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On Linearization of $N = 1$ Nonlinear Supersymmetry

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Abstract

The $N = 1$ Volkov-Akulov model of nonlinear supersymmetry is explicitly related to a vector supermultiplet model with a Fayet-Iliopoulos D term of linear supersymmetry. The physical significance of the results is discussed briefly.

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Spontaneous breakdown of supersymmetry (SUSY) produces inevitably Nambu-Goldstone (N-G) fermions [1], as demonstrated in the Fayet-Iliopoulos model [2] and the O'Raifeartaigh model [3]. Dynamics of N-G fermions is described by the Volkov-Akulov action [4]. When N-G fermions are coupled to supergravity [5] under a local SUSY invariant way, they are converted to the longitudinal components of spin 3/2 fields by the super Higgs mechanism [6] as demonstrated for the V-A model.

This may be always the case if we adhere to the coset space G/H interpretation of the nonlinear realization of SUSY and to the assumption of the existence of the invariant action under the initial larger symmetry group G with local SUSY. Most of the SUSY unified theories adopt this mechanism and N-G fermions disappear at low energy, which gives an explanation of the absence of free (bare) N-G fermions in nature.

However if we consider seriously the distinguished character of SUSY [7], i.e., SUSY and its spontaneous breakdown are profoundly connected to the noncompact spacetime (Poincaré) symmetry, it may be worthwhile regarding the V-A model as a nonlinear realization of SUSY originated not necessarily from specific Lagrangian models of G and G/H expressed by field operators but from a spontaneous breakdown of the higher symmetry of spacetime by itself in terms of the geometrical arguments.

In ref. [8] one of the authors has proposed the *superon-graviton model* (SGM) as an attempt along this idea. The fundamental action of the SGM is an Einstein-Hilbert action analogue of general relativity, which is obtained by the geometrical arguments of the local GL(4,$\mathbf{R}$) invariance of the SGM spacetime, where there exist fermionic degrees of freedom (N-G fermions) at every four-dimensional curved spacetime point. It consists of the Einstein-Hilbert action, the V-A action with a global SO(10) and their interactions and is invariant under a new SUSY [9]. All observed (low energy) elementary particles except graviton are regarded as (composite) eigenstates of the *linear* representation of the SO(10) super Poincaré algebra composed of fundamental objects *superons* (N-G fermions) with spin 1/2 [10]. For deriving the low energy physical contents of the SGM action it is often useful to linearize such a highly nonlinear theory and obtain a low energy effective theory. Toward the linearization of the SGM we investigate the linearization of the V-A model in detail.

The linearization of the V-A model was investigated by many authors [11, 12, 13, 14, 15]. Ivanov and Kapustnikov [11] have established the general relations between linear and nonlinear realizations of global SUSY. In ref. [12] Roček constructed irreducible and SUSY invariant constraints on a scalar supermultiplet in terms of the N-G field and showed explicitly that the V-A model of nonlinear SUSY was related to a scalar supermultiplet of the linear SUSY of Wess and Zumino [7]. In ref. [11] a relationship between the V-A model and a vector supermultiplet is studied in terms of a constrained gauge superfield in the context of the coupling of the V-A action to the gauge multiplet action with the Fayet-Iliopoulos D term of linear SUSY. Although the relation between the action of linear SUSY and the V-A action is established as expected from the viewpoint that they are equally responsible to spontaneous SUSY breaking. The explicit representation of all component fields of the vector superfield in terms of the N-G fermion, which is crucial for the SGM scenario, is remained to be studied.

In this letter we construct the complete form of the SUSY invariant constraints and show explicitly that the V-A model is related to the total action of a U(1) gauge supermultiplet [16] of the linear SUSY with the Fayet-Iliopoulos D term indicating a spontaneous SUSY breaking. We find that a U(1) gauge field can be constructed explicitly from the N-G fermion fields although it is an axial vector.

An $N = 1$ U(1) gauge supermultiplet is given by a real superfield

$$V(x,\theta,\bar\theta) = C + i\theta\chi - i\bar\theta\bar\chi + \frac{1}{2}i\theta^2(M+iN) - \frac{1}{2}i\bar\theta^2(M-iN) \\ -\theta\sigma^m\bar\theta v_m + i\theta^2\bar\theta\left(\bar\lambda + \frac{1}{2}i\bar\sigma^m\partial_m\chi\right) - i\bar\theta^2\theta\left(\lambda + \frac{1}{2}i\sigma^m\partial_m\bar\chi\right) \\ +\frac{1}{2}\theta^2\bar\theta^2\left(D + \frac{1}{2}\Box C\right), \tag{1}$$

where $C(x)$, $M(x)$, $N(x)$, $D(x)$ are real scalar fields, $\chi_\alpha(x)$, $\lambda_\alpha(x)$ and $\bar\chi_{\dot\alpha}(x)$, $\bar\lambda_{\dot\alpha}(x)$ are Weyl spinors and their complex conjugates, and $v_m(x)$ is a real vector field. We use the two-component spinor notation in ref. [17]. Spacetime vector indices are denoted by $m, n, \cdots = 0, 1, 2, 3$, and spinor indices by $\alpha, \beta, \cdots = 1, 2$ and $\dot\alpha, \dot\beta, \cdots = 1, 2$. For more details of the notations see ref. [17]. Only λ, $\bar\lambda$, D and $v_{mn} = \partial_m v_n - \partial_n v_m$ are gauge invariant. Other component fields can be set to zero by a gauge transformation in the superspace. The supertransformation of V with transformation parameters ϵ_α, $\bar\epsilon_{\dot\alpha}$ is given by

$$\delta V = \left(\epsilon Q + \bar\epsilon\bar Q\right)V, \tag{2}$$

where

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i(\sigma^m \bar{\theta})_\alpha \partial_m, \qquad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i(\theta \sigma^m)_{\dot{\alpha}} \partial_m. \tag{3}$$

We introduce an N-G fermion field $\zeta_\alpha(x)$ and its complex conjugate $\bar{\zeta}_{\dot{\alpha}}(x)$. Their supertransformations are

$$\begin{aligned} \delta\zeta &= \frac{1}{\kappa}\epsilon - i\kappa\left(\zeta\sigma^m\bar{\epsilon} - \epsilon\sigma^m\bar{\zeta}\right)\partial_m\zeta, \\ \delta\bar{\zeta} &= \frac{1}{\kappa}\bar{\epsilon} - i\kappa\left(\zeta\sigma^m\bar{\epsilon} - \epsilon\sigma^m\bar{\zeta}\right)\partial_m\bar{\zeta}, \end{aligned} \tag{4}$$

where κ is a constant whose dimension is $(\text{mass})^{-2}$. Following refs. [11], [13] we define the superfield $\tilde{V}(x,\theta,\bar{\theta})$ by

$$\tilde{V}(x,\theta,\bar{\theta}) = V(x',\theta',\bar{\theta}'), \tag{5}$$

where

$$\begin{aligned} x'^m &= x^m + i\kappa\left(\zeta(x)\sigma^m\bar{\theta} - \theta\sigma^m\bar{\zeta}(x)\right), \\ \theta' &= \theta - \kappa\zeta(x), \qquad \bar{\theta}' = \bar{\theta} - \kappa\bar{\zeta}(x). \end{aligned} \tag{6}$$

$\tilde{V}$ may be expanded in component fields as

$$\begin{aligned} \tilde{V}(x,\theta,\bar{\theta}) = {} & \tilde{C} + i\theta\tilde{\chi} - i\bar{\theta}\bar{\tilde{\chi}} + \frac{1}{2}i\theta^2(\tilde{M}+i\tilde{N}) - \frac{1}{2}i\bar{\theta}^2(\tilde{M}-i\tilde{N}) \\ & -\theta\sigma^m\bar{\theta}\tilde{v}_m + i\theta^2\bar{\theta}\left(\bar{\tilde{\lambda}} + \frac{1}{2}i\bar{\sigma}^m\partial_m\tilde{\chi}\right) - i\bar{\theta}^2\theta\left(\tilde{\lambda} + \frac{1}{2}i\sigma^m\partial_m\bar{\tilde{\chi}}\right) \\ & +\frac{1}{2}\theta^2\bar{\theta}^2\left(\tilde{D} + \frac{1}{2}\Box\tilde{C}\right), \end{aligned} \tag{7}$$

where $\tilde{C}, \tilde{\chi}, \bar{\tilde{\chi}}, \cdots$ can be expressed by $C, \chi, \bar{\chi}, \cdots$ and ζ, $\bar{\zeta}$ by using the relation (5). From eqs. (2), (4) it can be shown that supertransformations of these component fields $\tilde{\phi}_i(x) = (\tilde{C}, \tilde{\chi}, \bar{\tilde{\chi}}, \cdots)$ have a form

$$\delta\tilde{\phi}_i = -i\kappa\left(\zeta\sigma^m\bar{\epsilon} - \epsilon\sigma^m\bar{\zeta}\right)\partial_m\tilde{\phi}_i. \tag{8}$$

Therefore, a condition $\tilde{\phi}_i(x) = \text{constant}$ is invariant under supertransformations.

The explicit form of the relation between $\tilde{C}, \tilde{\chi}, \bar{\tilde{\chi}}, \cdots$ and $C, \chi, \bar{\chi}, \cdots$ is given by

$$
\begin{aligned}
\tilde{C} &= C' - i\kappa\zeta\chi' + i\kappa\bar{\zeta}\bar{\chi}' + \frac{1}{2}i\kappa^2\zeta^2(M' + iN') - \frac{1}{2}i\kappa^2\bar{\zeta}^2(M' - iN') \\
&\quad -\kappa^2\zeta\sigma^m\bar{\zeta}v'_m - i\kappa^3\zeta^2\bar{\zeta}\bar{\lambda}' + i\kappa^3\bar{\zeta}^2\zeta\lambda' + \frac{1}{2}\kappa^4\zeta^2\bar{\zeta}^2D', \\
\tilde{\chi} &= \chi' - \kappa\zeta(M' + iN') - i\kappa\sigma^m\bar{\zeta}v'_m + 2\kappa^2\zeta\bar{\zeta}\bar{\lambda}' - \kappa^2\lambda'\bar{\zeta}^2 + i\kappa^3\zeta\bar{\zeta}^2D', \\
\tilde{M} + i\tilde{N} &= M' + iN' - 2\kappa\bar{\zeta}\bar{\lambda}' - i\kappa^2\bar{\zeta}^2D', \\
\tilde{v}_m &= v'_m - i\kappa\zeta\sigma_m\bar{\lambda}' + i\kappa\lambda'\sigma_m\bar{\zeta} + \kappa^2\zeta\sigma_m\bar{\zeta}D', \\
\tilde{\lambda} + \frac{1}{2}i\sigma^m\partial_m\bar{\tilde{\chi}} &= \lambda' - i\kappa\zeta D', \\
\tilde{D} + \frac{1}{2}\Box\tilde{C} &= D', \qquad (9)
\end{aligned}
$$

where

$$
\begin{aligned}
C' &= C, \\
\chi' &= \chi - \kappa\sigma^m\bar{\zeta}\partial_m C, \\
M' + iN' &= M + iN + i\kappa\partial_m\chi\sigma^m\bar{\zeta} - \frac{1}{2}i\kappa^2\bar{\zeta}^2\Box C, \\
v'_m &= v_m + \frac{1}{2}\kappa\zeta\sigma^n\bar{\sigma}_m\partial_n\chi + \frac{1}{2}\kappa\partial_n\bar{\chi}\bar{\sigma}_m\sigma^n\bar{\zeta} - \frac{1}{2}\kappa^2\zeta\sigma^k\bar{\sigma}_m\sigma^l\bar{\zeta}\partial_k\partial_l C, \\
\lambda' &= \lambda + \frac{1}{2}i\sigma^m\partial_m\bar{\chi} - \frac{1}{2}i\kappa\sigma^m\bar{\zeta}\partial_m(M - iN) + \frac{1}{2}\kappa\sigma^m\bar{\sigma}^n\zeta\partial_n v_m \\
&\quad -\frac{1}{2}\kappa^2\sigma^n\bar{\zeta}\zeta\sigma^m\partial_m\partial_n\bar{\chi} - \frac{1}{4}\kappa^2\Box\chi\zeta^2 + \frac{1}{4}\kappa^3\sigma^m\bar{\zeta}\zeta^2\partial_m\Box C, \\
D' &= D + \frac{1}{2}\Box C + \kappa\zeta\sigma^n\partial_n\left(\bar{\lambda} + \frac{1}{2}i\bar{\sigma}^m\partial_m\chi\right) - \kappa\bar{\zeta}\bar{\sigma}^n\partial_n\left(\lambda + \frac{1}{2}i\sigma^m\partial_m\bar{\chi}\right) \\
&\quad +\frac{1}{4}i\kappa^2\zeta^2\Box(M + iN) - \frac{1}{4}i\kappa^2\bar{\zeta}^2\Box(M - iN) + \frac{1}{2}\kappa^2\zeta\sigma^k\bar{\sigma}^m\sigma^l\bar{\zeta}\partial_k\partial_l v_m \\
&\quad -\frac{1}{4}\kappa^3\zeta^2\partial_m\Box\chi\sigma^m\bar{\zeta} - \frac{1}{4}\kappa^3\bar{\zeta}^2\zeta\sigma^m\partial_m\Box\bar{\chi} + \frac{1}{8}\kappa^4\bar{\zeta}^2\zeta^2\Box^2 C. \qquad (10)
\end{aligned}
$$

As in refs. [11], [13] it is possible to solve eq. (9) and express $C, \chi, \bar{\chi}, \cdots$ in terms of $\tilde{C}, \tilde{\chi}, \bar{\tilde{\chi}}, \cdots$ and ζ, $\bar{\zeta}$. By imposing a SUSY and gauge invariant constraint on $\tilde{\lambda}$ the original fields $C, \chi, \bar{\chi}, \cdots$ become functions of $\tilde{C}$, $\tilde{\chi}$, $\bar{\tilde{\chi}}$, $\tilde{M}$, $\tilde{N}$, $\tilde{v}_m$, $\tilde{D}$ and ζ, $\bar{\zeta}$. Substituting these expressions into an action one obtains an action of the N-G fields ζ, $\bar{\zeta}$ interacting with other fields. Indeed, the couplings of ζ, $\bar{\zeta}$ to $\tilde{v}_m$ were obtained in ref. [11]. Here, we are only interested in the sector which only depends on the N-G fields.

To eliminate other degrees of freedom than the N-G fields we impose SUSY invariant constraints

$$\tilde{C} = \tilde{\chi} = \tilde{M} = \tilde{N} = \tilde{v}_m = \tilde{\lambda} = 0, \qquad \tilde{D} = \frac{1}{\kappa}. \tag{11}$$

Solving these constraints we find that the original component fields C, χ, $\bar{\chi}$, $\cdots$ can be expressed by the N-G fields ζ, $\bar{\zeta}$. We find

$$\begin{aligned}
C &= \frac{1}{2}\kappa^3\zeta^2\bar{\zeta}^2, \\
\chi &= -i\kappa^2\zeta\bar{\zeta}^2 + \kappa\sigma^m\bar{\zeta}\partial_m C, \\
M + iN &= -i\kappa\bar{\zeta}^2 - i\kappa\partial_m\chi\sigma^m\bar{\zeta} + \frac{1}{2}i\kappa^2\bar{\zeta}^2\Box C, \\
v_m &= \kappa\zeta\sigma_m\bar{\zeta} - \frac{1}{2}\kappa\zeta\sigma^n\bar{\sigma}_m\partial_n\chi - \frac{1}{2}\kappa\partial_n\bar{\chi}\bar{\sigma}_m\sigma^n\bar{\zeta} + \frac{1}{2}\kappa^2\zeta\sigma^k\bar{\sigma}_m\sigma^l\bar{\zeta}\partial_k\partial_l C, \\
\lambda &= i\zeta - \frac{1}{2}i\sigma^m\partial_m\bar{\chi} + \frac{1}{2}i\kappa\sigma^m\bar{\zeta}\partial_m(M - iN) - \frac{1}{2}\kappa\sigma^m\bar{\sigma}^n\zeta\partial_n v_m \\
&\quad + \frac{1}{2}\kappa^2\sigma^n\bar{\zeta}\zeta\sigma^m\partial_m\partial_n\bar{\chi} + \frac{1}{4}\kappa^2\Box\chi\zeta^2 - \frac{1}{4}\kappa^3\sigma^m\bar{\zeta}\zeta^2\partial_m\Box C, \\
D &= \frac{1}{\kappa} - \frac{1}{2}\Box C - \kappa\zeta\sigma^n\partial_n\left(\bar{\lambda} + \frac{1}{2}i\bar{\sigma}^m\partial_m\chi\right) + \kappa\bar{\zeta}\bar{\sigma}^n\partial_n\left(\lambda + \frac{1}{2}i\sigma^m\partial_m\bar{\chi}\right) \\
&\quad - \frac{1}{4}i\kappa^2\zeta^2\Box(M + iN) + \frac{1}{4}i\kappa^2\bar{\zeta}^2\Box(M - iN) - \frac{1}{2}\kappa^2\zeta\sigma^k\bar{\sigma}^m\sigma^l\bar{\zeta}\partial_k\partial_l v_m \\
&\quad + \frac{1}{4}\kappa^3\zeta^2\partial_m\Box\chi\sigma^m\bar{\zeta} + \frac{1}{4}\kappa^3\bar{\zeta}^2\zeta\sigma^m\partial_m\Box\bar{\chi} - \frac{1}{8}\kappa^4\zeta^2\bar{\zeta}^2\Box^2 C.
\end{aligned} \tag{12}$$

The first equation gives C in terms of ζ, $\bar{\zeta}$. Substituting this into the second equation gives χ in terms of ζ, $\bar{\zeta}$. By substituting these results into the third equation gives $M + iN$ in terms of ζ, $\bar{\zeta}$, and so on. By the supertransformation of ζ, $\bar{\zeta}$ in eq. (4) these C, χ, $\bar{\chi}$, $\cdots$ transform exactly as in eq. (2). The leading terms in the expansion of the fields v_m, λ, $\bar{\lambda}$ and D, which contain gauge invariant degrees of freedom, in κ are

$$\begin{aligned}
v_m &= \kappa\zeta\sigma_m\bar{\zeta} + \cdots, \\
\lambda &= i\zeta - \frac{1}{2}\kappa^2\zeta\left(\zeta\sigma^m\partial_m\bar{\zeta} - \partial_m\zeta\sigma^m\bar{\zeta}\right) + \kappa^2\sigma^{mn}\zeta\partial_m\left(\zeta\sigma_n\bar{\zeta}\right) + \cdots, \\
D &= \frac{1}{\kappa} + i\kappa\left(\zeta\sigma^m\partial_m\bar{\zeta} - \partial_m\zeta\sigma^m\bar{\zeta}\right) + \cdots,
\end{aligned} \tag{13}$$

where $\cdots$ are higher order terms in κ. In the four-component spinor notation the first equation becomes $v_m \sim \kappa\bar{\zeta}\gamma_m\gamma_5\zeta + \cdots$, which is an axial vector.

Our discussion so far does not depend on a particular form of the action. We now consider a free action of a U(1) gauge supermultiplet with a Fayet-Iliopoulos D term

$$S = \frac{1}{4}\int d^4xd^2\theta\, W^\alpha W_\alpha + \frac{1}{4}\int d^4xd^2\bar{\theta}\, \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}} - \frac{2}{\kappa}\int d^4xd^2\theta d^2\bar{\theta}\, V, \tag{14}$$

where

$$\begin{aligned} W_\alpha &= -\frac{1}{4}\bar{D}_{\dot{\beta}}\bar{D}^{\dot{\beta}}D_\alpha V, \qquad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^\beta D_\beta \bar{D}_{\dot{\alpha}}V, \\ D_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i(\sigma^m\bar{\theta})_\alpha\partial_m, \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i(\theta\sigma^m)_{\dot{\alpha}}\partial_m. \end{aligned} \tag{15}$$

The last term proportional to κ^{-1} is the Fayet-Iliopoulos D term. In component fields we have

$$S = \int d^4x \left[-\frac{1}{4}v_{mn}v^{mn} - i\lambda\sigma^m\partial_m\bar{\lambda} + \frac{1}{2}D^2 - \frac{1}{\kappa}D \right]. \tag{16}$$

The field equation for D gives $D = \frac{1}{\kappa} \neq 0$ in accordance with eq. (13), which shows that supersymmetry is spontaneously broken.

We substitute eq. (12) into the action (14) and obtain an action for the N-G fields ζ, $\bar{\zeta}$. To do this it is more convenient to use a different form of the action equivalent to eq. (14) [17]

$$S = \int d^4xd^2\theta d^2\bar{\theta}\, \mathcal{L}(x,\theta,\bar{\theta}), \tag{17}$$

where

$$\mathcal{L} = -\frac{1}{16}\left(\bar{D}^2D^\alpha V D_\alpha V + D^2\bar{D}_{\dot{\alpha}}V\bar{D}^{\dot{\alpha}}V\right) - \frac{2}{\kappa}V. \tag{18}$$

Changing the integration variables $(x,\theta,\bar{\theta}) \to (x',\theta',\bar{\theta}')$ by eq. (6) we obtain

$$\begin{aligned} S &= \int d^4x'd^2\theta'd^2\bar{\theta}'\, \mathcal{L}(x',\theta',\bar{\theta}') \\ &= \int d^4xd^2\theta d^2\bar{\theta} J(x,\theta,\bar{\theta})\tilde{\mathcal{L}}(x,\theta,\bar{\theta}), \end{aligned} \tag{19}$$

where $J(x,\theta,\bar{\theta})$ is the Jacobian for the change of variables and

$$\tilde{\mathcal{L}}(x,\theta,\bar{\theta}) = -\frac{1}{16}\left(\bar{D}'^2D'^\alpha\tilde{V}D'_\alpha\tilde{V} + D'^2\bar{D}'_{\dot{\alpha}}\tilde{V}\bar{D}'^{\dot{\alpha}}\tilde{V}\right) - \frac{2}{\kappa}\tilde{V}. \tag{20}$$

From eqs. (7), (11) we have

$$\tilde{V} = \frac{1}{2\kappa}\theta^2\bar{\theta}^2. \tag{21}$$

In terms of the transformation matrix for the change of variables (6)

$$\begin{aligned} M &= \frac{\partial(x', \theta', \bar{\theta}')}{\partial(x, \theta, \bar{\theta})} \\ &= \begin{pmatrix} \delta^n_m - i\kappa\left(\theta\sigma^n\partial_m\bar{\zeta} - \partial_m\zeta\sigma^n\bar{\theta}\right) & -\kappa\partial_m\zeta^\beta & -\kappa\partial_m\bar{\zeta}^{\dot{\beta}} \\ -i\kappa(\sigma^n\bar{\zeta})_\alpha & \delta^\beta_\alpha & 0 \\ -i\kappa(\zeta\sigma^n)_{\dot{\alpha}} & 0 & \delta^{\dot{\beta}}_{\dot{\alpha}} \end{pmatrix} \end{aligned} \tag{22}$$

the Jacobian and the transformation of derivatives are given by

$$J(x, \theta, \bar{\theta}) = \mathrm{sdet} M, \qquad \begin{pmatrix} \frac{\partial}{\partial x'} \\ \frac{\partial}{\partial \theta'} \\ \frac{\partial}{\partial \bar{\theta}'} \end{pmatrix} = M^{-1} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial \theta} \\ \frac{\partial}{\partial \bar{\theta}} \end{pmatrix}, \tag{23}$$

where sdet is the superdeterminant. More explicitly, we obtain

$$\begin{aligned} J &= \det\left(V_m{}^n\right), \\ \frac{\partial}{\partial x'^m} &= V_m{}^n\left(\frac{\partial}{\partial x^n} + \kappa\partial_n\zeta^\beta\frac{\partial}{\partial\theta^\beta} + \kappa\partial_n\bar{\zeta}^{\dot{\beta}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}}\right), \\ D'_\alpha &= \frac{\partial}{\partial\theta^\alpha} + i(\sigma^n\bar{\theta})_\alpha\frac{\partial}{\partial x'^n}, \\ \bar{D}'_{\dot{\alpha}} &= -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i(\theta\sigma^n)_{\dot{\alpha}}\frac{\partial}{\partial x'^n}, \end{aligned} \tag{24}$$

where

$$V_m{}^n = \delta^n_m - i\kappa\left(\theta\sigma^n\partial_m\bar{\zeta} - \partial_m\zeta\sigma^n\bar{\theta}\right) + i\kappa^2\left(\zeta\sigma^n\partial_m\bar{\zeta} - \partial_m\zeta\sigma^n\bar{\zeta}\right). \tag{25}$$

Substituting eqs. (21), (24) into eq. (19) and integrating over θ, $\bar{\theta}$ we obtain an action for the N-G fields

$$S = -\frac{1}{2\kappa^2}\int d^4x\ \det\left[\delta^n_m + i\kappa^2\left(\zeta\sigma^n\partial_m\bar{\zeta} - \partial_m\zeta\sigma^n\bar{\zeta}\right)\right]. \tag{26}$$

This is exactly the V-A action.

Now we summarize the results as follows. All component fields of the vector gauge supermultiplet of linear SUSY are represented uniquely in terms of the N-G spinor field, and the V-A action of nonlinear SUSY is reproduced by just substituting the representations into the action of the vector gauge supermultiplet of linear SUSY. It is remarkable that the coefficients of all terms including the Fayet-Iliopoulos D term in the linear SUSY action is determined uniquely by the SUSY (constraints). As for the axial vector nature of the U(1) gauge field we speculate that the adopted constraints may cut out implicitly the dyonic (electric and magnetic) aspect of the dynamics of the V-A action. All these phenomena are favorable to the SGM scenario [8].

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