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Long life stau in the minimal supersymmetric standard model

Toshifumi Jittoh,* Joe Sato,† Takashi Shimomura,‡ and Masato Yamanaka§

Department of Physics, Saitama University, Shimo-okubo, Sakura-ku, Saitama, 338-8570, Japan

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We study the stau lifetime in a scenario with the LSP taken to be a neutralino and the NLSP being a stau, based on the minimal supersymmetric Standard Model. The mass difference between the LSP and NLSP, δm, must satisfy $\delta m/m_{\tilde{\chi}} \sim$ a few % or less for coannihilation to occur, where $m_{\tilde{\chi}}$ is the neutralino mass. We calculate the stau lifetime from the decay modes $\tilde{\tau} \to \tilde{\chi}\tau$, $\tilde{\chi}\nu_\tau\pi$, and $\tilde{\chi}\nu_\tau\mu(e)\nu_{\mu(e)}$ and discuss its dependence on various parameters. We find that the lifetime is in the range 10^{-22}–10^{16} sec for $10^{-2} \leq \delta m \leq 10$ GeV. We also discuss the connection with lepton flavor violation if there is mixing between sleptons.

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I. INTRODUCTION

The existence of nonbarionic dark matter is now confirmed and its density has been quantitatively estimated [1,2]. However its identity is still unknown. One of the most prominent candidates is the weakly interacting massive particle (WIMP) [3–6].

As is well known, the supersymmetric extension of the Standard Model provides a stable exotic particle, the lightest supersymmetric particle (LSP), if R parity is conserved. Among LSP candidates, the neutralino LSP is the most suitable for nonbarionic dark matter since its nature fits that of the WIMP . Neutralinos are a linear combination of the supersymmetric partners of the U(1) and SU(2) gauge bosons (bino and wino) and the Higgs bosons (Higgsino). They have mass in a range from 100 GeV to several TeV and are electrically neutral. The lightest neutralino is stable if R parity is conserved.

Since the supersymmetric extension of the standard model is the most attractive theory, the nature of neutralino dark matter has been studied extensively [9]. In many scenarios of the supersymmetric model, the LSP neutralino consists mainly of the bino, the so-called binolike neutralino. In this case, naive calculations show that the predicted density in the current universe is too high and it is necessary to find a way to reduce it. One mechanism to suppress the density is coannihilation [10]. If the next lightest supersymmetric particle (NLSP) is nearly degenerate in mass with the LSP, the interaction of the LSP with the NLSP is important in calculating the LSP annihilation process. For coannihilation to occur tight degeneracy is necessary, since without coannihilation the LSP decouples from the thermal bath at $T \sim m/20$ [11], where m is the LSP mass. Therefore the mass difference δm must satisfy $\delta m/m <$ a few %, otherwise the NLSP decouples before coannihilation becomes dominant. Furthermore, if the degeneracy is much tighter, we would observe a line spectrum of photons from pair annihilation of dark matter [12,13], since the annihilation cross section of dark matter would be strongly enhanced due to the threshold correction.

A candidate for the NLSP is the stau or stop in many class of MSSM, and in this paper we study the lifetime of the staulike slepton having mass degenerate with the LSP neutralino. For the neutralino LSP to be dark matter, very tight degeneracy is required in mass between the NLSP and the LSP neutralino. In particular the heavier the LSP is, the tighter the degeneracy must be [9]. For such a degeneracy, the NLSP is expected to have a long lifetime due to phase space suppression [14,15].

An alternative scenario for a long-lived scalar particle is the gravitino LSP. Considerable work has been devoted to the long life NLSP in the context of the gravitino LSP. In this case, due to the small coupling between a superWIMP (including the gravitino) and the NLSP, the lifetime of the NLSP becomes very long [16–18]. To determine the most likely candidate for the LSP, we can accumulate and identify [19,20] the candidate for long-lived NLSPs and compare the nature of the particles including couplings.

An implication on the fundamental feature of quantum mechanics would be brought by a long life stau. As noted in Ref. [21], a Small-Q-value S-wave (SQS) decay can exhibit nonexponential decay. A small Q value naively implies a small mass difference. For a gravitino LSP, the decay must occur in a P wave since the gravitino has spin $3/2$. In contrast, in our case the decay can be S wave since all daughter particles have spin 0 or spin $1/2$. Therefore, to extract fundamental parameters from observations we must take special care in interpreting the results. This study offers the opportunity to examine a fundamental problem of quantum physics in collider physics. Hence it is worthwhile studying the proposition that the stau is the NLSP and the lightest neutralino is the LSP, and that their masses are tightly degenerate.

In Sec. II, we present the relevant Lagrangian and calculate the decay rate of the stau. In Sec. III, we study the

*Electronic address: jittoh@krishna.th.phy.saitama-u.ac.jp
†Electronic address: joe@phy.saitama-u.ac.jp
‡Electronic address: takashi@krishna.th.phy.saitama-u.ac.jp
§Electronic address: masa@krishna.th.phy.saitama-u.ac.jp

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parameter dependences of the lifetime. We then consider the connection with lepton flavor violation (LFV) in Sec. IV. Finally we summarize our results in Sec. V.

II. DECAY RATE

In this section, we calculate the decay rate of the stau NLSP. The stau is a mass eigenstate consisting of superpartneres of left- and right-handed taus,

$$\tilde{\tau} = \cos\theta_\tau \tilde{\tau}_L + \sin\theta_\tau e^{-i\gamma_\tau}\tilde{\tau}_R. \tag{1}$$

Here, θ_τ is the mixing angle between $\tilde{\tau}_L$ and $\tilde{\tau}_R$, and γ_τ is the CP violating phase. The decay mode is governed by the mass difference, $\delta m \equiv m_{\rm NLSP} - m_{\rm LSP}$, according to kinematics. That is, the lifetime of the stau depends strongly on δm.

We consider the following four decay modes, $\tilde{\tau} \to \tilde{\chi}\tau$, $\tilde{\tau} \to \tilde{\chi}\nu_\tau\pi$, $\tilde{\tau} \to \tilde{\chi}\mu\nu_\tau\nu_\mu$, and $\tilde{\tau} \to \tilde{\chi}e\nu_\tau\nu_e$ (see Fig. 1). Note that the NLSP can decay into other particles, for example, if $\delta m > 1.86$ GeV, a D meson can be produced in the stau decay but $\tilde{\tau} \to \tilde{\chi}\tau$ is dominant in this δm region since the D meson production process is suppressed by couplings and propagators. In the 3-body and 4-body decay processes, $\tilde{\tau} \to \tilde{\chi}\nu_\tau\pi$, $\tilde{\tau} \to \tilde{\chi}\mu\nu_\tau\nu_\mu$, and $\tilde{\tau} \to \tilde{\chi}e\nu_\tau\nu_e$, diagrams can be formulated with charginos as intermediate states, however, such processes are strongly suppressed by the chargino propagator and we can safely ignore them [14].

In this paper, we consider the small mass difference case and hence we can ignore the momentum in the W boson propagator. Thus the interaction Lagrangian is given by

$$\mathcal{L}_{\rm int} = \tilde{\tau}^*\bar{\tilde{\chi}}(g_L P_L + g_R P_R)\tau + \frac{G}{\sqrt{2}}\nu_\tau\gamma_\mu P_L \tau J^\mu + \frac{4G}{\sqrt{2}}(\bar{l}\gamma^\mu P_L \nu_l)(\bar{\nu}_\tau\gamma_\mu P_L\tau) + \text{h.c.} \tag{2}$$

The first term describes stau decay into a neutralino and a tau. Here, P_L and P_R are the projection operators and g_L and g_R are the coupling constants given by, for example, in the binolike neutralino case,

$$g_L = \frac{g}{\sqrt{2}\cos\theta_W}\sin\theta_W\cos\theta_\tau, \qquad g_R = \frac{\sqrt{2}g}{\cos\theta_W}\sin\theta_W\sin\theta_\tau e^{i\gamma_\tau}, \tag{3}$$

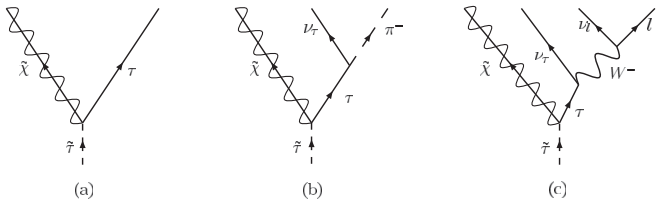


FIG. 1. Feynmann diagrams of stau decay: (a) $\tilde{\tau} \to \tilde{\chi}\tau$, (b) $\tilde{\tau} \to \tilde{\chi}\nu_\tau\pi$, (c) $\tilde{\tau} \to \tilde{\chi}l\nu_\tau\nu_l$.

where g is the weak coupling constant and θ_W is the Weinberg angle. The second and third terms describe tau decays into a pion and/or leptons, where G is the Fermi constant.

The detailed calculation of the stau decay rate is given in the appendix A. Here we show approximate formulae. In the region $\delta m > m_\tau$, 2-body decay [see Fig. 1(a)] is allowed kinematically and is dominant. The decay rate of the 2-body final state is approximately

$$\Gamma_{\text{2-body}} = \frac{1}{4\pi m_{\tilde{\chi}}}\sqrt{(\delta m)^2 - m_\tau^2}((g_L^2 + |g_R|^2)\delta m - 2\,\mathrm{Re}[g_L g_R]m_\tau), \tag{4}$$

where $m_{\tilde{\tau}}$, $m_{\tilde{\chi}}$, and m_τ are the masses of $\tilde{\tau}$, $\tilde{\chi}$, and τ, respectively.

The 2-body decay is forbidden kinematically for $\delta m < m_\tau$, and pion production [see Fig. 1(b)] is dominant if δm is larger than the pion mass m_π. The pion production 3-body decay rate has the approximate form

$$\Gamma_{\text{3-body}} = \frac{G^2 f_\pi^2 \cos^2\theta_c}{210(2\pi)^3 m_{\tilde{\chi}} m_\tau^4}((\delta m)^2 - m_\pi^2)^{5/2} \times [g_L^2\delta m(4(\delta m)^2 + 3m_\pi^2) - 2\,\mathrm{Re}[g_L g_R] \times m_\tau(4(\delta m)^2 + 3m_\pi^2) + 7|g_R|^2 m_\tau^2 \delta m]. \tag{5}$$

Here f_π is the pion decay constant and θ_c is the Cabbibo angle.

Incidentally, we note that a quark cannot appear alone in any physical processes, a point that was missed in Ref. [14]. Hence, u and d quarks appear only as mesons and the u, d production process is relevant only for $\delta m > m_\pi$.

Finally, when the mass difference is smaller than the pion mass, 4-body decay processes, $\tilde{\tau} \to \tilde{\chi}\mu\nu_\tau\nu_\mu$ and $\tilde{\tau} \to \tilde{\chi}e\nu_\tau\nu_e$, are significant [see Fig. 1(c)]. The approximate decay rate is calculated as

$$\Gamma_{\text{4-body}} = \frac{G^2}{945(2\pi)^5 m_{\tilde{\chi}} m_\tau^4}((\delta m)^2 - m_l^2)^{5/2} \times [2g_L^2(\delta m)^3(2(\delta m)^2 - 19m_l^2) - 4\,\mathrm{Re}[g_L g_R]m_\tau(\delta m)^2(2(\delta m)^2 - 19m_l^2) + 3|g_R|^2 m_\tau^2 \delta m(2(\delta m)^2 - 23m_l^2)]. \tag{6}$$

Here m_l is the charged lepton (e or μ) mass.

III. PARAMETER DEPENDENCE

In this section, we discuss the parameter dependence of the stau lifetime and the cosmological constraints for the parameters in the binolike LSP case. From Eqs. (4)–(6), we see that the stau lifetime depends on δm, θ_τ, $m_{\tilde{\chi}}$, and γ_τ.

A. $m_{\tilde{\chi}}$

First, we examine the neutralino mass dependence of the lifetime. We consider the cosmological constraints for dark matter to get the mass range of the LSP. It is well known that the dark matter relic density is reduced by the coannihilation process and hence the neutralino mass can be heavier than it would be without coannihilation. Accounting for the coannihilation process gives a neutralino mass range (the first Ref. of [9]) of

$$200\ \text{GeV} \leq m_{\tilde{\chi}} \leq 600\ \text{GeV}. \tag{7}$$

Here, we use CMSSM bound as a reference value of $m_{\tilde{\chi}}$, though we study the stau lifetime in a general MSSM framework, since it is not strongly dependent on $m_{\tilde{\chi}}$ as noted below. This is consistent with the cosmological constraint, $0.094 \leq \Omega_{DM}h^2 \leq 0.129$.

It is clear from Eqs. (4)–(6) that the stau lifetime is proportional to the neutralino mass and therefore in the mass range given in Eq. (7), the stau lifetime varies by a factor of 3. Taking this into account, we use only $m_{\tilde{\chi}} = 300$ GeV in our figures.

B. δm

Next, we consider the δm dependence of the stau lifetime. For coannihilation to occur, the mass difference must satisfy $\delta m/m_{\tilde{\chi}} \sim$ a few % or smaller [10]. Therefore, δm must be smaller than a few GeV. The δm dependence of the total stau lifetime and the partial lifetimes of each decay mode in this δm region is shown in Fig. 2. In Fig. 2, we set the values as follows: $m_{\tilde{\chi}} = 300$ GeV, $\theta_\tau = \pi/3$, and $\gamma_\tau = 0$.

From Fig. 2, we can see which mode is dominant for a certain δm. The figure shows that the lifetime increases drastically as δm becomes smaller than the tau mass. This is because taus are produced in the region $\delta m > m_\tau$, while in the region $\delta m < m_\tau$, taus cannot be produced and instead pions appear in the final state. At $\delta m = m_\pi$ the lifetime increases slightly. This is due to the fact that the dominant mode changes from 3-body to 4-body decay. In contrast, at $\delta m = m_\mu$ the lifetime does not increase much, even though above this mass, muons can be created. This is because at the pion mass, the muon production process is already kinematically suppressed and the electron production process governs the stau decay.

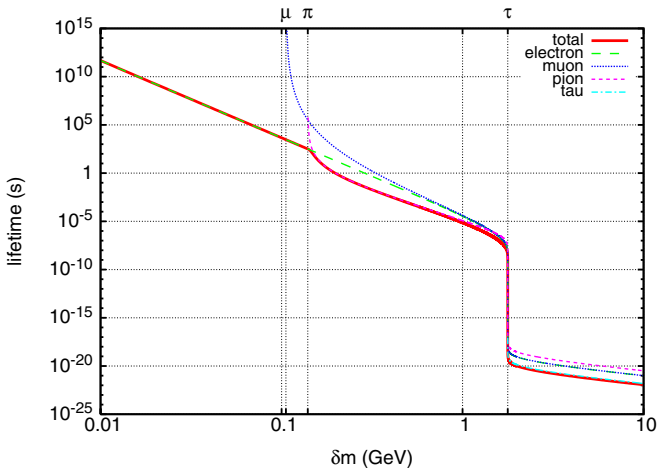


FIG. 2 (color online). Total lifetime and partial lifetimes of each decay mode as a function of δm. The lines label electron, muon, pion, and tau correspond to the processes $\tilde{\tau} \to \tilde{\chi} e \nu_\tau \nu_e$, $\tilde{\tau} \to \tilde{\chi} \mu \nu_\tau \nu_\mu$, $\tilde{\tau} \to \tilde{\chi} \nu_\tau \pi$, and $\tilde{\tau} \to \tilde{\chi} \tau$, respectively. Here we take $m_{\tilde{\chi}} = 300$ GeV, $\theta_\tau = \pi/3$, and $\gamma_\tau = 0$.

To understand the δm dependence of the lifetime quantitatively and intuitively we determine the power of δm in the decay rate, considering stau decay into neutralino and $n-1$ massless particles. The δm dependence of the decay rate is determined by the phase space and the squared amplitude [14].

First, we examine the δm dependence by considering the phase space. For 2-body decay, a phase space consideration gives

$$d\phi^{(2)} = \frac{d\Omega}{32\pi^2}\left(1 - \left(\frac{m_{\tilde{\chi}}}{m_{\tilde{\chi}} + \delta m}\right)^2\right) \propto \delta m. \tag{8}$$

By using a recursion relation between $d\phi^{(n)}$ and $d\phi^{(n-1)}$, the phase space of n-body decay renders the δm dependence as

$$d\phi^{(n)} \propto d\phi^{(n-1)} \int^{\delta m} d\mu (d\phi^{(2)}) \propto (\delta m)^{2(n-2)+1}. \tag{9}$$

Second, we consider the δm dependence from the squared amplitude. If all of the $n-1$ massless particles are fermions, the squared amplitude depends on δm as

$$\mathcal{M}^{(n)} \propto (\delta m)^{n-1}, \tag{10}$$

since it depends linearly on the massless fermion momentum. Thus, we obtain the dependence of the decay rate on δm for a final state of only fermions,

$$\Gamma^{(n)} \propto \mathcal{M}^{(n)} d\phi^{(n)} \propto (\delta m)^{3n-4}. \tag{11}$$

In contrast, if one pion(NG-boson) appears in the stau decay process, the δm dependence of the squared amplitude becomes

$$\mathcal{M}^{(n)} \propto (\delta m)^{n}. \tag{12}$$

This change in the decay process is due to the fact that the amplitude of the pion production is proportional to the pion momentum. Namely, the squared amplitude of the pion production process is proportional to the pion momentum squared. Thus the δm dependence of the process, in which one pion is involved, is

$$\Gamma^{(n)} \propto (\delta m)^{3(n-1)}. \tag{13}$$

In the massless limit of external line particles, the δm dependences of our results, calculated in appendix A, are

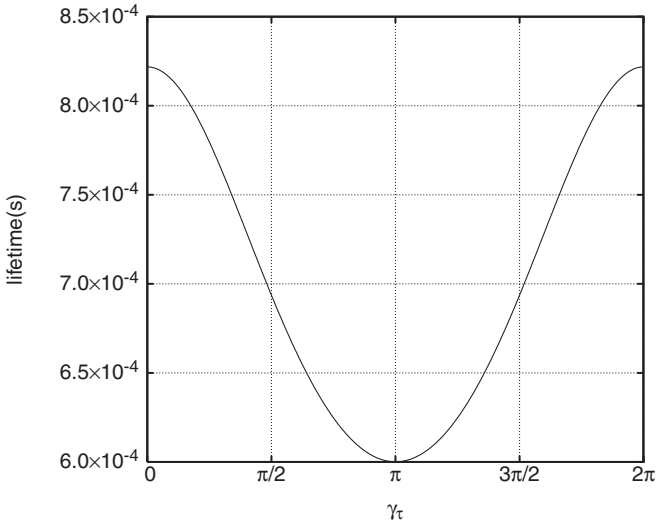


FIG. 3. *CP* violating phase and stau lifetime for $m_{\tilde{\chi}} = 300$ GeV, $\delta m = 0.5$ GeV, and $\theta_\tau = \pi/3$.

$$\Gamma_{2\text{-body}} \propto (\delta m)^2, \qquad \Gamma_{3\text{-body}} \propto (\delta m)^6, \qquad \Gamma_{4\text{-body}} \propto (\delta m)^8, \tag{14}$$

which are consistent with Eq. (11) and (13).

More precisely, by taking into account the masses of the produced particles, we get the complicated δm dependences

$$\begin{aligned} \Gamma_{2\text{-body}} &\propto (\delta m)((\delta m)^2 - m_\tau^2)^{1/2}, \\ \Gamma_{3\text{-body}} &\propto (\delta m)((\delta m)^2 - m_\pi^2)^{5/2}, \\ \Gamma_{4\text{-body}} &\propto (\delta m)^3((\delta m)^2 - m_l^2)^{5/2}. \end{aligned} \tag{15}$$

These equations clearly show the δm dependence of the stau lifetime.

C. γ_τ

We next consider the γ_τ dependence of the stau lifetime. Figure 3 shows the lifetime as a function of the *CP* violating phase. We set the other parameters to be $m_{\tilde{\chi}} = 300$ GeV, $\delta m = 0.5$ GeV, and $\theta_\tau = \pi/3$.

From Fig. 3, it is clear that the *CP* violating phase does not greatly affect the stau lifetime, and so we fix $\gamma_\tau = 0$ (no *CP* violation).

As expressed in Eq. (6), the effect of *CP* violation appears in the $\mathrm{Re}[g_L g_R]$ terms only. Since the coefficients of the $\mathrm{Re}[g_L g_R]$ terms are smaller than those of $|g_R|^2$, it is again clear that the *CP* violating phase does not greatly affect the stau lifetime.

D. θ_τ

The θ_τ dependence of the stau lifetime is as strong as the δm dependence. Figure 4 shows the θ_τ dependence of the stau lifetime with $m_{\tilde{\chi}} = 300$ GeV and $\gamma_\tau = 0$.

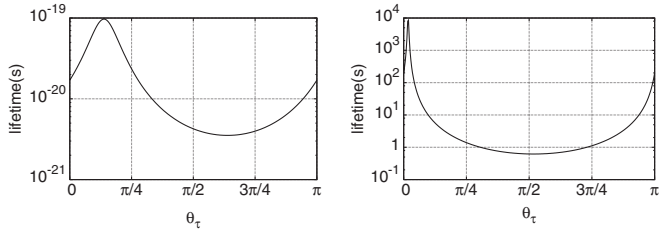


FIG. 4. Stau mixing angle and stau lifetime with $m_{\tilde{\chi}} = 300$ GeV and $\gamma_\tau = 0$. Left panel: $\delta m = 2.0$ GeV, right panel: $\delta m = 0.2$.

We can see from the right panel in Fig. 4 that for $\delta m \ll m_\tau$, $\tilde{\tau}_R$ decays much more quickly than $\tilde{\tau}_L$. This can be understood by considering two steps. First, we note that only left-handed virtual taus contribute to the final state ν_τ. Second, $\tilde{\tau}_R$ converts to τ_L by picking up m_τ in the tau propagator, while $\tilde{\tau}_L$ converts by picking up the momentum p_τ in the propagator. Since $p_\tau \sim \delta m \ll m_\tau$, the former contribution is much larger and hence there is a strong dependence on θ_τ.

E. Constraint from big bang nucleosynthesis

Finally, we consider the constraints from Big Bang nucleosynthesis (BBN). The stau lifetime will be constrained by the standard BBN scenario. Particularly, in the case where a stau decays into hadrons, the constraint on the stau lifetime is more stringent [22,23]. At $t \geq 100$ (sec) (t is the age of the universe), pions emitted from a stau decay also decay before they interact with the background nucleons, so they do not affect BBN. However, if pions are emitted at $t \leq 100$ (sec), they interconvert a background proton and neutron into each other, even after the normal freeze-out time of the n/p ratio. Therefore we must take careful consideration to the parameter region in which pions are emitted for (BBN start) $\leq t \leq$ (BBN end),

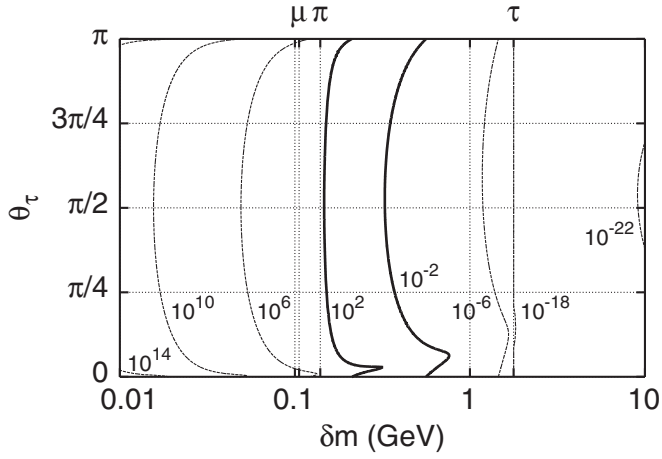


FIG. 5. Contour plot of stau lifetime in sec as a function of δm and θ_τ. To emphasize the BBN era, we draw thick lines for 10^{-2} and 10^2 sec.

i.e. 10^{-2} (sec) $\leq t \leq 10^{2}$ (sec). Naively interpreting Fig. 38 in Ref. [23], we can conclude that the corresponding parameter region can neglect the constraints from BBN. This is inferred from the fact that the $\tilde{\tau}$ number density is roughly two orders lower than that of baryons, $Y_{\tilde{\tau}} \sim 10^{-12}$, since $m_{\tilde{\chi}}$ is of the order of several hundreds of GeV, and the released energy is less than 1 GeV. Here $Y_{\tilde{\tau}}$ is the ratio of the stau number density to the entropy density. Thus $E_{vis} Y_{\tilde{\tau}} < 10^{-12}$ GeV, which does not influence the BBN for $t < 10^{2}$ sec. Incidentally, it is apparent from Fig. 5 that stau lifetime can not exceed the age of universe (about 4.1×10^{17} sec). It is consistent with the fact that an exotic heavy charged particle has not been observed yet.

IV. CONNECTION WITH LEPTON FLAVOR VIOLATION

We next consider LFV [24]. The NLSP slepton might be a linear combination of flavor eigenstates. It is expected that lepton flavor violating events will be observed due to this mixing, such as $\mu \rightarrow e\gamma$. If we observe $\tau \rightarrow e\gamma$ or $\tau \rightarrow \mu\gamma$ events, then within the context of the minimal supersymmetric standard model (MSSM), we would conclude that the selectron forms part of the NLSP:

$$\phi_{\mathrm{NLSP}} = N_1 \tilde{e} + \sqrt{1 - N_1^2}\,\tilde{\tau}. \tag{16}$$

The branching ratio of $\tau \rightarrow e\gamma$ is roughly proportional to N_1^2. The current upper bound on the branching ratio, $< O(10^{-7})$, gives a poor constraint, at most $N_1 < 0.1$. If $N_1 \neq 0$, the NLSP slepton can decay,

$$\phi_{\mathrm{NLSP}} \rightarrow \tilde{\chi} + e, \tag{17}$$

with a lifetime of

$$1.666 \times 10^{-18} \left(\frac{m_{\tilde{\chi}}}{300\ \mathrm{GeV}}\right) \left(\frac{1.0\ \mathrm{GeV}}{\delta m}\right)^2 \left(\frac{0.1}{N_1}\right)^2\ \mathrm{sec}. \tag{18}$$

We may interpret the result as implying that the NLSP slepton consists only of a scalar electron since it decays only into an electron. The lifetime is so short, even if $\delta m < m_\tau$, that we cannot accumulate the NLSP. However if we can measure the lifetime, we can determine the fraction of the scalar electron in the NLSP slepton. Further evidence is given by flavor violating processes of both charged leptons [25–28] and the neutrino [29]. It is important to interpret all the LFV processes together.

V. SUMMARY

We have studied an MSSM scenario in which the LSP and NLSP are a binolike neutralino and a stau, respectively. Since the mass difference is, in many cases, assumed to be degenerate, from the requirement of coannihilation, we paid special attention to the very small δm case. We calculated the partial lifetimes for the decay modes shown in Fig. 1.

We have investigated the stau lifetime dependence on δm, θ_τ, γ_τ, and $m_{\tilde{\chi}}$, considering cosmological constraints. The lifetime strongly depends on δm and θ_τ, while it is almost independent of γ_τ and $m_{\tilde{\chi}}$.

The stau lifetime dependence on δm changes as each threshold is crossed. When δm is larger than m_τ, the lifetime increases in proportion to $(\delta m)^{-2}$ as δm decreases. In the range $m_\tau > \delta m > m_\pi$ the lifetime obeys the scaling $\sim(\delta m)^{-6}$. Below m_π, it grows with $(\delta m)^{-8}$. The δm dependence of the stau lifetime can be largely understood by counting the mass dimension of phase space and the squared amplitude in the massless limit of Standard Model particles. While the massless limit is a good approximation in regions far from the thresholds, the δm dependence near the thresholds are given by Eq. (15).

The lifetime also strongly depends on θ_τ, as shown in Fig. 4. $\tilde{\tau}_R$ contributes to 3- and 4-body decay processes by picking up the m_τ term in the intermediate τ propagator, while $\tilde{\tau}_L$ picks up the p_τ term. Since $p_\tau \sim \delta m \ll m_\tau$, the contribution for $\tilde{\tau}_R$ is much larger and hence there is a strong dependence on θ_τ.

As is seen in Fig. 2, if δm is smaller than m_τ, a stau can be very long-lived. This fact may need to be taken into account in studies at the Large Hadron Collider (LHC) and the International Linear Collider (ILC). The superWIMP case is similar. However, in the gravitino scenario an energetic τ is apparently produced, while in our scenario we should observe very low energy π, μ, or e. Therefore a clean experiment is required. The ILC would be most suitable for investigating the nature of the NLSP slepton.

Another candidate for the NLSP is the stop. However, it is more complicated to investigate this possibility. A stop must always be accompanied by quark(s); it forms a mesonlike fermion with one quark or a baryonlike boson with two quarks. It is almost impossible to calculate the exact mass eigenvalue using QCD. It is beyond the scope of this paper and is left for future work.

We have also discussed lepton flavor violation due to slepton mixing. If there is even a tiny component of a scalar electron or a scalar muon in the NLSP "stau", the decay signal of the NLSP will be completely different from the pure stau case. The NLSP slepton undergoes 2-body decay into the accompanying electron or muon. Since it is a 2-body process, it occurs very quickly, $\sim(10^{-20})N_1^{-2}$ sec where N_1 represents the portion of the scalar electron or scalar muon, as shown in Eq. (16). As this mixing causes charged/neutral lepton flavor violation, it is very important to compare the NLSP slepton decay with other processes such as $\tau \rightarrow e(\mu)\gamma$. To fully clarify the nature of the NLSP, we need to interpret all the LFV processes together.

In contrast, if there is no LFV in slepton mixing, the pure stau has a very long lifetime and it is then possible to experimentally observe SQS decay [21]. SQS decay takes

place in a case of small mass difference. It will appear strongly, if decay rate is small. Since long lifetime means small decay rate, long-lived stau will give a good chance to observe a nonexponential decay. This is important to check the fundamental feature of quantum mechanics.

Hence, the small δm case is very interesting and its study is very important.

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APPENDIX

In this appendix, we show the exact and approximate stau decay rates. We use the exact decay rate in formulating the figures. We make the following approximations: we keep only the leading order term of $(\delta m/m_{\tilde{\chi}})$ and we replace the denominator of the τ propagator by m_τ^2 in the 3- and 4-body cases. We calculate the decay rates of the three processes shown in Fig. 1 with the Lagrangian of Eq. (2).

1. 2-body decay

The decay rate of the 2-body decay process [see Fig. 1(a)] is given by

$$\Gamma_{\text{2-body}} = \frac{1}{16\pi m_{\tilde{\tau}}^3}(m_{\tilde{\tau}}^4 + m_{\tilde{\chi}}^4 + m_\tau^4 - 2m_{\tilde{\tau}}^2 m_{\tilde{\chi}}^2 - 2m_{\tilde{\tau}}^2 m_\tau^2 - 2m_{\tilde{\chi}}^2 m_\tau^2)^{1/2}\{(g_L^2 + |g_R|^2)(m_{\tilde{\tau}}^2 - m_{\tilde{\chi}}^2 - m_\tau^2) - 4\,\mathrm{Re}[g_L g_R] m_\tau m_{\tilde{\chi}}\}. \tag{A1}$$

For the analysis discussed in Sec. II, we approximate the decay rate as

$$\Gamma_{\text{2-body}} = \frac{1}{4\pi m_{\tilde{\chi}}}\sqrt{(\delta m)^2 - m_\tau^2}((g_L^2 + |g_R|^2)\delta m - 2\,\mathrm{Re}[g_L g_R] m_\tau). \tag{A2}$$

2. 3-body decay

The decay rate of the 3-body decay process [see Fig. 1(b)] is calculated as

$$\Gamma_{\text{3-body}} = \frac{G^2 f_\pi^2 \cos^2\theta_c((\delta m)^2 - m_\pi^2)}{64\pi^3 m_{\tilde{\tau}}^3}\int_0^1 dx \sqrt{((\delta m)^2 - q_f^2)((\delta m + 2m_{\tilde{\chi}})^2 - q_f^2)}\,\frac{1}{(q_f^2 - m_\tau^2)^2 + (m_\tau\Gamma_\tau)^2}(q_f^2 - m_\pi^2) \times\left[\frac{1}{4}(g_L^2 q_f^2 + |g_R^2|m_\tau^2)((\delta m)^2 + 2m_{\tilde{\chi}}\delta m - q_f^2) - \mathrm{Re}[g_L g_R] m_{\tilde{\chi}} m_\tau q_f^2\right]. \tag{A3}$$

Here q_f^2 is given as

$$q_f^2 = (\delta m)^2 - ((\delta m)^2 - m_f^2)x, \tag{A4}$$

where the index $f(=\pi, e, \mu)$ denotes a massive particle, except the neutralino, in the final states; $f = \pi$ in the 3-body case. Γ_τ is the tau decay width and $(m_\tau\Gamma_\tau)^2$ is added to the denominator of the tau propagator for the region $\delta m \geq m_\tau$.

The approximate decay rate is

$$\Gamma_{\text{3-body}} = \frac{G^2 f_\pi^2 \cos^2\theta_c}{210(2\pi)^3 m_{\tilde{\chi}} m_\tau^4}((\delta m)^2 - m_\pi^2)^{5/2}[g_L^2\delta m(4(\delta m)^2 + 3m_\pi^2) - 2\,\mathrm{Re}[g_L g_R] m_\tau(4(\delta m)^2 + 3m_\pi^2) + 7|g_R|^2 m_\tau^2 \delta m]. \tag{A5}$$

3. 4-body decay

In the 4-body decay processes [see Fig. 1(c)], the decay rate is given by

$$\Gamma_{\text{4-body}} = \frac{G^2((\delta m)^2 - m_l^2)}{24(2\pi)^5 m_{\tilde{\tau}}^3}\int_0^1 dx \sqrt{((\delta m)^2 - q_f^2)((\delta m + 2m_{\tilde{\chi}})^2 - q_f^2)}\,\frac{1}{(q_f^2 - m_\tau^2)^2 + (m_\tau\Gamma_\tau)^2}\,\frac{1}{q_f^4} \times\left[\left\{\frac{1}{4}(g_L^2 q_f^2 + |g_R^2|m_\tau^2)((\delta m)^2 + 2m_{\tilde{\chi}}\delta m - q_f^2) - \mathrm{Re}[g_L g_R] m_{\tilde{\chi}} m_\tau q_f^2\right\} \times\left\{12m_l^4 q_f^4 \log[\frac{q_f^2}{m_l^2}] + (q_f^4 - m_l^4)(q_f^4 - 8m_l^2 q_f^2 + m_l^4)\right\}\right], \tag{A6}$$

where $l = e, \mu$ and q_l^2 is given by Eq. (A4).

We can approximate the decay rate as

$$\Gamma_{4\text{-body}} = \frac{G^2}{945(2\pi)^5 m_{\tilde{\chi}} m_\tau^4}((\delta m)^2 - m_l^2)^{5/2}[2g_L^2(\delta m)^3(2(\delta m)^2 - 19m_l^2) - 4\,\mathrm{Re}[g_L g_R]m_\tau(\delta m)^2(2(\delta m)^2 - 19m_l^2) + 3|g_R|^2 m_\tau^2 \delta m(2(\delta m)^2 - 23m_l^2)]. \quad \text{(A7)}$$

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