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# Perturbations and Supersymmetries in $\text{AdS}_3/\text{CFT}_2$

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## Abstract

Symmetry breaking by perturbations in the  $\text{AdS}/\text{CFT}$  correspondence is discussed. Perturbations of vector fields to the  $\text{AdS}_3 \times \text{S}^3$  solution of the six-dimensional  $\mathcal{N} = (4, 4)$  supergravity are considered. These perturbations are identified as descendants of chiral primary operators of a two-dimensional  $\mathcal{N} = (4, 4)$  CFT with conformal weight  $(2, 2)$  or  $(1, 1)$ . We examine unbroken symmetries by the perturbations in the CFT side as well as in the supergravity side and find the same result: the  $\mathcal{N} = (4, 2)$  or  $\mathcal{N} = (2, 4)$  Poincaré supersymmetry for the  $(2, 2)$  perturbation and the  $\mathcal{N} = (0, 4)$  or  $\mathcal{N} = (4, 0)$  superconformal symmetry for the  $(1, 1)$  perturbation.

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## 1. Introduction

In the original context of the AdS/CFT correspondence [1, 2, 3] string theories or supergravities in the AdS space describe field theories on the boundary with the conformal symmetry and a large extended supersymmetry. (For a review, see ref. [4].) To apply it to more realistic models one has to consider theories with lower supersymmetries. One should understand supersymmetry breaking in both of the supergravity side and the field theory side.

One of the approaches to obtain the AdS/CFT correspondence for lower supersymmetric cases is to modify AdS solutions of supergravities by adding a perturbation. In ref. [5], for instance, a perturbation of the three-form flux was added to the  $\text{AdS}_5 \times \text{S}^5$  solution, which breaks  $\mathcal{N} = 4$  supersymmetry to  $\mathcal{N} = 1$ . The perturbation is a solution of the linearized field equation around the  $\text{AdS}_5 \times \text{S}^5$  background. This perturbation corresponds to fermion mass terms of the three  $\mathcal{N} = 1$  chiral multiplets in the  $\mathcal{N} = 4$  super Yang-Mills theory and polarizes D3-branes into 5-branes [6, 7]. It is easy to see how these mass terms break the supersymmetry in the field theory side. Furthermore, supersymmetry breaking by the perturbation was also studied in the supergravity side [8, 9] by examining supertransformations of the fermionic fields. The results of supersymmetry breaking are consistent in the field theory side and in the supergravity side.

A similar supersymmetry breaking by a perturbation was discussed for a two-dimensional CFT and its dual supergravity solution. In ref. [10] solutions of the linearized field equations of vector fields around the  $\text{AdS}_3 \times \text{S}^3$  solution of the six-dimensional  $\mathcal{N} = (4, 4)$  supergravity were obtained. This six-dimensional supergravity is an effective theory of the type IIB superstring compactified on  $\text{T}^4$  with the size of  $\text{T}^4$  much smaller than those of  $\text{AdS}_3$  and  $\text{S}^3$ . By adding these solutions of vector fields as a perturbation the  $\mathcal{N} = (4, 4)$  superconformal symmetry of the two-dimensional dual field theory is broken. A preliminary analysis in the supergravity side showed that there are cases in which it is broken to  $\mathcal{N} = (4, 0)$ . In contrast to the above  $\text{AdS}_5 \times \text{S}^5$  case the physical meaning of the perturbations in the field theory side is not clear in this case. By this reason supersymmetry breaking in the field theory side was not studied in ref. [10].

The purpose of the present paper is to study supersymmetry breaking by the perturbations in this model in more detail. We first identify operators of the two-dimensional CFT corresponding to the perturbations of the supergravity solution.

The relation between operators of the CFT and linearized solutions of the supergravity was studied in refs. [11, 12, 13, 14]. We use these results to find that the perturbations correspond to certain descendents of chiral primary operators of the  $\mathcal{N} = (4, 4)$  superconformal field theory, which have conformal weight  $(h, \bar{h}) = (2, 2)$  or  $(1, 1)$ . We then examine breaking of supersymmetry as well as of bosonic symmetries by perturbations of these operators in the CFT side. We find that the unbroken symmetry is the  $\mathcal{N} = (4, 2)$  or  $\mathcal{N} = (2, 4)$  Poincaré supersymmetry for the  $(2, 2)$  perturbation, while it is the  $\mathcal{N} = (0, 4)$  or  $\mathcal{N} = (4, 0)$  superconformal symmetry for the  $(1, 1)$  perturbation. Finally, we examine symmetry breaking in the supergravity side by studying supertransformations of the fermionic fields. The unbroken symmetries are in complete agreement with those in the CFT side. The result in this paper may be regarded as another non-trivial evidence in support of the AdS/CFT correspondence.

In the next section we review the  $\text{AdS}_3 \times \text{S}^3$  solution of the six-dimensional  $\mathcal{N} = (4, 4)$  supergravity and its symmetries. In sect. 3 we give the perturbations around this solution obtained in ref. [10]. In sect. 4 we first identify operators in the CFT corresponding to these perturbations. Then, we examine unbroken symmetries by the perturbations in the CFT side. In sect. 5 we examine unbroken symmetries in the supergravity side and show that they precisely coincide with those in the CFT side. In Appendix we give our conventions of  $\text{SO}(4)$  and  $\text{SO}(5)$  gamma matrices used in the text.

## 2. $\text{AdS}_3 \times \text{S}^3$ background

We first recall the  $\text{AdS}_3 \times \text{S}^3$  solution of the six-dimensional supergravity. The six-dimensional  $\mathcal{N} = (4, 4)$  supergravity [15, 16] has a rigid  $\text{SO}(5, 5)$  symmetry and a local  $\text{SO}(5) \times \text{SO}(5)$  symmetry. The field content of the theory is a vielbein  $e_M^{\hat{M}}$ , five antisymmetric tensor fields  $B_{MN}^m$ , 16 vector fields  $A_M^{\tilde{\mu}\dot{\mu}}$ , 25 scalar fields  $\phi_{\tilde{\mu}\dot{\mu}}^{\alpha\dot{\alpha}}$ , eight Rarita-Schwinger fields  $\psi_{+M\alpha}$ ,  $\psi_{-M\dot{\alpha}}$  and 40 spinor fields  $\chi_{+a\dot{\alpha}}$ ,  $\chi_{-\dot{a}\alpha}$ . The indices  $M, N, \dots$  and  $\hat{M}, \hat{N}, \dots$  are six-dimensional world and local Lorentz indices, respectively. Other indices take values  $m, a, \dot{a} = 1, \dots, 5$  and  $\tilde{\mu}, \dot{\mu}, \alpha, \dot{\alpha} = 1, \dots, 4$ . A pair of indices  $\tilde{\mu}\dot{\mu}$  represent a spinor index of  $\text{SO}(5, 5)$ , while  $a, \dot{a}$  and  $\alpha, \dot{\alpha}$  represent vector and spinor indices of  $\text{SO}(5) \times \text{SO}(5)$ , respectively. The field strengths of the antisymmetric tensor fields and their duals belong to **10** of  $\text{SO}(5, 5)$ . The fermionic

fields are SO(5)-symplectic Majorana-Weyl spinors and the signs on the fields denote the chiralities. The scalar fields take values on the coset space SO(5,5)/(SO(5) × SO(5)).

We are only interested in the fields  $e_M^{\hat{M}}$ ,  $B_{MN}^m$  and  $A_M^{\tilde{\mu}\dot{\mu}}$  and set other fields to zero except  $\phi_{\tilde{\mu}\dot{\mu}}^{\alpha\dot{\alpha}} = \delta_{\tilde{\mu}}^{\alpha}\delta_{\dot{\mu}}^{\dot{\alpha}}$ . By this scalar field background the rigid SO(5,5) and local SO(5) × SO(5) symmetries are broken to a rigid SO(5) × SO(5) symmetry, and the indices  $\tilde{\mu}$  and  $\alpha$ ,  $\dot{\mu}$  and  $\dot{\alpha}$  are identified, respectively. The local supertransformation of the fermionic fields [16]\* in this background becomes

$$\begin{aligned}\delta\psi_{+M\alpha} &= D_M\epsilon_{+\alpha} + \frac{1}{4}H_{+MNP}^a(\gamma_a)_{\alpha}{}^{\beta}\Gamma^{NP}\epsilon_{+\beta} + \frac{3}{4}G_{MN\alpha\dot{\beta}}\Gamma^N\epsilon_{-}^{\dot{\beta}} - \frac{1}{8}G_{NP\alpha\dot{\beta}}\Gamma_M{}^{NP}\epsilon_{-}^{\dot{\beta}}, \\ \delta\psi_{-M\dot{\alpha}} &= D_M\epsilon_{-\dot{\alpha}} - \frac{1}{4}H_{-MNP}^{\dot{a}}(\gamma_{\dot{a}})_{\dot{\alpha}}{}^{\dot{\beta}}\Gamma^{NP}\epsilon_{-\dot{\beta}} + \frac{3}{4}G_{MN\beta\dot{\alpha}}\Gamma^N\epsilon_{+}^{\beta} - \frac{1}{8}G_{NP\beta\dot{\alpha}}\Gamma_M{}^{NP}\epsilon_{+}^{\beta}, \\ \delta\chi_{+a\alpha} &= -\frac{1}{12}H_{+aMNP}\Gamma^{MNP}\epsilon_{-\dot{\alpha}} - \frac{1}{4}G_{MN\beta\dot{\alpha}}\Gamma^{MN}\epsilon_{+}^{\alpha}(\gamma_a)_{\alpha}{}^{\beta}, \\ \delta\chi_{-\dot{a}\dot{\alpha}} &= \frac{1}{12}H_{-\dot{a}MNP}\Gamma^{MNP}\epsilon_{+\alpha} - \frac{1}{4}G_{MN\alpha\dot{\beta}}\Gamma^{MN}\epsilon_{-}^{\dot{\alpha}}(\gamma_{\dot{a}})_{\dot{\alpha}}{}^{\dot{\beta}},\end{aligned}\tag{2.1}$$

where the transformation parameter  $\epsilon_{+\alpha}$  and  $\epsilon_{-\dot{\alpha}}$  are SO(5)-symplectic Majorana-Weyl spinors.  $\Gamma^{\hat{M}}$  are gamma matrices of the six-dimensional Lorentz group SO(1,5), while  $\gamma^a$ ,  $\gamma^{\dot{a}}$  are those of SO(5) × SO(5). The field strengths of the tensor and vector fields are defined as

$$\begin{aligned}H_{MNP}^m &= 3\partial_{[M}B_{NP]}^m + \frac{3}{2}G_{[MN}^{\alpha\dot{\alpha}}A_{P]\beta\dot{\alpha}}(\gamma^m)_{\alpha}{}^{\beta} - \frac{3}{2}G_{[MN}^{\alpha\dot{\alpha}}A_{P]\alpha\dot{\beta}}(\gamma^m)_{\dot{\alpha}}{}^{\dot{\beta}}, \\ G_{MN}^{\alpha\dot{\alpha}} &= 2\partial_{[M}A_{N]}^{\alpha\dot{\alpha}}.\end{aligned}\tag{2.2}$$

$H_{+}^a$  and  $H_{-}^{\dot{a}}$  are self-dual and anti self-dual part of  $H^m$  with  $m = a$  or  $m = \dot{a}$ , and transform as **(5, 1)** and **(1, 5)** under the rigid SO(5) × SO(5) respectively.  $G_{MN}^{\alpha\dot{\alpha}}$  satisfies a doubly-symplectic reality condition

$$(G_{MN}^{\alpha\dot{\alpha}})^* = (\Omega^{-1})_{\alpha\beta}(\Omega^{-1})_{\dot{\alpha}\dot{\beta}}G_{MN}^{\beta\dot{\beta}},\tag{2.3}$$

where  $\Omega^{\alpha\beta}$  and  $\Omega^{\dot{\alpha}\dot{\beta}}$  are antisymmetric SO(5) charge conjugation matrices (See Appendix.).

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\* There are several misprints in ref. [16]. The left-hand sides of eq. (17) should be  $\tilde{H}_{1\pm MNP}^m$ . The right-hand side of the first equation of eq. (24) should be  $F^m - \tilde{H}_{1+}^m - \tilde{H}_{1-}^m$ . There should be a minus sign on the right-hand side of the third equation of eq. (24). The coefficient of the fifth term of eq. (21) should not be  $-\frac{1}{4}i$  but  $-\frac{1}{2}i$ . The eighth term of eq. (21) should be  $-\frac{1}{3}(F_{+}^a \cdot H_{0+}^a + F_{-}^{\dot{a}} \cdot H_{0-}^{\dot{a}})$ .

The  $\text{AdS}_3 \times \text{S}^3$  solution has a metric

$$\begin{aligned} ds^2 &= Z(r)^{-1} dx^\mu dx^\nu \eta_{\mu\nu} + Z(r) dx^i dx^j \delta_{ij} \\ &= Z(r)^{-1} dx^\mu dx^\nu \eta_{\mu\nu} + Z(r) dr^2 + R^2 d\Omega_3^2 \end{aligned} \quad (2.4)$$

and a self-dual field strength with non-vanishing components

$$H_{\mu\nu i}^a = R^{-2} \mathcal{S}^a \epsilon_{\mu\nu} x^i, \quad H_{ijk}^a = -r^{-4} R^2 \mathcal{S}^a \epsilon_{ijkl} x^l, \quad (2.5)$$

where  $r^2 = x^i x^i$ ,  $Z(r) = \frac{R^2}{r^2}$ , and  $d\Omega_3^2$  is the metric of  $\text{S}^3$  of unit radius. We have split the six-dimensional world index as  $M = (\mu, i)$  ( $\mu = 0, 1$ ;  $i = 2, 3, 4, 5$ ). The antisymmetric  $\epsilon_{\mu\nu}$  and  $\epsilon_{ijkl}$  are chosen as  $\epsilon_{01} = +1 = \epsilon_{2345}$ .  $\mathcal{S}^a$  is a constant vector of unit length  $\mathcal{S}^a \mathcal{S}^a = 1$ . We choose  $\mathcal{S}^5 = 1$  and  $\mathcal{S}^a = 0$  ( $a = 1, 2, 3, 4$ ) without losing generality. The constant parameter  $R$  denotes the radius of  $\text{AdS}_3$  and  $\text{S}^3$ . The metric (2.4) gives a vielbein and a spin connection as

$$\begin{aligned} e_\mu^{\hat{\mu}} &= \delta_\mu^{\hat{\mu}} Z^{-\frac{1}{2}}, & e_i^{\hat{i}} &= \delta_i^{\hat{i}} Z^{\frac{1}{2}}, \\ \omega_\mu^{\hat{\nu}\hat{i}} &= \frac{x^i}{R^2} \delta_\mu^{\hat{\nu}} \delta_i^{\hat{i}}, & \omega_i^{\hat{j}\hat{k}} &= -\frac{x^l}{r^2} (\delta_i^{\hat{j}} \delta_l^{\hat{k}} - \delta_i^{\hat{k}} \delta_l^{\hat{j}}), \end{aligned} \quad (2.6)$$

where  $\hat{\mu}, \hat{\nu}, \dots$  and  $\hat{i}, \hat{j}, \dots$  denote local Lorentz indices. It is convenient to decompose the six-dimensional gamma matrices  $\Gamma^{\hat{M}}$  as

$$\begin{aligned} \Gamma^{\hat{\mu}} &= \hat{\gamma}^{\hat{\mu}} \otimes \bar{\gamma}_{4\text{D}}, \\ \Gamma^{\hat{i}} &= 1 \otimes \hat{\gamma}^{\hat{i}}, \end{aligned} \quad (2.7)$$

where  $\hat{\gamma}^{\hat{\mu}}$  and  $\hat{\gamma}^{\hat{i}}$  are gamma matrices of  $\text{SO}(1,1)$  and  $\text{SO}(4)$  respectively, and we have defined

$$\bar{\gamma}_{2\text{D}} = \hat{\gamma}^{\hat{0}} \hat{\gamma}^{\hat{1}}, \quad \bar{\gamma}_{4\text{D}} = \hat{\gamma}^{\hat{2}} \hat{\gamma}^{\hat{3}} \hat{\gamma}^{\hat{4}} \hat{\gamma}^{\hat{5}}. \quad (2.8)$$

We use the explicit representations of the  $\text{SO}(4)$  gamma matrices given in Appendix.

The  $\text{AdS}_3 \times \text{S}^3$  solution (2.4), (2.5) has bosonic and fermionic symmetries. The rigid  $\text{SO}(5) \times \text{SO}(5)$  symmetry is broken to a rigid  $\text{SO}(4) \times \text{SO}(5)$  by the non-vanishing value of  $H_+^a \propto \mathcal{S}^a$  in **(5,1)**. The first factor  $\text{SO}(4) \sim \text{SU}(2) \times \text{SU}(2)$  corresponds to the automorphism group of the  $\mathcal{N} = (4, 4)$  superconformal algebra to be discussed in sect. 4, while the second factor  $\text{SO}(5)$  will not play important role in the following discussion. The solution is also invariant under the isometry of  $\text{AdS}_3 \times \text{S}^3$ . The isometry of  $\text{S}^3$  is  $\text{SO}(4) \sim \text{SU}(2) \times \text{SU}(2)$ , which acts on  $x^i$  as  $\text{SO}(4)$  rotations. This symmetry corresponds to the  $\text{SU}(2) \times \text{SU}(2)$  generators  $J_0^I$ ,

$\tilde{J}'_0$  in the  $\mathcal{N} = (4, 4)$  superconformal algebra in sect. 4. The isometry of  $\text{AdS}_3$  is  $\text{SO}(2, 2) \sim \text{SO}(2, 1) \times \text{SO}(2, 1)$ . It is generated by Killing vectors  $\xi^M$

$$\xi^\mu = \zeta^\mu - \frac{1}{4} \frac{R^4}{r^2} \eta^{\mu\nu} \partial_\nu \partial_\rho \zeta^\rho, \quad \xi^i = -\frac{1}{2} x^i \partial_\rho \zeta^\rho, \quad (2.9)$$

where  $\zeta^\mu(x^\nu)$  is an arbitrary vector satisfying

$$\partial_\mu \zeta^\rho \eta_{\rho\nu} + \partial_\nu \zeta^\rho \eta_{\rho\mu} = \partial_\rho \zeta^\rho \eta_{\mu\nu}, \quad \partial_\mu \partial_\nu \partial_\rho \zeta^\rho = 0. \quad (2.10)$$

Thus,  $\zeta^\mu$  is a two-dimensional conformal Killing vector which is quadratic in  $x^\mu$ . This symmetry corresponds to Virasoro generators  $L_m, \tilde{L}_m$  ( $m = \pm 1, 0$ ) in the superconformal algebra.

Finally, supersymmetries preserved by this solution are given by the parameters  $\epsilon_- = 0$  and  $\epsilon_+$  satisfying

$$D_M \epsilon_{+\alpha} + \frac{1}{4} H_{+MNP}^a (\gamma_a)_\alpha{}^\beta \Gamma^{NP} \epsilon_{+\beta} = 0. \quad (2.11)$$

This condition comes from the vanishing of the supertransformations of the fermionic fields (2.1). Substituting eqs. (2.5), (2.6) into eq. (2.11) it becomes

$$\begin{aligned} \left[ \partial_\mu - \frac{1}{2R} \hat{\gamma}_\mu \hat{\gamma}^{\hat{r}} \bar{\gamma}_{4D} (1 - \bar{\gamma}_{2D} \gamma_5) \right] \epsilon_+ &= 0, \\ \left[ \partial_i - \frac{x^i}{2r^2} \bar{\gamma}_{2D} \gamma_5 - \frac{x^j}{2r^2} \hat{\gamma}_{ij} (1 - \bar{\gamma}_{2D} \gamma_5) \right] \epsilon_+ &= 0, \end{aligned} \quad (2.12)$$

where  $\hat{\gamma}^{\hat{r}} = \frac{x^i}{r} \hat{\gamma}^{\hat{i}}$ , and  $\gamma_5$  is the fifth matrix of the  $\text{SO}(5)$  gamma matrices  $\gamma_a$ . The general  $\epsilon_+$  satisfying these equations is a sum of

$$\begin{aligned} \epsilon_+^{(++)} &= r^{\frac{1}{2}} \eta^{(++)}, \\ \epsilon_+^{(-+)} &= -\frac{1}{2} r^{\frac{1}{2}} \hat{\gamma}^{\hat{r}} \hat{\gamma}^\mu \partial_\mu \eta^{(++)}, \\ \epsilon_+^{(--)} &= r^{\frac{1}{2}} \eta^{(--)}, \\ \epsilon_+^{(+-)} &= \frac{1}{2} r^{\frac{1}{2}} \hat{\gamma}^{\hat{r}} \hat{\gamma}^\mu \partial_\mu \eta^{(--)}, \end{aligned} \quad (2.13)$$

where the suffix  $(\pm\pm)$  on  $\epsilon_+$  and  $\eta$  denotes eigenvalues of  $\bar{\gamma}_{4D}$  and  $\gamma_5$ .  $\eta^{(\pm\pm)}$  are two-dimensional conformal Killing spinors

$$\begin{aligned} \eta^{(++)}(x^+) &= \epsilon_0^{(++)} + \frac{\sqrt{2}}{R} \epsilon_1^{(++)} x^+, \\ \eta^{(--)}(x^-) &= \epsilon_0^{(--)} + \frac{\sqrt{2}}{R} \epsilon_1^{(--)} x^-, \end{aligned} \quad (2.14)$$

where  $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$ , and  $\epsilon_0^{(\pm\pm)}$  and  $\epsilon_1^{(\pm\pm)}$  are arbitrary constant spinors with given  $\bar{\gamma}_{4D}$  and  $\gamma_5$  eigenvalues. Note that  $\eta^{(\pm\pm)}$  must be linear in  $x^\pm$ .

The boundary of  $\text{AdS}_3$  at infinity, on which the CFT is defined, is a cylinder. We will use the coordinates of the cylinder when we discuss the CFT in sect. 4. To compare the supergravity side and the CFT side we need a relation between the coordinates  $x^\mu$  in eq. (2.4) and the coordinates of the cylinder  $\tau, \sigma$  ( $-\infty < \tau < \infty$ ,  $0 \leq \sigma \leq 2\pi$ ), which is given by (see, e.g. ref. [4])

$$e^{i(\tau \pm \sigma)} = \frac{R + i(x^0 \pm x^1)}{R - i(x^0 \pm x^1)}. \quad (2.15)$$

Going to the Euclidean signature this relation becomes

$$z = \frac{1 + w}{1 - w}, \quad (2.16)$$

where

$$z = e^{\tau_E + i\sigma}, \quad w = \frac{1}{R}(t_E + ix^1), \quad (2.17)$$

and  $\tau_E = i\tau$ ,  $t_E = ix^0$  are Euclidean time coordinates. In terms of the coordinate  $z$  the conformal Killing spinors (2.14) become

$$\begin{aligned} \eta^{(++)}(z) &= \frac{1}{\sqrt{2}}(1 + z)\epsilon_0^{(++)} + \frac{1}{\sqrt{2}}i(1 - z)\epsilon_1^{(++)}, \\ \eta^{(--)}(\bar{z}) &= \frac{1}{\sqrt{2}}(1 + \bar{z})\epsilon_0^{(--)} + \frac{1}{\sqrt{2}}i(1 - \bar{z})\epsilon_1^{(--)}, \end{aligned} \quad (2.18)$$

where we have used the fact that  $\eta^{(\pm\pm)}$  transforms as a primary field of weight  $-\frac{1}{2}$  under the conformal transformation (2.16) in the same way as for superconformal ghosts in the superstring [17]. From the expression (2.18) we see that the parameters  $\epsilon_0^{(++)}$  and  $\epsilon_1^{(++)}$  correspond to combinations of the supercharges  $G_{-\frac{1}{2}} + G_{\frac{1}{2}}$  and  $G_{-\frac{1}{2}} - G_{\frac{1}{2}}$  in the superconformal algebra in sect. 4 respectively. Similarly,  $\epsilon_0^{(--)}$  and  $\epsilon_1^{(--)}$  correspond to  $\tilde{G}_{-\frac{1}{2}} + \tilde{G}_{\frac{1}{2}}$  and  $\tilde{G}_{-\frac{1}{2}} - \tilde{G}_{\frac{1}{2}}$  respectively.

### 3. Perturbations in supergravity

In ref. [10] perturbations of the vector fields were obtained which satisfy the linearized field equations around the  $\text{AdS}_3 \times \text{S}^3$  solution (2.4), (2.5). We consider

the linear order in these perturbations. There is no back reaction to other fields to this order.

To show the perturbations we introduce self-dual and anti self-dual two-forms  $T_2 = \frac{1}{2}T_{ij}dx^i \wedge dx^j$  satisfying

$$*_4 T_2 = \pm T_2, \quad (3.1)$$

where  $*_4$  is the Hodge dual for the flat metric  $\delta_{ij}$ . The general forms of these two-forms are

$$T_2 = m_1 dz^1 \wedge d\bar{z}^2 + m_2 d\bar{z}^1 \wedge dz^2 + m_3 (dz^1 \wedge d\bar{z}^1 - dz^2 \wedge d\bar{z}^2) \quad (3.2)$$

for the self-dual case and

$$T_2 = m_1 dz^1 \wedge dz^2 + m_2 d\bar{z}^1 \wedge d\bar{z}^2 + m_3 (dz^1 \wedge d\bar{z}^1 + dz^2 \wedge d\bar{z}^2). \quad (3.3)$$

for the anti self-dual case, where  $m_1, m_2, m_3$  are constant coefficients and we have introduced the complex coordinates

$$z^1 = \frac{1}{\sqrt{2}}(x^2 + ix^4), \quad z^2 = \frac{1}{\sqrt{2}}(x^3 + ix^5). \quad (3.4)$$

Under the isometry  $SU(2) \times SU(2)$  of  $S^3$  these two-forms transform as  $(\mathbf{3}, \mathbf{1})$  and  $(\mathbf{1}, \mathbf{3})$  respectively. In particular, the  $m_1$  terms in eqs. (3.2), (3.3) represent the highest weight state of each  $SU(2)$ . We also define a two-form  $V_2$  from  $T_2$  with components

$$V_{ij} = \frac{x^k}{r^2}(x^i T_{kj} + x^j T_{ik}). \quad (3.5)$$

The perturbations satisfying the field equations were given in terms of  $T_2$  and  $V_2$  in ref. [10]. We give the field strengths  $G_2$  of the vector fields in eq. (2.2). For the self-dual case  $*_4 T_2 = T_2$  there are two pairs of solutions

$$\begin{aligned} G_2^{(+)} &= \frac{1}{2}T_2(1 - \gamma_5), \\ G_2^{(-)} &= \frac{1}{2}r^{-6}(T_2 - 3V_2)(1 - \gamma_5) \end{aligned} \quad (3.6)$$

and

$$\begin{aligned} G_2^{(+)} &= \frac{1}{2}r^{-2}(T_2 - V_2)(1 + \gamma_5), \\ G_2^{(-)} &= \frac{1}{2}r^{-4}(T_2 - 2V_2)(1 + \gamma_5). \end{aligned} \quad (3.7)$$



Similarly, for the anti self-dual case  $*_4 T_2 = -T_2$  there are two pairs of solutions

$$\begin{aligned} G_2^{(+)} &= \frac{1}{2} T_2 (1 + \gamma_5), \\ G_2^{(-)} &= \frac{1}{2} r^{-6} (T_2 - 3V_2) (1 + \gamma_5) \end{aligned} \quad (3.8)$$

and

$$\begin{aligned} G_2^{(+)} &= \frac{1}{2} r^{-2} (T_2 - V_2) (1 - \gamma_5), \\ G_2^{(-)} &= \frac{1}{2} r^{-4} (T_2 - 2V_2) (1 - \gamma_5). \end{aligned} \quad (3.9)$$

By the reality condition of  $G_{ij}$  (2.3) the coefficients of  $T_2$  must satisfy

$$(m_1^{\alpha\dot{\alpha}})^* = (\Omega^{-1})_{\alpha\beta} (\Omega^{-1})_{\dot{\alpha}\dot{\beta}} m_2^{\beta\dot{\beta}}, \quad (m_3^{\alpha\dot{\alpha}})^* = -(\Omega^{-1})_{\alpha\beta} (\Omega^{-1})_{\dot{\alpha}\dot{\beta}} m_3^{\beta\dot{\beta}}. \quad (3.10)$$

For each pair  $G_2^{(+)}$  represents a perturbation in the CFT by a operator, while  $G_2^{(-)}$  represents the vacuum expectation value of the operator [18, 19]. We will examine symmetries preserved by  $G_2^{(+)}$ .

## 4. Perturbations in CFT

The  $\text{AdS}_3 \times \text{S}^3 \times \text{T}_4$  supergravity background corresponds to a two-dimensional  $\mathcal{N} = (4, 4)$  superconformal field theory [1]. This CFT is described as a deformation of the supersymmetric sigma model with a target space  $T_4^N/S_N$  [20, 21]. For details of this theory see, e.g. ref. [22]. The perturbations of the supergravity solution discussed in the previous section correspond to certain operators in the CFT. In this section we identify these operators and examine unbroken symmetries by these operators. We do not need the detailed properties of the operators but the fact that they are operators corresponding to descendents of chiral primary states.

$\mathcal{N} = (4, 4)$  superconformal field theories have two copies of the  $\mathcal{N} = 4$  super Virasoro algebra for the holomorphic and the anti-holomorphic parts. The  $\mathcal{N} = 4$  super Virasoro algebra for the holomorphic part consists of Virasoro generators  $L_n$ ,  $\text{SU}(2)$  currents  $J_n^I$  ( $I = 1, 2, 3$ ) and supercharges  $G_r^{A\dot{A}}$  ( $A = 1, 2; \dot{A} = \dot{1}, \dot{2}$ ). The mode indices take values  $n \in \mathbf{Z}$  and  $r \in \mathbf{Z} + \frac{1}{2}$  corresponding to the anti-periodic

boundary condition on fermionic fields. The (anti-)commutation relations of this algebra are

$$\begin{aligned}
[L_m, L_n] &= (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}, \\
\{G_r^{A\dot{A}}, G_s^{B\dot{B}}\} &= \epsilon^{\dot{A}\dot{B}}\epsilon^{AB}L_{r+s} + (r-s)\epsilon^{\dot{A}\dot{B}}(\sigma^I)^{AB}J_{r+s}^I + \frac{c}{6}\left(r^2 - \frac{1}{4}\right)\epsilon^{\dot{A}\dot{B}}\epsilon^{AB}\delta_{r+s,0}, \\
[J_m^I, J_n^J] &= i\epsilon^{IJK}J_{m+n}^K + \frac{c}{12}m\delta^{IJ}\delta_{m+n,0}, \\
[L_m, G_r^{A\dot{A}}] &= \left(\frac{1}{2}m - r\right)G_{m+r}^{A\dot{A}}, \\
[L_m, J_n^I] &= -nJ_{m+n}^I, \\
[J_m^I, G_r^{A\dot{A}}] &= -\frac{1}{2}(\sigma^I)^A{}_B G_{m+r}^{B\dot{A}},
\end{aligned} \tag{4.1}$$

where  $\epsilon^{AB}$  and  $\epsilon^{\dot{A}\dot{B}}$  are antisymmetric in the indices with  $\epsilon^{12} = 1 = \epsilon^{\dot{1}\dot{2}}$ , and  $(\sigma^I)^A{}_B$  are components of the Pauli matrices. We use  $\epsilon$ 's to raise and lower indices, e.g.  $(\sigma^I)^{AB} = \epsilon^{BC}(\sigma^I)^A{}_C$ . For unitary representations the generators satisfy hermiticity conditions

$$(L_n)^\dagger = L_{-n}, \quad (J_n^I)^\dagger = J_{-n}^I, \quad (G_r^{A\dot{A}})^\dagger = \epsilon_{AB}\epsilon_{\dot{A}\dot{B}}G_{-r}^{B\dot{B}}. \tag{4.2}$$

The  $\mathcal{N} = 4$  super Virasoro algebra for the anti-holomorphic part has generators  $\tilde{L}_n$ ,  $\tilde{J}_n^{I'}$  and  $\tilde{G}_r^{A'\dot{A}'}$  ( $A' = 1', 2'; \dot{A}' = \dot{1}', \dot{2}'$ ), which satisfy the similar (anti-)commutation relations and hermiticity conditions.

A chiral primary state  $|\phi_0\rangle$  of the  $\mathcal{N} = 4$  superconformal algebra by definition satisfies

$$\begin{aligned}
L_0 |\phi_0\rangle &= j |\phi_0\rangle, \\
J_0^3 |\phi_0\rangle &= j |\phi_0\rangle, \\
J_0^+ |\phi_0\rangle &= 0, \\
L_n |\phi_0\rangle &= 0 \quad (n > 0), \\
J_n^I |\phi_0\rangle &= 0 \quad (n > 0), \\
G_r^{A\dot{A}} |\phi_0\rangle &= 0 \quad (r > 0), \\
G_{-\frac{1}{2}}^{2\dot{A}} |\phi_0\rangle &= 0,
\end{aligned} \tag{4.3}$$

where we have defined  $J_0^\pm = J_0^1 \pm iJ_0^2$ .  $L_0$  and  $J_0^3$  must have the same eigenvalue  $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$ . One can construct descendent states by applying other generators

on  $|\phi_0\rangle$ . A chiral primary state and its descendent states are grouped into highest weight representations of the Virasoro and SU(2) Kac-Moody algebras. For  $j \geq 1$  the corresponding highest weight states are

$$\begin{aligned} |\phi_0\rangle \\ |\phi_1^A\rangle &= G_{-\frac{1}{2}}^{1A} |\phi_0\rangle, \\ |\phi_2\rangle &= \left( G_{-\frac{1}{2}}^{12} G_{-\frac{1}{2}}^{11} + \frac{1}{2j} L_{-1} J_0^- \right) |\phi_0\rangle. \end{aligned} \quad (4.4)$$

The second term in  $|\phi_2\rangle$  is needed so that  $|\phi_2\rangle$  becomes a highest weight state of the Virasoro and SU(2) Kac-Moody algebras. The eigenvalues of  $L_0$  and  $J_0^3$  for these states are

$$\begin{array}{ccc} & L_0 & J_0^3 \\ |\phi_0\rangle & j & j \\ |\phi_1^A\rangle & j + \frac{1}{2} & j - \frac{1}{2} \\ |\phi_2\rangle & j + 1 & j - 1. \end{array} \quad (4.5)$$

For  $j = \frac{1}{2}$  there exist only  $|\phi_0\rangle$  and  $|\phi_1^A\rangle$ , and no  $|\phi_2\rangle$ .

In refs. [11, 12, 13, 14] the Kaluza-Klein spectrum of six-dimensional supergravities for the compactification on  $\text{AdS}_3 \times \text{S}^3$  was obtained and compared to the spectrum of chiral primary states of two-dimensional superconformal field theories. We can identify the perturbations (3.6)–(3.9) in this spectrum. The Kaluza-Klein spectrum of the six-dimensional  $\mathcal{N} = (4, 4)$  supergravity on  $\text{AdS}_3 \times \text{S}^3$  obtained in ref. [13] is

$$\begin{aligned} & \bigoplus_{\mathbf{m}=2}^{\infty} [(\mathbf{m}, \mathbf{m} + \mathbf{2})_S + (\mathbf{m} + \mathbf{2}, \mathbf{m})_S + 4(\mathbf{m}, \mathbf{m} + \mathbf{1})_S + 4(\mathbf{m} + \mathbf{1}, \mathbf{m})_S] \\ & + \bigoplus_{\mathbf{m}=3}^{\infty} [6(\mathbf{m}, \mathbf{m})_S] + 5(\mathbf{2}, \mathbf{2})_S. \end{aligned} \quad (4.6)$$

Here,  $(\mathbf{m}, \mathbf{m}')_S$  represents a short representation of the superalgebra  $\text{SU}(2|1, 1) \times \text{SU}(2|1, 1)$ . It is a product representation of two short representations  $\mathbf{m}_S$  and  $\mathbf{m}'_S$  for each  $\text{SU}(2|1, 1)$ . The superalgebra  $\text{SU}(2|1, 1)$  is a subalgebra of the  $\mathcal{N} = 4$  superconformal algebra (4.1) consisting of the  $\text{SO}(2, 1)$  generators  $L_0, L_{\pm 1}$ , the SU(2) generators  $J_0^I$  and the supersymmetry generators  $G_{\pm\frac{1}{2}}^{AA}$ . The representation  $\mathbf{m}_S$  consists of four irreducible representations of  $\text{SO}(2, 1) \times \text{SU}(2)$  whose highest

perturbation	$(h, \bar{h})$	$SU(2) \times SU(2)$	multiplicity	supermultiplet
eq. (3.6)	(2, 2)	<b>(3, 1)</b>	8	<b>(3, 4)<sub>S</sub></b>
eq. (3.7)	(1, 1)	<b>(3, 1)</b>	8	<b>(3, 2)<sub>S</sub></b>
eq. (3.8)	(2, 2)	<b>(1, 3)</b>	8	<b>(4, 3)<sub>S</sub></b>
eq. (3.9)	(1, 1)	<b>(1, 3)</b>	8	<b>(2, 3)<sub>S</sub></b>

Table 1: Perturbations.

weight states are given in eq. (4.4) with  $j = \frac{1}{2}(m - 1)$ . Namely, there is a one-to-one correspondence between a representation  $\mathbf{m}_S$  and a chiral primary state with  $j = \frac{1}{2}(m - 1)$ .

The conformal weights  $(h, \bar{h})$  of the perturbations  $G_2^{(+)}$  are determined by their spins and  $r$ -dependences. Since the perturbations (3.6)–(3.9) are spin 0 scalars on the two-dimensional boundary of  $AdS_3$  we have  $h = \bar{h}$ . When  $G_{ij}^{(+)} \sim r^s$  in the coordinate frame, we have  $G_{ij}^{(+)} \sim r^s \times (Z^{-\frac{1}{2}})^2 \sim r^{s+2}$  in the inertial (local Lorentz) frame. We obtain a relation  $h + \bar{h} = d + (s + 2) = s + 4$  ( $d = 2$ ). Under the isometry  $SO(4) = SU(2) \times SU(2)$  of  $S^3$  the self-dual and the anti self-dual two-forms in eqs. (3.2) and (3.3) transform as **(3, 1)** and **(1, 3)**. Therefore, the perturbations  $G_2^{(+)}$  in eqs. (3.6)–(3.9) have the quantum numbers in Table 1. The multiplicity is 8 since  $G_2^{(+)\alpha\dot{\alpha}}$  has two internal indices  $\alpha = 1, \dots, 4$ ,  $\dot{\alpha} = 1, \dots, 4$  and the projections  $\frac{1}{2}(1 \pm \gamma_5)$  reduce the 16 components by half. Looking for short supermultiplets in eq. (4.6) which contain states having these quantum numbers we find that only supermultiplets shown in the last column of Table 1 contain those states. Explicitly, the perturbations correspond to the following states in the CFT

$$\begin{aligned}
\text{eq. (3.6)} : & \quad \left| \phi_1^{\dot{A}}(j = \tfrac{3}{2}) \right\rangle \otimes \left| \tilde{\phi}_2(\bar{j} = 1) \right\rangle, \\
\text{eq. (3.7)} : & \quad \left| \phi_1^{\dot{A}}(j = \tfrac{1}{2}) \right\rangle \otimes \left| \tilde{\phi}_0(\bar{j} = 1) \right\rangle, \\
\text{eq. (3.8)} : & \quad \left| \phi_2(j = 1) \right\rangle \otimes \left| \tilde{\phi}_1^{\dot{A}'}(\bar{j} = \tfrac{3}{2}) \right\rangle, \\
\text{eq. (3.9)} : & \quad \left| \phi_0(j = 1) \right\rangle \otimes \left| \tilde{\phi}_1^{\dot{A}'}(\bar{j} = \tfrac{1}{2}) \right\rangle.
\end{aligned} \tag{4.7}$$

To examine unbroken symmetries in the CFT side we need to know the action of supercharges on these states. From eqs. (4.4), (4.3), (4.1) we obtain

$$G_{\frac{1}{2}}^{2\dot{A}} |\phi_0\rangle = G_{\frac{1}{2}}^{1\dot{A}} |\phi_0\rangle = G_{-\frac{1}{2}}^{2\dot{A}} |\phi_0\rangle = 0,$$

$$\begin{aligned}
G_{-\frac{1}{2}}^{1\dot{A}} |\phi_0\rangle &= |\phi_1^{\dot{A}}\rangle, \\
G_{\frac{1}{2}}^{2\dot{A}} |\phi_1^{\dot{B}}\rangle &= -2j\epsilon^{\dot{A}\dot{B}} |\phi_0\rangle, \\
G_{\frac{1}{2}}^{1\dot{A}} |\phi_1^{\dot{B}}\rangle &= \epsilon^{\dot{A}\dot{B}} J_0^- |\phi_0\rangle, \\
G_{-\frac{1}{2}}^{2\dot{A}} |\phi_1^{\dot{B}}\rangle &= -\epsilon^{\dot{A}\dot{B}} L_{-1} |\phi_0\rangle, \\
G_{-\frac{1}{2}}^{1\dot{A}} |\phi_1^{\dot{B}}\rangle &= -\epsilon^{\dot{A}\dot{B}} \left( |\phi_2\rangle - \frac{1}{2j} L_{-1} J_0^- |\phi_0\rangle \right), \\
G_{\frac{1}{2}}^{2\dot{A}} |\phi_2\rangle &= \left( \frac{1}{2j} - 2j \right) |\phi_1^{\dot{A}}\rangle, \\
G_{\frac{1}{2}}^{1\dot{A}} |\phi_2\rangle &= \left( 1 + \frac{1}{2j} \right) J_0^- |\phi_1^{\dot{A}}\rangle, \\
G_{-\frac{1}{2}}^{2\dot{A}} |\phi_2\rangle &= - \left( 1 - \frac{1}{2j} \right) L_{-1} |\phi_1^{\dot{A}}\rangle, \\
G_{-\frac{1}{2}}^{1\dot{A}} |\phi_2\rangle &= \frac{1}{2j} L_{-1} J_0^- |\phi_1^{\dot{A}}\rangle.
\end{aligned} \tag{4.8}$$

We can express these relations in terms of local operators and currents. For each state we introduce a local operator  $\phi(z)$  which create the state as

$$|\phi\rangle = \phi(0) |0\rangle, \tag{4.9}$$

where  $|0\rangle$  is the  $SU(2|1,1)$  invariant vacuum. We also introduce currents for the  $\mathcal{N} = 4$  superconformal generators

$$\begin{aligned}
T(z) &= \sum_{n \in \mathbf{Z}} z^{-n-2} L_n, \\
G^{A\dot{A}}(z) &= \sum_{r \in \mathbf{Z} + \frac{1}{2}} z^{-r-\frac{3}{2}} G_r^{A\dot{A}}, \\
J^I(z) &= \sum_{n \in \mathbf{Z}} z^{-n-1} J_n^I.
\end{aligned} \tag{4.10}$$

Then, the relations (4.8) lead to the OPEs

$$\begin{aligned}
G^{2\dot{A}}(z_1) \phi_0(z_2) &\sim \text{regular}, \\
G^{1\dot{A}}(z_1) \phi_0(z_2) &\sim \frac{1}{z_1 - z_2} \phi_1^{\dot{A}}(z_2), \\
G^{2\dot{A}}(z_1) \phi_1^{\dot{B}}(z_2) &\sim -\frac{2j}{(z_1 - z_2)^2} \epsilon^{\dot{A}\dot{B}} \phi_0(z_2) - \frac{1}{z_1 - z_2} \epsilon^{\dot{A}\dot{B}} \partial \phi_0(z_2),
\end{aligned}$$

$$\begin{aligned}
G^{1\dot{A}}(z_1)\phi_1^{\dot{B}}(z_2) &\sim \frac{1}{(z_1 - z_2)^2} \epsilon^{\dot{A}\dot{B}} [J_0^-, \phi_0(z_2)] - \frac{1}{z_1 - z_2} \epsilon^{\dot{A}\dot{B}} \left( \phi_2 - \frac{1}{2j} \partial [J_0^-, \phi_0] \right) (z_2), \\
G^{2\dot{A}}(z_1)\phi_2(z_2) &\sim - \left( 2j - \frac{1}{2j} \right) \frac{1}{(z_1 - z_2)^2} \phi_1^{\dot{A}}(z_2) - \left( 1 - \frac{1}{2j} \right) \frac{1}{z_1 - z_2} \partial \phi_1^{\dot{A}}(z_2), \\
G^{1\dot{A}}(z_1)\phi_2(z_2) &\sim \left( 1 + \frac{1}{2j} \right) \frac{1}{(z_1 - z_2)^2} [J_0^-, \phi_1^{\dot{A}}(z_2)] + \frac{1}{2j} \frac{1}{z_1 - z_2} \partial [J_0^-, \phi_1^{\dot{A}}(z_2)]. \quad (4.11)
\end{aligned}$$

From these OPEs one can obtain commutators of the generators  $L_n$ ,  $J_n^I$ ,  $G_r^{AA}$  and the local operators  $\phi_0(z)$ ,  $\phi_1^{\dot{A}}(z)$ ,  $\phi_2(z)$  by computing contour integrals of  $z_1$  around  $z_2$ .

The operators corresponding to the perturbations in the supergravity solution are integrated operators. There is an arbitrariness in the choice of the integration measure. Since the perturbation in the supergravity side is invariant under translations of  $x^\mu$ , we choose the measure invariant under the translation of  $w$  in eq. (2.17) (Euclidean version of  $x^\mu$ ). Thus, we consider the integrated operators

$$\begin{aligned}
\Phi_0 &= \int d^2w \phi_0(w) \tilde{\phi}(\bar{w}) = \int d^2z \left| \frac{1}{2}(z+1)^2 \right|^{2(j-1)} \phi_0(z) \tilde{\phi}(\bar{z}), \\
\Phi_1^{\dot{A}} &= \int d^2w \phi_1^{\dot{A}}(w) \tilde{\phi}(\bar{w}) = \int d^2z \left| \frac{1}{2}(z+1)^2 \right|^{2(j-\frac{1}{2})} \phi_1^{\dot{A}}(z) \tilde{\phi}(\bar{z}), \\
\Phi_2 &= \int d^2w \phi_2(w) \tilde{\phi}(\bar{w}) = \int d^2z \left| \frac{1}{2}(z+1)^2 \right|^{2j} \phi_2(z) \tilde{\phi}(\bar{z}), \quad (4.12)
\end{aligned}$$

where we have used the transformation property of the Virasoro primary field of conformal weight  $h$ :  $\phi(w) = \left( \frac{\partial z}{\partial w} \right)^h \phi(z)$ . In eq. (4.12) we have specified only the holomorphic part of the local operators. The anti-holomorphic part  $\tilde{\phi}$  can be  $\tilde{\phi}_0$ ,  $\tilde{\phi}_1^{\dot{A}}$ ,  $\tilde{\phi}_2$  with the same conformal weight as the holomorphic part. Using the fact that the local operators  $\phi_0$ ,  $\phi_1^{\dot{A}}$ ,  $\phi_2$  and  $\tilde{\phi}$  are primary fields of the Virasoro algebra it is easy to see that these integrated operators indeed commute with the translation generators of  $w$ ,  $\bar{w}$

$$P = L_{-1} + 2L_0 + L_1, \quad \tilde{P} = \tilde{L}_{-1} + 2\tilde{L}_0 + \tilde{L}_1. \quad (4.13)$$

By integrating the commutation relations between the generators and the local operators we obtain

$$\begin{aligned}
[G_r^{2\dot{A}}, \Phi_0] &= 0, \\
[G_r^{1\dot{A}}, \Phi_0] &= \int d^2z \left| \frac{1}{2}(z+1)^2 \right|^{2(j-1)} z^{r+\frac{1}{2}} \phi_1^{\dot{A}}(z) \tilde{\phi}(\bar{z}),
\end{aligned}$$

$$\begin{aligned}
[G_r^{2\dot{A}}, \Phi_1^{\dot{B}}] &= -2 \left(j - \frac{1}{2}\right) \epsilon^{\dot{A}\dot{B}} \int d^2z \left| \frac{1}{2}(z+1)^2 \right|^{2(j-\frac{1}{2})} \frac{z^{r-\frac{1}{2}}}{z+1} \left[ \left(r - \frac{1}{2}\right)z + \left(r + \frac{1}{2}\right) \right] \\
&\quad \times \phi_0(z) \tilde{\phi}(\bar{z}), \\
[G_r^{1\dot{A}}, \Phi_1^{\dot{B}}] &= \frac{1}{j} \left(j - \frac{1}{2}\right) \epsilon^{\dot{A}\dot{B}} \int d^2z \left| \frac{1}{2}(z+1)^2 \right|^{2(j-\frac{1}{2})} \frac{z^{r-\frac{1}{2}}}{z+1} \left[ \left(r - \frac{1}{2}\right)z + \left(r + \frac{1}{2}\right) \right] \\
&\quad \times [J_0^-, \phi_0(z)] \tilde{\phi}(\bar{z}) - \epsilon^{\dot{A}\dot{B}} \int d^2z \left| \frac{1}{2}(z+1)^2 \right|^{2(j-\frac{1}{2})} z^{r+\frac{1}{2}} \phi_2(z) \tilde{\phi}(\bar{z}), \\
[G_r^{2\dot{A}}, \Phi_2] &= -2 \left(j - \frac{1}{2}\right) \int d^2z \left| \frac{1}{2}(z+1)^2 \right|^{2j} \frac{z^{r-\frac{1}{2}}}{z+1} \left[ \left(r - \frac{1}{2}\right)z + \left(r + \frac{1}{2}\right) \right] \phi_1^{\dot{A}}(z) \tilde{\phi}(\bar{z}), \\
[G_r^{1\dot{A}}, \Phi_2] &= \int d^2z \left| \frac{1}{2}(z+1)^2 \right|^{2j} \frac{z^{r-\frac{1}{2}}}{z+1} \left[ \left(r - \frac{1}{2}\right)z + \left(r + \frac{1}{2}\right) \right] [J_0^-, \phi_1^{\dot{A}}(z)] \tilde{\phi}(\bar{z}). \quad (4.14)
\end{aligned}$$

Let us find unbroken supersymmetries by the perturbations. The  $(h, \bar{h}) = (2, 2)$  operators corresponding to the first and third states in eq. (4.7) are

$$\begin{aligned}
\Phi_{\dot{A}} &= \int d^2w \phi_{1\dot{A}}(w; j = \tfrac{3}{2}) \tilde{\phi}_2(\bar{w}; \bar{j} = 1), \\
\Phi_{\dot{A}'} &= \int d^2w \phi_2(w; j = 1) \tilde{\phi}_{1\dot{A}'}(\bar{w}; \bar{j} = \tfrac{3}{2}). \quad (4.15)
\end{aligned}$$

Here, we have lowered the indices  $\dot{A}, \dot{A}'$  by using  $\epsilon_{\dot{A}\dot{B}}, \epsilon_{\dot{A}'\dot{B}'}$ . From eq. (4.14) we find that the supercharges which commute with  $\Phi_{\dot{A}'}$  for a given  $\dot{A}'$  are

$$G_{-\frac{1}{2}}^{B\dot{B}} + G_{\frac{1}{2}}^{B\dot{B}}, \quad \tilde{G}_{-\frac{1}{2}}^{2'\dot{A}'} + \tilde{G}_{\frac{1}{2}}^{2'\dot{A}'}, \quad \tilde{G}_{\pm\frac{1}{2}}^{B'\dot{B}'}, \quad (4.16)$$

where  $B, B', \dot{B}$  are arbitrary and  $\dot{B}' \neq \dot{A}'$ . The bosonic generators which commute with  $\Phi_{\dot{A}'}$  are  $P, \tilde{P}$  and  $J_0^I$ . To add these operators to the CFT Hamiltonian as a perturbation we should make a hermitian combination  $m\Phi_{\dot{A}'} + m^*\Phi_{\dot{A}'}^\dagger$ , where  $m$  is a complex constant. The supersymmetries preserved by this perturbation are those preserved by both of  $\Phi_{\dot{A}'}$  and  $\Phi_{\dot{A}'}^\dagger$ . Remembering the hermiticity condition (4.2) we find that the unbroken supercharges are

$$Q^{B\dot{B}} = G_{-\frac{1}{2}}^{B\dot{B}} + G_{\frac{1}{2}}^{B\dot{B}}, \quad \tilde{Q} = \tilde{G}_{-\frac{1}{2}}^{2'\dot{B}'} + \tilde{G}_{\frac{1}{2}}^{2'\dot{B}'}, \quad (4.17)$$

where  $B, \dot{B}$  are arbitrary and  $\dot{B}' \neq \dot{A}'$ . These supercharges together with the translation generators (4.13) satisfy the  $\mathcal{N} = (4, 2)$  Poincaré supersymmetry algebra

$$\{Q^{A\dot{A}}, Q^{B\dot{B}}\} = \epsilon^{AB} \epsilon^{\dot{A}\dot{B}} P, \quad \{\tilde{Q}, \tilde{Q}^\dagger\} = \tilde{P}. \quad (4.18)$$

The supersymmetries preserved by  $\Phi_{\dot{A}}$  in eq. (4.15) and its hermitian conjugate are similarly obtained and form the  $\mathcal{N} = (2, 4)$  Poincaré supersymmetry algebra.

The  $(h, \bar{h}) = (1, 1)$  operators corresponding to the second and fourth states in eq. (4.7) are

$$\begin{aligned}\Phi_{\dot{A}} &= \int d^2w \phi_{1\dot{A}}(w; j = \tfrac{1}{2}) \tilde{\phi}_0(\bar{w}; \bar{j} = 1), \\ \Phi_{\dot{A}'} &= \int d^2w \phi_0(w; j = 1) \tilde{\phi}_{1\dot{A}'}(\bar{w}; \bar{j} = \tfrac{1}{2}).\end{aligned}\tag{4.19}$$

From eq. (4.14) the supercharges which commute with  $\Phi_{\dot{A}'}$  for a given  $\dot{A}'$  are

$$G_{\pm\frac{1}{2}}^{1\dot{B}}, \quad \tilde{G}_{\pm\frac{1}{2}}^{B'\dot{B}'}, \tag{4.20}$$

where  $B', \dot{B}, \dot{B}'$  are arbitrary. The bosonic generators which commute with  $\Phi_{\dot{A}'}$  are  $P, \tilde{L}_{\pm 1}, \tilde{L}_0$  and  $\tilde{J}_0'$ . The supercharges which commute with a hermitian perturbation  $m\Phi_{\dot{A}'} + m^*\Phi_{\dot{A}'}^\dagger$  are

$$\tilde{G}_{\pm\frac{1}{2}}^{B'\dot{B}'}, \tag{4.21}$$

which together with  $\tilde{L}_{-1}, \tilde{L}_0, \tilde{L}_1$  and  $\tilde{J}_0'$  form the  $\mathcal{N} = (0, 4)$  superconformal algebra. The supersymmetries preserved by  $\Phi_{\dot{A}}$  and its hermitian conjugate are similarly obtained and form the  $\mathcal{N} = (4, 0)$  superconformal algebra.

In the next section we will show that the perturbations in the supergravity solution preserve the same supersymmetries as above.

## 5. Unbroken symmetries by perturbations in supergravity

Let us start from the bosonic symmetries, i.e., the isometry of  $\text{AdS}_3 \times \text{S}^3$ . The coordinate transformation of the perturbation  $G_2$  for  $\delta x^M = \xi^M$  is

$$\delta G_{MN} = \xi^P \partial_P G_{MN} + \partial_M \xi^P G_{PN} + \partial_N \xi^P G_{MP}. \tag{5.1}$$

Our perturbations (3.6)–(3.9) have only non-vanishing components  $G_{ij}$  and are independent of  $x^\mu$ .

In the case of the Killing vectors (2.9) of  $\text{AdS}_3$  this transformation gives

$$\begin{aligned}\delta G_{\mu\nu} &= 0, \\ \delta G_{\mu i} &= -\frac{1}{2} \partial_\mu \partial_\rho \zeta^\rho x^j G_{ji}, \\ \delta G_{ij} &= -\frac{1}{2} \partial_\rho \zeta^\rho (r \partial_r + 2) G_{ij}.\end{aligned}\tag{5.2}$$



For Poincaré transformations, for which  $\partial_\rho \zeta^\rho = 0$ , they automatically vanish. However, the invariance under all of the Killing vectors (2.9) requires

$$x^i G_{ij} = 0, \quad (r\partial_r + 2) G_{ij} = 0, \quad (5.3)$$

namely,  $G_{ij}$  should lie along the  $S^3$  directions and have  $r$ -dependence  $r^{-2}$ . These conditions are satisfied by the perturbations  $G_2^{(+)}$  in eqs. (3.7), (3.9) but not by those in eqs. (3.6), (3.8). This is consistent with the result in the CFT side that local operators corresponding to (3.7), (3.9) have conformal weight (1, 1) and the integrated operators are invariant under the conformal transformations while those corresponding to (3.6), (3.8) are only invariant under the translations.

The Killing vectors for the isometry  $SO(3) \sim SU(2) \times SU(2)$  of  $S^3$  are  $\xi^i = \lambda^i_j x^j$ , where  $\lambda_{ij} = -\lambda_{ji}$ . The net effect of the transformation (5.1) in this case is that the components  $T_{ij}$  in  $G_2$  are changed according to the  $SO(3)$  transformation. Since  $T_{ij}$  belong to a representation  $(\mathbf{3}, \mathbf{1})$  or  $(\mathbf{1}, \mathbf{3})$ ,  $SU(2) \times SU(2)$  symmetry is broken to  $1 \times SU(2)$  or  $SU(2) \times 1$ .

Finally, let us obtain supersymmetries preserved by the perturbations. We consider the case in which only the  $m_1$  terms in eqs. (3.2), (3.3) are present. These terms represent the highest weight state of  $(\mathbf{3}, \mathbf{1})$  or  $(\mathbf{1}, \mathbf{3})$  of  $SU(2) \times SU(2)$ , and corresponds to the CFT operator in eqs. (4.15), (4.19). Therefore, we should recover unbroken supersymmetries in eqs. (4.16), (4.20).

We shall obtain transformation parameter  $\epsilon$  for which the supertransformations of the fermionic fields (2.1) vanish to the first order in  $G_2$ . First, the condition  $\delta\chi_{+a\dot{\alpha}} = 0$  require

$$\delta\chi_{+a\dot{\alpha}} = \frac{1}{R} S_a \hat{\gamma}^{\hat{r}} \bar{\gamma}_{2D} \epsilon_{-\dot{\alpha}} - \frac{1}{4} G_{ij\alpha\dot{\alpha}} \hat{\gamma}^{ij} \epsilon_+^\beta (\gamma_a)_\beta{}^\alpha = 0. \quad (5.4)$$

Multiplying  $S^a$  to this equation we can express  $\epsilon_-$  in terms of  $\epsilon_+$  as

$$\epsilon_{-\dot{\alpha}} = \frac{1}{4} R G_{ij\alpha\dot{\alpha}} \hat{\gamma}^{\hat{r}} \hat{\gamma}^{ij} \bar{\gamma}_{2D} \epsilon_+^\beta (\gamma_5)_\beta{}^\alpha. \quad (5.5)$$

We see that  $\epsilon_-$  is non-vanishing and of order  $G_2$ . The condition  $\delta\psi_{+M\alpha} = 0$  gives the same condition as the unperturbed background (2.11) to the first order in  $G_2$ , whose solution is eq. (2.13). The condition  $\delta\chi_{-a\dot{\alpha}} = 0$  is also automatically satisfied. Substituting eq. (5.5) into  $\delta\psi_{-M\dot{\alpha}}$  and  $\delta\chi_{+a\dot{\alpha}}$  and using the differential equation on  $\epsilon_+$  (2.12) we obtain

$$\delta\psi_{-\mu}{}^{\dot{\alpha}} = \frac{1}{8} G_{kl}^{\alpha\dot{\alpha}} \hat{\gamma}_\mu \bar{\gamma}_{4D} [\hat{\gamma}^{\hat{r}}, \hat{\gamma}^{kl}] \left( \delta_\alpha^\beta - \bar{\gamma}_{4D} (\gamma_5)_\alpha{}^\beta \right) \epsilon_{+\beta},$$

$$\begin{aligned}
\delta\psi_{-i}^{\dot{\alpha}} &= \frac{1}{8}G_{kl}^{\alpha\dot{\alpha}} \left( \hat{\gamma}_i \hat{\gamma}^{kl} - \hat{\gamma}^{\hat{r}} \hat{\gamma}^{kl} \hat{\gamma}^{\hat{r}} \hat{\gamma}_i \right) \left( \delta_{\alpha}^{\beta} - \bar{\gamma}_{4D}(\gamma_5)_{\alpha}^{\beta} \right) \epsilon_{+\beta} \\
&\quad - \frac{R}{4r^2} \partial_i(r^2 G_{kl}^{\alpha\dot{\alpha}}) \hat{\gamma}^{\hat{r}} \hat{\gamma}^{kl} \bar{\gamma}_{4D}(\gamma_5)_{\alpha}^{\beta} \epsilon_{+\beta} - G_{ik}^{\alpha\dot{\alpha}} \hat{\gamma}^k \epsilon_{+\alpha}, \\
\delta\chi_{+a}^{\dot{\alpha}} &= \frac{1}{4} G_{ij}^{\alpha\dot{\alpha}} \hat{\gamma}^{ij} (\gamma_a)_{\alpha}^{\beta} \epsilon_{+\beta} \quad (a = 1, 2, 3, 4).
\end{aligned} \tag{5.6}$$

Supersymmetry parameters for which these transformations vanish correspond to unbroken supersymmetries. We examine the conditions (5.6) for each of the perturbations (3.6)–(3.9).

First, let us consider the perturbations  $G_2^{(+)}$  in eq. (3.6). In this case  $G_{ij}^{(+)}$  is constant and self-dual. The self-duality implies

$$G_{kl}^{(+)} \hat{\gamma}^{kl} \psi = 0 \tag{5.7}$$

when  $\bar{\gamma}_{4D}\psi = \psi$ . We first consider  $\delta\chi_+$ . When  $\epsilon_+$  satisfies  $\gamma_5\epsilon_+ = -\epsilon_+$  or  $\bar{\gamma}_{4D}\epsilon_+ = \epsilon_+$ , it vanishes because of  $G_2^{(+)}\gamma_5 = -G_2^{(+)}$  or the identity (5.7). When  $\bar{\gamma}_{4D}\epsilon_+ = -\epsilon_+$  and  $\gamma_5\epsilon_+ = \epsilon_+$ , it is proportional to

$$(m_1)^{\alpha\dot{\alpha}} \hat{\gamma}^{1\bar{2}} (\gamma_a)_{\alpha}^{\beta} \epsilon_{+\beta}. \tag{5.8}$$

By substituting  $\epsilon_+^{(-+)}$  in eq. (2.13) into this equation and using the explicit form of gamma matrices  $\hat{\gamma}^i$  and  $\gamma^a$  in Appendix we find that it does not vanish. Therefore,  $\delta\chi_+ = 0$  requires  $\epsilon_{1A\dot{A}}^{(++)} = 0$ . As for  $\delta\psi_{+\mu}$ , it vanishes when  $\bar{\gamma}_{4D}\epsilon_+ = -\epsilon_+$  or  $\gamma_5\epsilon_+ = \epsilon_+$  as seen from eq. (5.6). When  $\bar{\gamma}_{4D}\epsilon_+ = \epsilon_+$  and  $\gamma_5\epsilon_+ = -\epsilon_+$ ,  $\delta\psi_{+\mu}$  is proportional to

$$(m_1)^{\alpha\dot{\alpha}} \left( z^1 \hat{\gamma}^{\bar{2}} - \bar{z}^2 \hat{\gamma}^1 \right) \epsilon_{+\alpha}. \tag{5.9}$$

Substituting  $\epsilon_+^{(+-)}$  in eq. (2.13) we find that  $\delta\psi_{+\mu} = 0$  requires  $\epsilon_{11'\dot{2}'}^{(--)} = 0$  when  $(m_1)^{i'\dot{\alpha}} = 0$ , and  $\epsilon_{11'\dot{1}'}^{(--)} = 0$  when  $(m_1)^{2'\dot{\alpha}} = 0$ . Here, we have used two-component spinor index  $\dot{A}' = \dot{1}', \dot{2}'$  instead of the four-component one  $\alpha = 1, 2, 3, 4$  for the first index of  $(m_1)^{\alpha\dot{\alpha}}$  (See Appendix.). Finally, let us consider  $\delta\psi_{-i}$ . When  $\gamma_5\epsilon_+ = \epsilon_+$ , it automatically vanishes. When  $\gamma_5\epsilon_+ = -\epsilon_+$ , the condition  $\delta\psi_{-i} = 0$  is shown to be equivalent to  $G_{ij}^{(+)\alpha\dot{\alpha}} \hat{\gamma}^j \epsilon_{+\alpha} = 0$ , i.e.,

$$(m_1)^{\alpha\dot{\alpha}} \hat{\gamma}^1 \epsilon_{+\alpha} = 0, \quad (m_1)^{\alpha\dot{\alpha}} \hat{\gamma}^{\bar{2}} \epsilon_{+\alpha} = 0. \tag{5.10}$$

Substituting  $\epsilon_+^{(\pm-)}$  in eq. (2.13) we find that  $\delta\psi_{+i} = 0$  requires  $\epsilon_{01'\dot{2}'}^{(--)} = 0$ ,  $\epsilon_{1A'\dot{2}'}^{(--)} = 0$  when  $(m_1)^{i'\dot{\alpha}} = 0$ , and  $\epsilon_{01'\dot{1}'}^{(--)} = 0$ ,  $\epsilon_{1A'\dot{1}'}^{(--)} = 0$  when  $(m_1)^{2'\dot{\alpha}} = 0$ . To summarize, the unbroken supersymmetries by the perturbation, for which all of the transformations

in eq. (5.6) vanish are given as follows. When only  $G_2^{(+)\dot{A}'\dot{\alpha}}$  is non-vanishing for a given  $\dot{A}'$ , the transformation parameters of the unbroken supersymmetries are

$$\epsilon_{0B\dot{B}}^{(++)}, \quad \epsilon_{02'\dot{A}'}^{(--)}, \quad \epsilon_{0B'\dot{B}'}^{(--)}, \quad \epsilon_{1B'\dot{B}'}^{(--)}, \quad (5.11)$$

where  $B, B', \dot{B}$  are arbitrary and  $\dot{B}' \neq \dot{A}'$ . Remembering the correspondence between the supersymmetry parameters and the supercharges discussed at the end of sect. 2 we see that this result is in complete agreement with the unbroken supersymmetries in the CFT (4.16).

Next, let us consider the perturbations  $G_2^{(+)}$  in eq. (3.9). In this case  $G_2^{(+)}$  satisfies eq. (5.3). We easily find that  $\delta\psi_{-\mu} = 0$  for an arbitrary  $\epsilon_+$  since

$$G_{kl}^{(+)}[\hat{\gamma}^{\hat{r}}, \hat{\gamma}^{kl}] = \frac{4}{R} x^k G_{kl}^{(+)} \hat{\gamma}^l \quad (5.12)$$

vanishes because of eq. (5.3). As for  $\delta\psi_{-i}$  we need a formula for derivative of  $G_{kl}^{(+)}$ . From the explicit form of  $G_{kl}^{(+)}$  in eq. (3.9) we find

$$\partial_i(r^2 G_{kl}^{(+)}) = -x^k G_{il}^{(+)} + \left( \delta_{ik} - \frac{x^i x^k}{r^2} \right) x^j (*_4 G^{(+)})_{jl} - (k \leftrightarrow l). \quad (5.13)$$

Substituting this into  $\delta\psi_{-i}$  in eq. (5.6) and doing some algebra we find that  $\delta\psi_{-i} = 0$  for an arbitrary  $\epsilon_+$ . Non-trivial conditions on  $\epsilon_+$  only come from  $\delta\chi_+ = 0$ . When  $\gamma_5 \epsilon_+ = -\epsilon_+$ ,  $\delta\chi_+$  automatically vanishes. When  $\gamma_5 \epsilon_+ = \epsilon_+$ , we find that  $\delta\chi_+$  vanishes only if  $G_{ij}^{(+)} \hat{\gamma}^{ij} \epsilon_+ = 0$ . Substituting  $\epsilon_+^{(\pm)}$  in eq. (2.13) into this equation and using the explicit form

$$G_{ij}^{(+)} \hat{\gamma}^{ij} = \frac{1}{2} \alpha r^{-2} m_1 (1 - \gamma_5) \left[ \frac{1}{2} \hat{\gamma}^{12} - \frac{z^1 z^2}{r^2} (\hat{\gamma}^{1\bar{1}} - \hat{\gamma}^{2\bar{2}}) - \frac{(z^2)^2}{r^2} \hat{\gamma}^{1\bar{2}} - \frac{(z^1)^2}{r^2} \hat{\gamma}^{\bar{1}2} \right] \quad (5.14)$$

we find that  $\delta\chi_+ = 0$  requires  $\epsilon_{02\dot{A}}^{(++)} = 0$ ,  $\epsilon_{12\dot{A}}^{(++)} = 0$ . Thus, the unbroken supersymmetries for the perturbation (3.9) are

$$\epsilon_{01\dot{B}}^{(++)}, \quad \epsilon_{11\dot{B}}^{(++)}, \quad \epsilon_{0B'\dot{B}'}^{(--)}, \quad \epsilon_{1B'\dot{B}'}^{(--)}, \quad (5.15)$$

where  $B', \dot{B}, \dot{B}'$  are arbitrary. This result is again in complete agreement with the unbroken supersymmetries in the CFT (4.20).

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## Appendix: SO(4) and SO(5) gamma matrices

Our representation of the SO(4) gamma matrices is

$$\begin{aligned}\hat{\gamma}^{\hat{2}} &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, & \hat{\gamma}^{\hat{3}} &= \begin{pmatrix} 0 & -\sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}, \\ \hat{\gamma}^{\hat{4}} &= \begin{pmatrix} 0 & -\sigma^3 \\ -\sigma^3 & 0 \end{pmatrix}, & \hat{\gamma}^{\hat{5}} &= \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix},\end{aligned}\tag{A.1}$$

where  $\sigma^1, \sigma^2, \sigma^3$  are the  $2 \times 2$  Pauli matrices. The chirality matrix in eq. (2.8) then becomes

$$\bar{\gamma}_{4D} = \hat{\gamma}^{\hat{2}}\hat{\gamma}^{\hat{3}}\hat{\gamma}^{\hat{4}}\hat{\gamma}^{\hat{5}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.\tag{A.2}$$

An SO(4) spinor  $\psi$  has components

$$\psi = \begin{pmatrix} \psi_A \\ \psi_{A'} \end{pmatrix} \quad (A = 1, 2; A' = 1', 2').\tag{A.3}$$

Our representation of the SO(5) gamma matrices  $(\gamma^a)_{\alpha}{}^{\beta}$  is

$$\begin{aligned}\gamma^1 &= \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, & \gamma^2 &= \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}, & \gamma^3 &= \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \\ \gamma^4 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, & \gamma^5 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.\end{aligned}\tag{A.4}$$

A four-component SO(5) spinor  $\psi_{\alpha}$  ( $\alpha = 1, 2, 3, 4$ ) is decomposed into two two-component spinors

$$\psi = \begin{pmatrix} \psi_{\dot{A}} \\ \psi_{\dot{A}'} \end{pmatrix} \quad (\dot{A} = \dot{1}, \dot{2}; \dot{A}' = \dot{1}', \dot{2}').\tag{A.5}$$

The  $SO(5)$  charge conjugation matrix  $\Omega^{\alpha\beta}$  satisfies

$$\Omega\gamma^a\Omega^{-1} = (\gamma^a)^T \quad (A.6)$$

and is given by

$$\Omega = \begin{pmatrix} \epsilon^{\dot{A}\dot{B}} & 0 \\ 0 & \epsilon^{\dot{A}'\dot{B}'} \end{pmatrix}, \quad (A.7)$$

where antisymmetric  $\epsilon^{\dot{A}\dot{B}}$  and  $\epsilon^{\dot{A}'\dot{B}'}$  are chosen as  $\epsilon^{\dot{1}\dot{2}} = +1 = \epsilon^{\dot{1}'\dot{2}'}$ . We also use antisymmetric  $\epsilon_{\dot{A}\dot{B}}$  and  $\epsilon_{\dot{A}'\dot{B}'}$  with  $\epsilon_{\dot{1}\dot{2}} = +1 = \epsilon_{\dot{1}'\dot{2}'}$ .

In eq. (2.13) the supertransformation parameter  $\epsilon_+$  is decomposed according to eigenvalues of  $\bar{\gamma}_{4D}$  and  $\gamma_5$ . Each of the components has indices as

$$\epsilon_{+A\dot{A}}^{(++)}, \quad \epsilon_{+A\dot{A}'}^{(+-)}, \quad \epsilon_{+A'\dot{A}}^{(-+)}, \quad \epsilon_{+A'\dot{A}'}^{(--)}. \quad (A.8)$$

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