lower MSE level than the other LTEs with a delay being set to half of the equalizer length. Furthermore, we have shown that the optimum delay for the LTE on nonminimum phase channels depends on

1. channel impulse response
2. equalizer length
3. SNR

and the proposed scheme succeeds in estimating the optimum delay. The proposed ABC equalizer will be reinforced by using more excellent channel estimators than the LMS estimator.

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