Multirate Sampling Method for Acceleration Control System

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***Abstract*— This paper focuses on realization of high performance motion control based on acceleration control. A high sampling frequency is known to be effective for improving the performance. Characteristics of acceleration control are investigated and the relationship between the performance and the sampling frequency of the system is discussed. Based on these considerations, a new multirate sampling method for the acceleration control system is proposed. Disturbance observer is redesigned for application in the multirate system. Stability analysis is performed to verify the validity of the proposal. Feasibility of the proposal and influence on the performance are also verified by experimental results.**

I. Introduction

Motion control is one of the most important elements for industrial application of robot control. Due to recent rapid progress of robotics, requirement of complicated motion has been increasing. The more complicated the motion becomes, the more robustness and responsibility required. Acceleration control is the control that gives acceleration reference and makes the system realize desired acceleration. It enables acquisition of higher robustness compared to position and velocity control. Acceleration control also makes it possible to treat force and position in the same dimension. For these reasons, acceleration control is inevitable for motion control. Disturbance observer[1], which estimates disturbance torque, has been proposed to make the system to be a robust acceleration control system. Shortening the sampling period is effective to widen the bandwidth in which acceleration control is realized.

On the other hand, the sampling periods have limitations relating to hardware performances even with recent dramatical development of hardware. In general, one constant sampling period is selected for input ($u(t)$), output ($y(t)$), and controller ($r(t)$). Due to the limitations of sampling periods, the sampling period for the system is selected so as to be equal to the longest of those three. In order to acquire better performance despite such hardware limitations, methods for setting sampling periods individually have been proposed. These methods are called multirate sampling control[2]. The system involves more than one sampling period by introducing this method. Many studies have been performed on the system in which output information cannot be acquired fast enough[3], [4]. The computer hard disk drives or the systems utilizing visual camera are the examples.

An aim of this study is realization of acceleration control in wide bandwidth. This paper focuses on the relationship between the performance and the sampling frequency of acceleration control. The needs of a higher sampling frequency for output than for input are described. From this point of view, this paper proposes a new multirate sampling method with a shorter output sampling period for improvement of acceleration control performance. Disturbance observer is redesigned to fit to the multirate system with a new definition of disturbance torque. Stability analysis is performed to make a comparison with single-rate control and to verify the validity of the proposal. Experimental results support the feasibility of the proposed method and improvement of the performance. This study considers the system in the 1st dimension for simplicity.

II. Acceleration Control

In this section, characteristics of acceleration control are discussed with a focus on its sampling periods. Figs. 1 and 2 are the block diagrams of position control and disturbance observer, respectively. Here, K_p denotes the position gain, K_v denotes the velocity gain, τ_l denotes the mechanical load, $\hat{\tau}_{dis}$ denotes the estimated disturbance torque, G_{dis} denotes the cut-off frequency of disturbance observer, I_a^{ref} denotes the current reference, K_t denotes the torque constant, J denotes the inertia, and the subscript n denotes the nominal value.

The total disturbance torque τ_{dis} contains mechanical load τ_l, varied self-inertia torque $\Delta J\ddot{\theta}$, and torque ripple from motor $\Delta K_t I_a^{ref}$. The disturbance torque τ_{dis} is represented as follows:

$$\tau_{dis} = \tau_l + \Delta J\ddot{\theta} - \Delta K_t I_a^{ref}. \quad (1)$$

Disturbance torque is calculated by the equation below.

$$\tau_{dis} = K_{tn} I_a^{ref} - J_n \frac{d\omega}{dt} \quad (2)$$

The first term $K_{tn} I_a^{ref}$ in (2) is based on input information, and the second $J_n \frac{d\omega}{dt}$ is based on output information. In other words, the first corresponds to the left side of disturbance observer in Fig. 2, while the second corresponds to the right side. Considering derivative calculation in the second term, the estimated disturbance torque is obtained through low-pass filter (LPF) as shown in the equation below.

$$\hat{\tau}_{dis} = \frac{G_{dis}}{s + G_{dis}} \tau_{dis} \quad (3)$$

Introduction of disturbance observer realizes acceleration control and improves the robustness of the system. In fact, the robustness is not assured in the frequency range higher than the cut-off frequency of disturbance observer G_{dis}.

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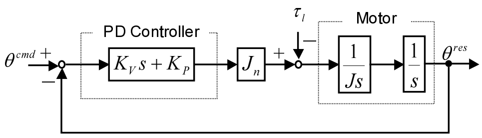

Fig. 1. Position Control

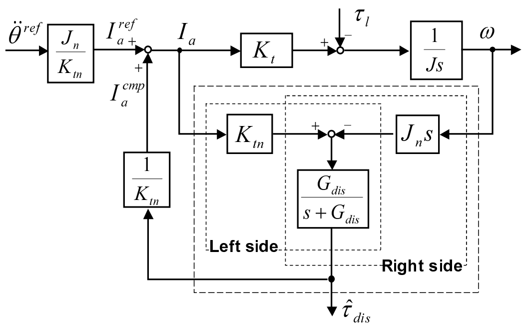

Fig. 2. Disturbance Observer

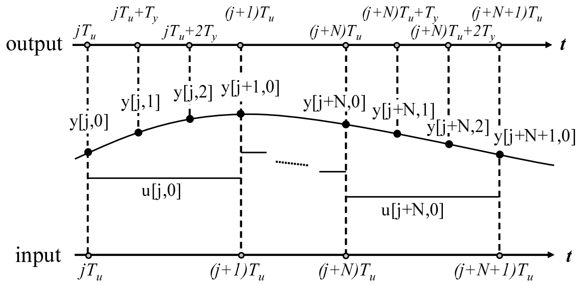

Fig. 3. Multirate Sampling

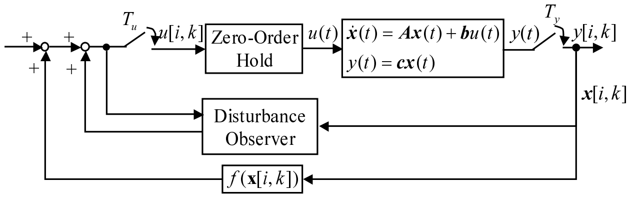

Fig. 4. Multirate Control System

G_{dis} can be set high by shortening a sampling period. Here, the data actually acquired is angular information from a rotary encoder. Thus, two times of derivative calculation are performed in the right side. Since it is usually difficult in experiments to perform derivative calculation due to data noise, pseudo-derivative with LPF is introduced. The velocity is calculated as follow:

$$\hat{\dot{\theta}} = s\frac{G_v}{s + G_v}\theta \tag{4}$$

where, G_v denotes the cut-off frequency of LPF.

It means that two LPFs are introduced into the right side of disturbance observer to realize acceleration control, considering pseudo-derivative and disturbance observer. The data acquired through LPF are delayed. For this reason, acceleration information in the second term of (2) calculated from encoder data is delayed compared with the current reference in the first term. It is better to reduce the delay by acquiring more output information than to input delayed values in a fast rate. In other words, it is preferable to acquire the output information faster than the renewal of actuation input to minimize the delay.

III. Multirate Sampling

A. Multirate Sampling Method for Acceleration Control

This section proposes a new multirate sampling method for the acceleration control system. As described in INTRODUCTION, multirate sampling methods to update input faster than acquisition of output information have been proposed. However, as mentioned in the previous section, it is important in acceleration control systems to acquire output information in a shorter sampling period than renewal of actuation input. The authors, therefore, propose a new multirate sampling method shown in Fig. 3, in which output information is acquired several times in one input sampling period. The sampling period of input T_u and that of the controller T_r are selected to satisfy the following equations.

$$T_u = nT_y \tag{5}$$
$$T_r = T_y \tag{6}$$

where, T_y is the sampling period of output and n is the integer number.

The limitations on sampling periods also support the adequacy of the proposal. The limitation on the input sampling period is generally more severe than that on the output. In motor control, output information is mainly acquired from encoders. Many studies have been performed to increase the rate of acquisition of encoder information[5]. A sampling period of acquisition can be selected in proportion to the clock time of DSP. On the other hand, a frequency of current input is limited by performance of an amplifier or a frequency of PWM. Therefore, the output sampling period can be set shorter than the input in many cases. Consider the continuous-time plant represented as follows:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + \boldsymbol{b}u(t) \tag{7}$$
$$y(t) = \boldsymbol{c}\boldsymbol{x}(t). \tag{8}$$

Assuming that the sampling periods of output and input are T and the input $u(\tau)$ also remains constant from t to $t + T$, the discrete-time plant is represented as follows:

$$\boldsymbol{x}[i+1] = A_d\boldsymbol{x}[i] + \boldsymbol{b}_d u[i] \tag{9}$$
$$\boldsymbol{y}[i] = \boldsymbol{c}_d\boldsymbol{x}[i] \tag{10}$$

where, $\boldsymbol{x}[i] = \boldsymbol{x}(iT)$. Matrix A_d and vectors $\boldsymbol{b}_d$ and $\boldsymbol{c}_d$ are given by

$$A_d = e^{AT}, \quad \boldsymbol{b}_d = \int_0^T e^{A\tau}d\tau\boldsymbol{b}, \quad \boldsymbol{c}_d = \boldsymbol{c}.$$

When a feedback control law is

$$u(t) = f(\boldsymbol{x}(t)) \tag{11}$$

then, it is rewritten into the equation below in discrete-time.

$$u[i] = f(\boldsymbol{x}[i]) \tag{12}$$

In this method, since the actuator input is updated only when $t = iT_u$ (i :integer number) is satisfied, the feedback control law (11) in the multirate system is given by the following equation.

$$u[i,k] = u[i,0] = f(\boldsymbol{x}[i,0]) \tag{13}$$

This equation shows that the actuator input remains constant from $t = iT_u$ to $t = (i+1)T_u$. In the proposed multirate method, therefore, the state-space equations (9) and (10) can be rewritten into the equations below, considering the relation of two sampling periods, T_y and T_u.

$$\boldsymbol{x}[i,k+1] = A_m\boldsymbol{x}[i,k] + \boldsymbol{b}_m u[i,0] : k \neq n-1 \tag{14}$$
$$\boldsymbol{x}[i+1,0] = A'_m\boldsymbol{x}[i,n-1] + \boldsymbol{b}'_m u[i,0] : k = n-1 \tag{15}$$
$$\boldsymbol{y}[i,k] = \boldsymbol{c}_m\boldsymbol{x}[i,k] \tag{16}$$

where,

$$\boldsymbol{x}[i,k] = \boldsymbol{x}((i+\frac{k}{n})T_u) = \boldsymbol{x}(iT_u + kT_y) \ \ (k=0,\cdots,n-1)$$
$$A_m = e^{AT_y}, \ \boldsymbol{b}_m = \int_0^{T_y} e^{A\tau}d\tau\boldsymbol{b}, \quad \boldsymbol{c}_m = \boldsymbol{c}.$$

B. Disturbance Observer in Multirate System

Application of disturbance observer in the multirate system is discussed in this section. In the proposed multirate system, there are two values of input, desired input value $I_m[i,k]$ and real input value $I_m^{real}[i,k]$. The former is calculated at an output sampling rate and the latter is a real input value to the robot, which is renewed at an input sampling rate. From (13), the following relation is obtained.

$$I_m^{real}[i,k] = I_m[i,0] \tag{17}$$

1) Application of Conventional Disturbance Observer: Disturbance torque defined in conventional disturbance observer is represented by the following equation in the multirate system.

$$\tau_{dis}[i,k] = \tau_l[i,k] + \Delta J\ddot{\theta}[i,k] - \Delta K_t I_m^{real}[i,k] \tag{18}$$

The estimated disturbance torque is acquired from the real input value $I_a^{real}[i,k]$ and velocity.

2) Disturbance Observer for Multirate System: Equation (17) shows that there is a deviation between the desired input value and the real input value when $T \neq iT_u$ is satisfied. Although the deviation is not considered in the conventional disturbance observer, it may exert an influence on the system. With a focus on that, the total disturbance torque of the multirate system including the influence of the deviation of the input value is defined as the equation below.

$$\begin{aligned}\tau_{mdis}[i,k] = \tau_l[i,k] \ &+ \ \Delta J\ddot{\theta}[i,k] - \Delta K_t I_m[i,k] \\ &+ \ (K_{tn} + \Delta K_t)\Delta I_m[i,k]\end{aligned} \tag{19}$$

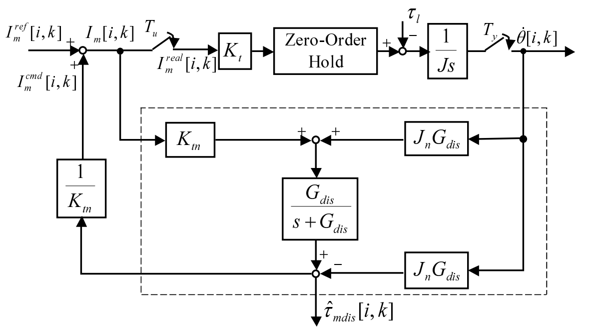

Fig. 5. Disturbance Observer for Multirate System

where,

$$\Delta I_m[i,k] = I_m[i,k] - I_m^{real}[i,k].$$

In order to estimate and compensate τ_{mdis}, disturbance observer in the multirate system is proposed as shown in Fig. 5. The desired input value $I_a[i,k]$ is utilized instead of $I_a^{real}[i,k]$.

C. Application of the Proposed Method

The following advantages are expected for the proposed multirate sampling method:

- Cut-off frequency can be set higher; and
- Information of disturbance can be acquired in a shorter sampling period.

As a result, responsibility against disturbance is improved and bandwidth of robust acceleration control is increased. On the other hand, the absence of updating of compensation input that occurs in the proposed method may deteriorate the performance. The proposed disturbance observer enables the system to estimate the disturbance including the influence of the absence of updating. By compensating τ_{mdis}, performance close to that achieved with a short sampling period for both output and input seems to be obtained.

IV. Stability Analysis

Stability analysis of both single-rate control and the proposed multirate control is performed to verify the validity of the proposed method. The limit of the input sampling period is assumed to be 0.1[msec] in this analysis. The block diagram of the whole system for analysis is shown in Fig. 6.

A. Modeling

A dynamic equation of a 1DOF manipulator in discrete-time is shown in the following equation.

$$\begin{bmatrix}\theta[i+1] \\ \dot{\theta}[i+1] \\ \tau_{dis}[i+1]\end{bmatrix} = \begin{bmatrix}1 & T & -\frac{T^2}{2J} \\ 0 & 1 & -\frac{T}{J} \\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\theta[i] \\ \dot{\theta}[i] \\ \tau_{dis}[i]\end{bmatrix} + \begin{bmatrix}\frac{T^2}{2J} \\ \frac{T}{J} \\ 0\end{bmatrix}\tau_m[i] \tag{20}$$

where, τ_m denotes the input torque and τ_{dis} denotes the disturbance torque, which is assumed to be constant. PD controller and disturbance observer are applied to the system. Sampling periods T_u and T_y are set to be equal in the single-rate control and set to satisfy $T_u = nT_y$ in the multirate control.

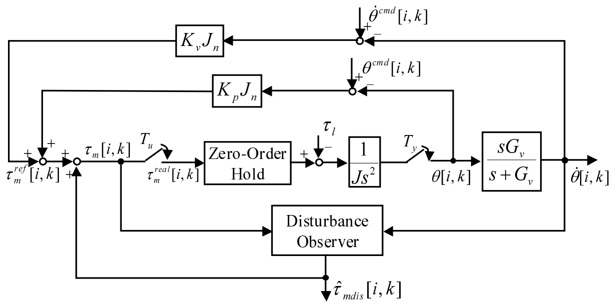

Fig. 6. Analysis Model

1) Single-rate Control: The state-space equation (20) is expanded so as to include state variables $w_1[i]$, $w_2[i]$ in disturbance observer, which is designed based on the Gopinath's method[6], and pseudo-derivative calculation, which is utilized to acquire velocity from position data. $\boldsymbol{x}[i]$, $u[i]$, A_d, $\boldsymbol{b}_d$ and $\boldsymbol{c}_d$ in (9) are represented as follows:

$$\boldsymbol{x}[i] = \begin{bmatrix} \theta[i] & \dot{\theta}[i] & \tau_{dis}[i] & w_1[i] & w_2[i] \end{bmatrix}^T, \ u[i] = \tau_m[i]$$

$$A_d = \begin{bmatrix} 1 & T & -\frac{T^2}{2J} & 0 & 0 \\ 0 & 1 & -\frac{T}{J} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \hat{b}G_v & 0 & 0 & \hat{a} & \hat{b}G_v(\beta-1) \\ 1 & 0 & 0 & 0 & \beta \end{bmatrix}, \ \boldsymbol{b}_d = \begin{bmatrix} \frac{T^2}{2J} \\ \frac{T}{J} \\ 0 \\ \hat{j} \\ 0 \end{bmatrix}$$

$$\boldsymbol{c}_d = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where,

$$\begin{aligned} \hat{a} &= \alpha, \ \hat{j} = 1-\alpha, \ \hat{b} = \frac{J}{T}(1-\alpha)^2, \ l_1 = \frac{J}{T}(1-\alpha) \\ \alpha &= e^{-G_{dis}T}, \ \beta = e^{-G_vT}. \end{aligned}$$

Velocity calculated with pseudo-derivative technique and estimated disturbance torque are given by the following equations.

$$\hat{\dot{\theta}}[i] = G_v(\beta-1)w_2[i] + G_v\theta[i] \qquad (21)$$

$$\hat{\tau}_{dis}[i] = w_1[i] - l_1\hat{\dot{\theta}}[i] \qquad (22)$$

The control law is expressed by the following equations.

$$\boldsymbol{w}[i+1] = \hat{A}\boldsymbol{w}[i] + \hat{\boldsymbol{b}}\theta[i] + \hat{\boldsymbol{j}}\tau_m[i] \qquad (23)$$

$$\hat{\boldsymbol{x}}[i] = \hat{C}\boldsymbol{w}[i] + \hat{\boldsymbol{d}}\theta[i] \qquad (24)$$

$$\tau_m[i] = K_d(\boldsymbol{r}[i] - \hat{\boldsymbol{x}}[i]) \qquad (25)$$

where,

$$\boldsymbol{w}[i] = \begin{bmatrix} w_1[i] & w_2[i] \end{bmatrix}^T, \ \hat{\boldsymbol{x}} = \begin{bmatrix} \theta[i] & \hat{\dot{\theta}}[i] & \hat{\tau}_m[i] \end{bmatrix}^T$$

$$\hat{A} = \begin{bmatrix} \hat{a} & \hat{b}G_v(\beta-1) \\ 0 & \beta \end{bmatrix}, \ \hat{\boldsymbol{b}} = \begin{bmatrix} \hat{b}G_v \\ 1 \end{bmatrix}, \ \hat{\boldsymbol{j}} = \begin{bmatrix} \hat{j} \\ 0 \end{bmatrix}$$

$$\hat{C} = \begin{bmatrix} 0 & 0 \\ 0 & G_v(\beta-1) \\ 1 & -l_1G_v(\beta-1) \end{bmatrix}, \ \hat{\boldsymbol{d}} = \begin{bmatrix} 1 \\ G_v \\ -l_1G_v \end{bmatrix}$$

$$K_d = \begin{bmatrix} K_pJ_n & K_vJ_n & -1 \end{bmatrix},$$

and $\boldsymbol{r}[i]$ denotes the reference. The following equation is obtained by transforming (23) and (24) into transfer function expression.

$$\hat{\boldsymbol{x}}[i] = K_y\theta[i] + K_u\tau_m[i] \qquad (26)$$

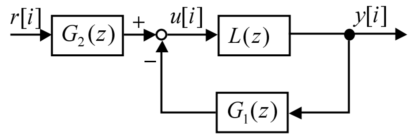

Fig. 7. Structure of System

The transfer functions of the system $L(z)$ and controller G_1 in Fig. 7 are obtained as follows:

$$L(z) = \boldsymbol{c}(zI - A_d)^{-1}B_d \qquad (27)$$

$$G_1(z) = (1 + K_dK_u)^{-1}K_dK_y. \qquad (28)$$

2) Multirate Control: The multirate sampling method and the disturbance observer proposed in the previous section are applied to the system. Considering that there are two values of the input torque, the state-space equation is represented as follows:

$$\boldsymbol{x}[i,k+1] = A_m\boldsymbol{x}[i,k] + B_m\begin{bmatrix} \tau_m^{real}[i,k] \\ \tau_m[i,k] \end{bmatrix} \qquad (29)$$

$$y[i,k] = \boldsymbol{c}_m\boldsymbol{x}[i,k] \qquad (30)$$

where,

$$A_m = A_d, \ \boldsymbol{c}_m = \boldsymbol{c}_d, \ T = T_y$$

$$B_m = \begin{bmatrix} \frac{T^2}{2J} & \frac{T}{J} & 0 & 0 & 0 \\ 0 & 0 & 0 & \hat{j} & 0 \end{bmatrix}^T.$$

The input torques are given by the following equations.

$$\tau_m[i,k] = J_n(-K_p\theta[i,k] - K_v\hat{\dot{\theta}}[i,k]) + \hat{\tau}_{mdis}[i,k] \qquad (31)$$

$$\tau_m^{real}[i,k] = \tau_m[i,0] \qquad (32)$$

Equation (29) is the state-space equation described for the shorter sampling period, the output sampling period. It is necessary, however, to describe the system for the longer sampling period for analysis. In order to rewrite the system for the longer sampling period, the method described in [7] is used. The state vectors are expanded as follows:

$$\boldsymbol{x}_M[i] = \begin{bmatrix} \boldsymbol{x}[i-1,1] \\ \vdots \\ \boldsymbol{x}[i-1,n-1] \\ \boldsymbol{x}[i,0] \end{bmatrix}, \boldsymbol{y}_M[i] = \begin{bmatrix} y[i,0] \\ \vdots \\ y[i,n-1] \end{bmatrix} \qquad (33)$$

The expanded reference $\boldsymbol{r}_M[i]$ and control signal $\tau_{mM}[i]$ are defined in parallel to $\boldsymbol{y}_M[i]$. The state-space equations of the expanded system are represented in the equations below.

$$\boldsymbol{x}_M[i+1] = A_M\boldsymbol{x}[i] + B_M\tau_{mM}[i] \qquad (34)$$

$$\boldsymbol{y}_M[i] = C_M(U_1\boldsymbol{x}_M[i+1] + U_2\boldsymbol{x}_M[i]) \qquad (35)$$

where,

$$U_1 = block\ diag\ (I_n, \ \cdots, \ I_n, \mathbf{0})$$

$$U_2 = block\ diag\ (\mathbf{0}, \cdots, \mathbf{0}, I_n)$$

and the subscript M denotes the expanded matrices. The control law also has to be rewritten for the longer

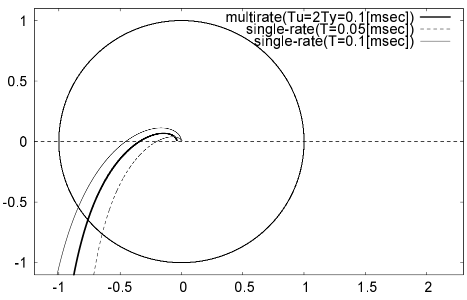

Fig. 8. Nyquist Diagram

TABLE I
SAMPLING PERIOD AND GAINS IN STABILITY ANALYSIS

Case	T_y[msec]	T_u[msec]
Single-rate(long)	0.1	0.1
Single-rate(short)	0.05	0.05
Multirate	0.05	0.1

sampling period. The expanded variable $\boldsymbol{w}_M[i]$ is defined in parallel to $\boldsymbol{x}_M[i]$, while $\hat{\boldsymbol{x}}_M[i]$ is in parallel to $\boldsymbol{y}_M[i]$. The expanded control law is given by the following equations.

$$\boldsymbol{w}_M[i+1]=\hat{A}_M\boldsymbol{w}_M[i]+\hat{B}_M\boldsymbol{y}_M[i]+\hat{J}_M\tau_{mM}[i] \quad (36)$$

$$\hat{\boldsymbol{x}}_M[i]=\hat{C}_M(z\bar{U}_1+\bar{U}_2)\boldsymbol{w}_M[i]+\hat{D}_M\boldsymbol{y}_M[i] \quad (37)$$

$$\tau_{mM}[i]=K_{dM}(\boldsymbol{r}_M[i]-\hat{\boldsymbol{x}}_M[i]) \quad (38)$$

where, $\bar{U}_1$ and $\bar{U}_2$ are defined in parallel to U_1 and U_2, respectively. The following equation is obtained by transforming (36) and (37) into transfer function expression.

$$\hat{\boldsymbol{x}}_M[i]=K_{yM}\boldsymbol{y}_M[i]+K_{uM}\tau_{mM}[i] \quad (39)$$

The transfer functions of the expanded system $L(z)$ and controller $G_1(z)$ are obtained as follows:

$$L(z) = C_M(zU_1+U_2)(zI-A_M)^{-1}B_M \quad (40)$$

$$G_1(z) = (1+K_{dM}K_{uM})^{-1}K_{dM}K_{yM}. \quad (41)$$

B. Stability Analysis

Nyquist criterion is obtained by drawing Nyquist diagram of $det[I+L(z)G_1(z)]-1$. Nyquist diagram of the system in the previous section is shown in Fig. 8. Tables I and II show the sampling periods and the gains used in the analysis. This analysis is performed with the assumption that the limit of the input sampling period is 0.1[msec]. The result of the proposed method shows improvement of stability compared with single-rate control with $T=0.1$[msec], although it does not come up with the result for $T=0.05$[msec]. The result indicates that shortening the output sampling period is effective to improve stability, especially in case that there are limitations on the input sampling period. As mentioned in III-A, limitation on the input sampling frequency is usually more severe than that on output. It means that the proposed method is effective for general systems.

Although the result of the stability analysis seems to indicate that the shorter sampling periods are, the higher the stability becomes, the problems involved in shortening the output sampling period are easily conceived. One

TABLE II
CONTROL PARAMETERS

K_p	Position Gain [rad/sec]	900
K_v	Velocity Gain [rad/sec]	60
G_v	Cut-off Frequency of Pseudo-derivative [rad/sec]	13000
G_{dis}	Cut-off Frequency of Disturbance Observer[rad/sec]	7000

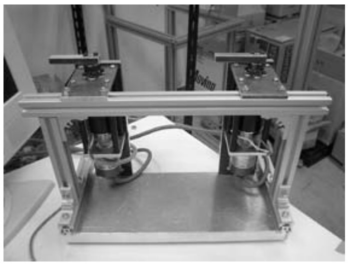

Fig. 9. Experimental Equipment

problem is that the shorter output sampling requires more computations. Another is the problem with encoder resolution. Encoder resolution in acceleration dimension R_a is calculated as follows:

$$R_a = \frac{2\pi}{P_e T^2} \quad (42)$$

where, P_e denotes the number of pulses of an encoder. As shown in (42), encoder resolution in acceleration dimension relates to the sampling period. In order to acquire the same resolution in acceleration dimension in a shorter sampling period, the number of pulses of an encoder must be higher. Although the quantization error is not considered in the stability analysis, it is necessary to consider the error for practical application. In order to verify the feasibility and the influence on performance of the proposed method in the system with quantization error, the experiments are performed in the next section.

V. EXPERIMENTS

Experimental results of the proposed multirate sampling method are demonstrated and compared with those of the single-rate sampling control in this section.

A. Experimental Setup

Experiments are performed with the robot shown in Fig. 9. The number of pulses of the encoder is $81,000$ [pulse/rev] and it is multiplied by four in counter board to improve resolution.

B. Experimental Results

In this experiment, 0.05[Nm] torque disturbance is added as step input from t=7.0[sec] to t=7.5[sec] and t=10.5[sec] to t=11.0[sec] while the manipulator is moved as a sine wave. Fig. 10 shows the position command and the response. Shaded areas show the response with the disturbance. The experiments are performed under assumption that limitation of the input sampling period is 0.3[msec]. In order to verify the effects of shortening the output sampling period and of the proposed disturbance observer, four patterns of experiments listed below are performed.

TABLE III

CONTROL PARAMETERS (EXPERIMENT)

	T_y[msec]	T_u[msec]	G_v	G_{dis}
Single-rate ($T = 0.3$)	0.3	0.3	1900	650
Single-rate ($T = 0.15$)	0.15	0.15	2500	1800
Multirate (τ_{dis})	0.1	0.3	2500	1800
Multirate (τ_{mdis})	0.1	0.3	2500	1800

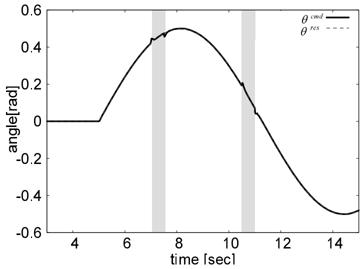

Fig. 10. Position Command and Response

- Single-rate with a short sampling period
- Single-rate with a long sampling period
- Multirate using conventional disturbance observer
- Multirate using proposed disturbance observer

TABLE III presents the sampling period and the gains in each experiment. Figs. 11 to 13 show the position error when the disturbance torque is added. Fig. 11 is the comparison between single-rate and the multirate controls with the same input period. In single-rate control, the manipulator oscillates and becomes unstable with G_{dis} larger than 700[rad/sec]. On the other hand, in the multirate control, G_{dis} can be set much higher. The influence of the disturbance is greatly reduced and convergence is also improved. In order to show the advantage of shortening the output sampling period more clearly, the result of the multirate control is compared with that of single-rate control with a shorter sampling period in Fig. 12. Note that the sampling period of the single-rate control is shorter than the assumed limitation. Although G_{dis} can be set as in case of the multirate control, oscillation is confirmed, which is not confirmed in the multirate control. The result indicates that better performance can be acquired even with a longer input sampling period by shortening an output sampling period. Therefore, the sampling period of output has a priority over that of input in acceleration control.

Fig. 13 compares the proposed disturbance observer with the conventional disturbance observer. Although both of them show almost the same response to the disturbance, they differ in the response without the disturbance. In the case of conventional disturbance observer, there is an error in a stationary state. The result shows superiority of the proposed disturbance observer.

VI. CONCLUSIONS

This paper showed the priority of the sampling period of output over that of input in acceleration control. From this point of view, the multirate sampling method for the acceleration control system was proposed. Effectiveness of the proposal was confirmed both in terms of stability and performance. Nyquist diagram shows improvement of stability by applying the proposed method. Considering all the experimental results, the merit of shortening the output sampling period is that better performance can be acquired even with a longer input sampling period. The proposed disturbance observer enables the system to perform as if it has a short sampling period not only for the output but also the input.

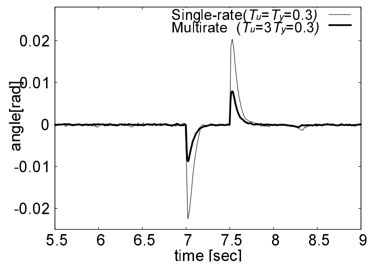

Fig. 11. Comparison of Single-rate and Multirate 1

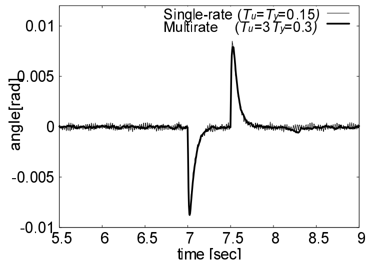

Fig. 12. Comparison of Single-rate and Multirate 2

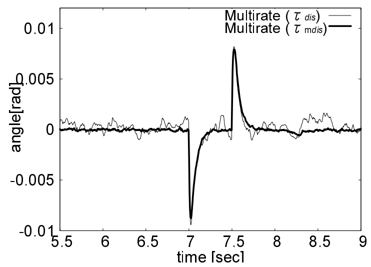

Fig. 13. Comparison of Disturbance Observer

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