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Phenomenological Description of Temperature and Frequency Dependence of Surface Resistance of High-T_c Superconductors by Improved Three-Fluid Model

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SUMMARY A calculation method by the improved three-fluid model is shown to describe phenomenologically temperature and frequency dependence of surface resistance R_s for high-T_c superconductors. It is verified that this model is useful to describe temperature dependence of R_s for such high-T_c superconducting films as Y-Ba-Cu-O (YBCO), Eu-Ba-Cu-O, and Tl-Ba-Ca-Cu-O films. For the frequency dependence of R_s of a YBCO bulk, furthermore, the measured results which have not depended on f^2 in the frequency range 10–25 GHz, can be described successfully by this model. Finally, a figure of merit is proposed to evaluate material quality for high-T_c superconductors from the values of electron densities and momentum relaxation time determined by the present model.

key words: *high-T_c superconductor, improved three-fluid model, surface resistance, material quality, frequency dependence*

1. Introduction

To design such transmission lines as microstrip and coplanar ones fabricated from high-T_c superconducting films, it is necessary to establish an appropriate model, by which we can explain microwave characteristics of these films. It is well known that the two-fluid model is useful to explain phenomenologically the microwave behavior of low-T_c metallic superconductors having lower values of residual surface resistance R_{sres}. However, this model is not valid for high-T_c superconductors having the greater values of R_{sres}. To overcome the disadvantage, recently, the authors have proposed an improved three-fluid model [1], where the concept of nonpairing residual normal electron density n_{res} which is independent of temperature is introduced and momentum relaxation time τ is assumed to depend on temperature both in the superconducting and normal states, on the basis of the experimental results for temperature dependence of τ presented by Romero et al. [2]. For a Y-Ba-Cu-O (YBCO) bulk, the present model was successfully introduced to explain the measured results of temperature dependence of surface impedance $\dot{Z}_s = R_s + jX_s$, where R_s is the surface resistance and X_s is the surface reactance, and of complex conductivity $\dot{\sigma} = \sigma_1 - j\sigma_2$ [1]. It was verified that the present model is valid also for other some YBCO bulk materials [3].

In this paper, the validity of the present model is discussed also for such high-T_c superconducting films as YBCO, Eu-Ba-Cu-O (EBCO), and Tl-Ba-Ca-Cu-O (TBCCO) films by comparing the calculation for R_s with measurements. Then, the frequency dependence of R_s for a YBCO bulk is evaluated by the present model. Furthermore, a figure of merit is proposed to evaluate material quality for high-T_c superconductors from the values of electron densities and τ determined by the present model.

2. Calculation Method of R_s

In the improved three-fluid model [1], R_s is expressed as

$$R_s = \sqrt{\frac{\omega\mu(\sqrt{\sigma_1^2+\sigma_2^2}-\sigma_2)}{2(\sigma_1^2+\sigma_2^2)}} \tag{1}$$

where

$$\sigma_1 = \frac{n_n e^2 \tau}{m(\omega^2\tau^2+1)} + \frac{n_{res} e^2 \tau}{m(\omega^2\tau^2+1)} \tag{2}$$

$$\sigma_2 = \frac{n_s e^2}{m\omega} + \frac{n_n e^2 \tau^2 \omega}{m(\omega^2\tau^2+1)} + \frac{n_{res} e^2 \tau^2 \omega}{m(\omega^2\tau^2+1)} \tag{3}$$

$$n_s = n\left[1-\left(\frac{T}{T_c}\right)^4\right], \quad n_n = n\left(\frac{T}{T_c}\right)^4 \tag{4}$$

$$n_t = n_s + n_n + n_{res} = n + n_{res} \tag{5}$$

$$\tau = \tau_c\left(\frac{T}{T_c}\right)^\alpha + \tau_0\left\{1-\left(\frac{T}{T_c}\right)^\alpha\right\} \tag{6}$$

and $\omega = 2\pi f$ is the angular frequency, f is the frequency, $\mu = \mu_0 = 4\pi\times10^{-7}$ H/m is the permeability in free space, T is the temperature, T_c is the critical temperature, n_n is the normal electron density, n_s is the super electron density, n_t is the total electron density which is independent of T, $e = -1.6022\times10^{-19}$ C is the electron charge, $m = 9.1096\times10^{-31}$ kg is the electron mass, τ_c is the value of τ at T_c K, τ_0 ($>\tau_c$) is one at 0 K, and α is a constant to be determined from fitting of the measured results, as described later. Also, the values of n_{res}, n, τ_c, and τ_0 are given by

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$$n_{res}=\frac{\sigma_{10}m\omega\{(\sigma_{1c}-\sigma_{10}A)^2+\sigma_{20}^2A^2\}}{e^2\sigma_{20}A(\sigma_{1c}-\sigma_{10}A)} \tag{7}$$

$$n=n_s+n_n=\frac{m\omega}{e^2(\sigma_{1c}-\sigma_{10}A)}\left[\sigma_{1c}\sigma_{20}-\frac{\sigma_{10}\{(\sigma_{1c}-\sigma_{10}A)^2+\sigma_{20}^2A^2\}}{\sigma_{20}A}\right] \tag{8}$$

$$\tau_c=\frac{\sigma_{1c}-\sigma_{10}A}{\sigma_{20}\omega} \tag{9}$$

$$\tau_0=\frac{\sigma_{1c}-\sigma_{10}A}{\sigma_{20}\omega A} \tag{10}$$

where

$$\sigma_{10}=\frac{2\omega\mu R_{s0}X_{s0}}{(R_{s0}^2+X_{s0}^2)^2} \tag{11}$$

$$\sigma_{20}=\frac{\omega\mu(X_{s0}^2-R_{s0}^2)}{(R_{s0}^2+X_{s0}^2)^2} \tag{12}$$

$$\sigma_{1c}=\frac{\omega\mu}{2R_{sn}^2} \tag{13}$$

$$A=\frac{\tau_c}{\tau_0} \tag{14}$$

In the above, R_{s0} and X_{s0} are R_s and X_s at $T=0$ K, respectively, R_{sn} is R_s at $T=T_c$ K.

The temperature dependence of R_s can be calculated from Eqs. (1)-(6), if the values of fitting parameter A and α are determined in accordance with a flow chart shown in Fig. 1; that is, the process of determination is as follows:

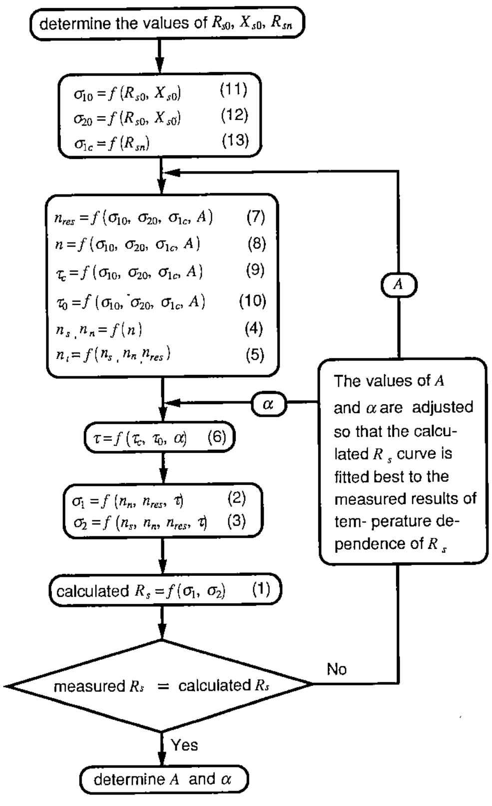

Fig. 1 A flow chart to determine the fitting parameter A and α.

1. For bulks, actual value of R_{sn} is obtained from the measured results of R_s at $T=T_c$ K, and ones of R_{s0} and X_{s0} are determined by extrapolating the measured results of R_s and X_s until $T=0$ K, respectively. For films, measured results of X_s are not obtained because the measurement is very difficult. The value of X_{s0} is determined from the relation $X_{s0}=\omega\mu\lambda_L$ using the London's penetration depth λ_L.
2. The values of σ_{10}, σ_{20}, and σ_{1c} are calculated from Eqs. (11)-(13) by using the R_{s0}, X_{s0}, and R_{sn} values.
3. The values of n_{res}, n, τ_c, τ_0, n_s, n_n, and n_t are calculated by substituting the σ_{10}, σ_{20}, and σ_{1c} values and arbitrary value of A into Eqs. (7)-(10), (4), and (5).
4. The temperature dependence of τ is calculated by substituting the values of τ_c, τ_0, and arbitrary value of α into Eq. (6).
5. The temperature dependence of σ_1 and σ_2 are calculated from Eqs. (2) and (3) by using the values of n_s, n_n, n_{res}, and τ.
6. The temperature dependence of R_s is calculated from Eq. (1) by using the values of σ_1 and σ_2.
7. If these R_s values do not agree with measured ones, the values of A and α are adjusted by a cut-and-try method so that the calculated ones are fitted best to the measured ones.

Furthermore, the frequency dependence of R_s is calculated from Eqs. (1)-(3) on the assumption of the values of n_s, n_n, n_{res}, and τ determined above are independent of f.

3. Temperature Dependence of R_s

For four YBCO bulks, it was verified [1], [3] that the present model is useful to explain the measured results of temperature dependence of R_s. The parameters using calculation of R_s are shown in Table 1. We discuss the validity of the present model for such high-T_c superconducting films as YBCO, EBCO, and TBCCO films. Figure 2(a) shows the measured results for the YBCO film at 18.9 GHz presented by N. Klein et al. [4]. By using the process described in Sect. 2, $R_{sn}=0.80\ \Omega$, $R_{s0}=0.43$ mΩ, $X_{s0}=15.0$ mΩ which is correspond to $\lambda_L=0.10\ \mu$m, $n_{res}=6.59\times10^{26}$ m^{-3}, $n=2.17\times10^{27}$ m^{-3}, $n_t=2.83\times10^{27}$ m^{-3}, $\tau_c=1.46\times10^{-15}$ s, $\tau_0=3.25\times10^{-11}$ s, $A=4.5\times10^{-5}$, and $\alpha=0.30$ are determined. These values are summarized in Table 2. Figures 2(b) and (c) show the temperature dependence of $1/\tau$, n_{res}, n_s, n_n, and n_t, respectively. The temperature dependence of R_s is calculated from Eqs. (1)-(3) by using the values in Figs. 2(b) and (c). The results are indicated by "3-fluid(τ)" in Fig. 2(a). As a result, these results fit well with the measured ones. In comparison with these, calculated results by the two-, three- [5], and "enhanced" two-fluid models proposed by Vendik [6] are also indicated by "2-fluid,"

Table 1 Parameters used to calculate R_s by the improved three-fluid model for the bulk materials.

	Tc(K)	f(GHz)	Rsn(Ω)	Rs0(mΩ)	Xs0(mΩ)	nres (m^{-3})	nt (m^{-3})	τ0 (s)	τc (s)	α	nres / nt	τ0 / τc
YBCO1 bulk	92	21.4	0.92	34.0	850	5.94×10^{23}	1.12×10^{24}	4.85×10^{-11}	3.15×10^{-12}	0.50	0.528	1.54×10^{1}
YBCO2 bulk	92	21.4	0.48	12.5	360	2.19×10^{24}	6.29×10^{24}	3.63×10^{-11}	2.07×10^{-12}	1.00	0.349	1.75×10^{1}
YBCO3 bulk	92	21.4	0.61	23.5	133	3.26×10^{25}	6.38×10^{25}	5.05×10^{-12}	1.26×10^{-13}	1.10	0.511	4.00×10^{1}
YBCO4 bulk	95	9.70	0.40	2.20	152	4.25×10^{23}	7.31×10^{24}	2.08×10^{-11}	1.19×10^{-12}	1.00	0.058	1.75×10^{1}

Table 2 Parameters used to calculate R_s by the improved three-fluid model for the films.

	Tc(K)	f(GHz)	Rsn(Ω)	Rs0(mΩ)	λL(μm)	Xs0(mΩ)	nres (m^{-3})	nt (m^{-3})	τ0 (s)	τc (s)	α	nres/nt	τ0/τc
YBCO film	90.0	18.9	0.80	0.43	0.10	15.0	6.59×10^{26}	2.83×10^{27}	3.25×10^{-11}	1.46×10^{-15}	0.30	0.233	2.22×10^{4}
EBCO film	90.0	9.69	0.80	0.12	0.13	10.0	3.48×10^{26}	1.66×10^{27}	1.42×10^{-10}	1.28×10^{-15}	0.13	0.210	1.11×10^{5}
TBCCO film	107.6	10.0	0.52	0.011	0.02	1.50	1.07×10^{28}	7.84×10^{28}	1.47×10^{-10}	6.61×10^{-17}	0.13	0.137	2.22×10^{6}

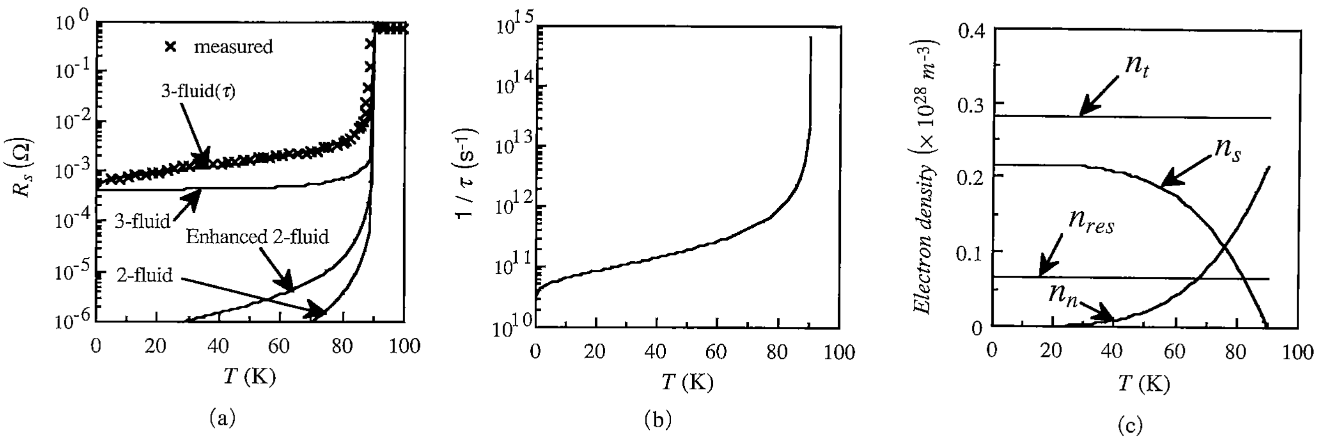

Fig. 2 Calculated results for a YBCO film.
(a) Measured results of R_s presented by Klein et al. [4] and calculated ones by improved three-, two-, three-, and enhanced two-fluid models.
(b) Calculated results of temperature dependence of $1/\tau$ by improved three-fluid model.
(c) Calculated results of temperature dependence of electron densities by improved three-fluid model.

"3-fluid," and "Enhanced 2-fluid," respectively. These results do not agree well with the measured ones. Recently, Kaparkov et al. [7] have proposed a new model where the concept of residual resistance has been introduced into the "enhanced" two-fluid model. However, we have not succeeded in calculating R_s, yet, as the above model has many fitting parameters. Figure 3 shows results for the EBCO film measured by authors and calculated ones by the presented model. The agreement of R_s between the calculation and measurement is good also in this case. On the other hand, calculated ones for other models do not agree well with the measured ones. Similarly, for the TBCCO film measured by W. L. Holstein et al. [8], the present model was successfully introduced as shown in Fig. 4. As a result, we can described phenomenologically that R_s decreases with decrease of T because n_n decreases with decrease of T, n_s increases with decrease of T, and τ increases with decrease of T. It is verified that this model is useful to describe temperature dependence of R_s for such as YBCO, EBCO, and TBCCO films as well as YBCO bulks.

4. Frequency Dependence of R_s

In this section, the frequency dependence of R_s is evaluated phenomenologically for the YBCO bulk (YBCO4 in Table 1) by the improved model. We have

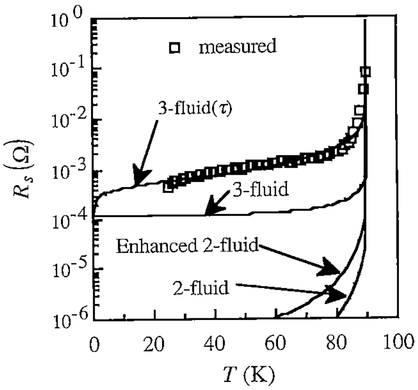

Fig. 3 Measured results of R_s for a EBCO film and calculated ones by improved three-, two-, three-, and enhanced two-fluid models.

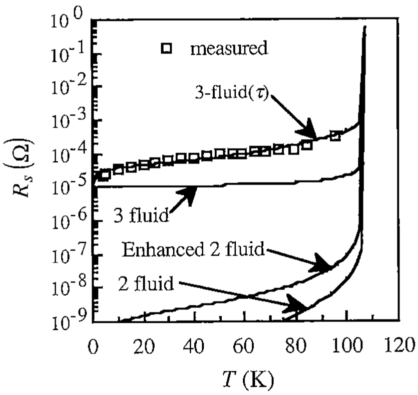

Fig. 4 Measured results of R_s for a TBCCO film presented by Holstein et al. [8] and calculated ones by improved three-, two-, three-, and enhanced two-fluid models.

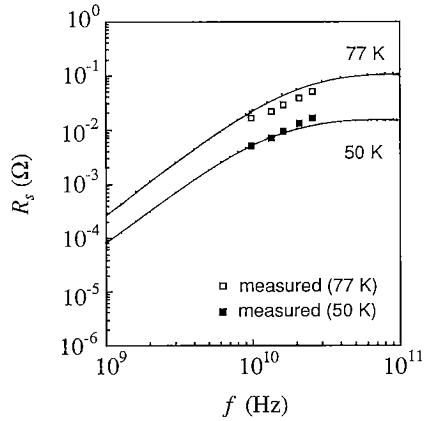

Fig. 5 Measured results of frequency dependence of R_s for a YBCO bulk (YBCO4 in Table 1) and calculated ones by the improved three-fluid model.

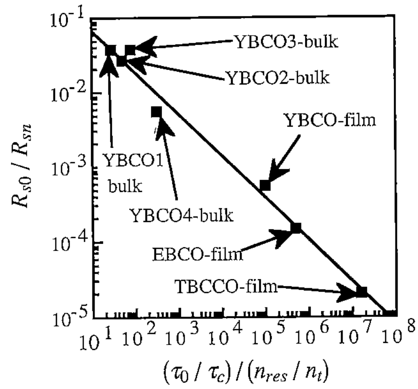

Fig. 6 Calculated results of R_{s0}/R_{sn} versus $(\tau_0/\tau_c)/(n_{res}/n_t)$ for seven materials.

measured the temperature dependence of R_s for the YBCO bulk at 9.7, 13.4, 16.2, 20.7, and 25.5 GHz [9]. These results at 50 K and 77 K are shown in Fig. 5 as a function of f. The results at 50 K are proportional to $f^{1.34}$, and the ones at 77 K are proportional to $f^{1.09}$. Also, the calculated results by the improved model are shown in this figure, which are obtained from Eqs. (1)-(6) by using the values of n_{res}, n_t, τ_c, τ_0, and α in Table 1. As a result, these results agree with measured ones. Therefore, it is verified that the improved three-fluid model is useful to explain the frequency dependence of R_s for the YBCO bulk which does not depend on f^2.

5. Evaluation of Material Quality

For application, it is necessary to evaluate material quality for high-T_c superconductors. In this section, a figure of merit is proposed using the values of electron densities and τ values derived from the present model. Generally, we think that the material having the lower value of R_{s0} is good one. However, it is difficult to evaluate material quality by R_{s0} which are measured in different frequency, because the value of R_{s0} depend on frequency. Therefore, we evaluate the quality by the normalized value R_{s0}/R_{sn}. The value of n_{res}/n_t has been proposed before as figure of merit of high-T_c superconductor bulk materials [5]. That is, it was assumed that the value of R_{s0}/R_{sn} decreases with decreases of n_{res}/n_t. In addition to n_{res}/n_t, in this paper, the normalized value τ_0/τ_c is introduced as another figure of merit. That is, it is assumed that R_{s0}/R_{sn} decreases with decrease of n_{res}/n_t and with increase of τ_0/τ_c. Thus, if this assumption is right, it is expected that R_{s0}/R_{sn} decreases with increase of $(\tau_0/\tau_c)/(n_{res}/n_t)$. Figure 6 shows the calculated results of R_{s0}/R_{sn} versus $(\tau_0/\tau_c)/(n_{res}/n_t)$ for the four bulk materials [1], [3] in Table 1 and three films in Table 2. As a results, we obtain the following relation

$$R_{s0}/R_{sn}=0.25[(\tau_0/\tau_c)/(n_{res}/n_t)]^{-0.56} \quad (15)$$

As is expected, it is verified that R_{s0}/R_{sn} decreases with increase of $(\tau_0/\tau_c)/(n_{res}/n_t)$. Therefore, the material quality becomes better with increase of $(\tau_0/\tau_c)/(n_{res}/n_t)$. Thus, we can use this value as figure of merit of high-T_c superconductor materials.

6. Conclusion

It was verified that the improved three-fluid model is useful to calculate the temperature dependence of R_s for the YBCO, EBCO, and TBCCO films as well as YBCO bulks. Furthermore, the frequency dependence of R_s for the YBCO bulk was evaluated by this model. Therefore, we expect that this model is used to describe phenomenologically the microwave characteristics of high-T_c superconductors. Also, the value of $(\tau_0/\tau_c)/(n_{res}/n_t)$ is useful as figure of merit to evaluate material quality for high-T_c superconductors.

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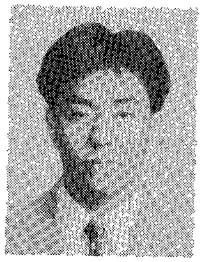

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