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Linearizing $N = 2$ Nonlinear Supersymmetry

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Abstract

We investigate for the $N = 2$ supersymmetry (SUSY) a relation between a vector supermultiplet of the linear SUSY and the Volkov-Akulov model of the nonlinear SUSY. We express component fields of the vector supermultiplet in terms of Nambu-Goldstone fermion fields at the leading orders in a SUSY invariant way, and show the vector nature of the U(1) gauge field explicitly. A relation of the actions for the two models is also discussed briefly.

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Spontaneous breaking of supersymmetry (SUSY) gives rise to Nambu-Goldstone (N-G) fermions [1], as shown in the breaking scheme of Fayet-Iliopoulos [2] and O'Raifeartaigh [3]. N-G fermions can be characterized by means of the nonlinear realization of global SUSY in the Volkov-Akulov (V-A) model [4]. Their coupling to supergravity [5] under a local SUSY invariant way was investigated for the V-A model in the framework of the super Higgs mechanism [6, 7], in which N-G fermions are converted to the longitudinal components of spin-3/2 fields.

On the other hand, the connection between the V-A model of the nonlinear SUSY and linear supermultiplets became clear from the early work by many authors [8, 9, 10]: Indeed, in Ref. [8] the general relationship between linear and nonlinear realizations of global SUSY was established. In Ref. [9] it was shown explicitly that the V-A model is related to a scalar supermultiplet of the linear SUSY of Wess and Zumino [15] by constructing irreducible and SUSY invariant relations on a scalar supermultiplet of the linear SUSY. The explicit connection between the V-A model and a scalar supermultiplet in two-dimensional spacetime is also discussed in Ref. [10]. As for a vector supermultiplet, its relationship to the V-A model was studied in Ref. [8] in the context of the coupling of the V-A action to the gauge multiplet action with the Fayet-Iliopoulos D term of the linear SUSY.

Recently, one of the authors proposed the superon-graviton model (SGM) based upon the SO(10) super-Poincaré algebra from a composite viewpoint of matters [11, 12], which may be a most economical supersymmetric unified model beyond the standard model. The fundamental action of the SGM is the Einstein-Hilbert type one obtained from the geometrical arguments of the local GL(4,R) invariance under new nonlinear SUSY transformations of the SGM spacetime, where there exist fermionic degrees of freedom (N-G fermions) at every four-dimensional curved spacetime point [12]. The expansion of the SGM action in terms of graviton and superons (N-G fermions) with spin-1/2 has a very complicated and rich structure [13]; indeed, it is a highly nonlinear one which consists of the Einstein-Hilbert action of the general relativity, the V-A action and their interactions. Also, the SGM action is invariant under at least [global nonlinear SUSY] $\otimes$ [local $\mathrm{GL}(4,\mathrm{R})$] $\otimes$ [local Lorentz] $\otimes$ [global SO(N)] as a whole [14], which is isomorphic to the global SO(N) super-Poincaré symmetry.

In the SGM the (composite) eigenstates of the *linear* representation of SO(10) super-Poincaré algebra which is composed of superons are regarded as all observed elementary particles at low energy except graviton [11, 12]. For deriving the low energy physical contents of the SGM action, it is important to linearize such a highly nonlinear theory. As a preliminary to do this, it is useful to investigate the linearization of the V-A model in detail. In this respect, in addition to the work by many authors [8, 9, 10], we have explicitly shown in Ref. [16] that the $N=1$ V-A model is related to the total action of a U(1) gauge supermultiplet [17] of the linear

SUSY with the Fayet-Iliopoulos D term indicating a spontaneous SUSY breaking. In the work of Ref. [16] it became clear that the representations of component fields of a U(1) gauge supermultiplet in terms of the N-G fermion fields indicate the axial vector nature of the U(1) gauge field. In order to see its vector nature at least, we have to investigate the linearization of the V-A model with an extended SUSY.

In this paper we restrict our attention to the $N = 2$ SUSY and discuss a connection between the V-A model and an $N = 2$ vector supermultiplet [18] of the linear SUSY in four-dimensional spacetime. In particular, we show that for the $N = 2$ theory a SUSY invariant relation between component fields of the vector supermultiplet and the N-G fermion fields can be constructed by means of the method used in Ref. [9] starting from an ansatz given below (Eq. (10)). We also briefly discuss a relation of the actions for the two models.

Let us denote the component fields of an $N = 2$ U(1) gauge supermultiplet [18], which belong to representations of a rigid SU(2), as follows; namely, ϕ for a physical complex scalar field, λ_R^i $(i = 1, 2)$ for two right-handed Weyl spinor fields and A_a for a U(1) gauge field in addition to D^I $(I = 1, 2, 3)$ for three auxiliary real scalar fields required from the mismatch of the off-shell degrees of freedom between bosonic and fermionic physical fields.[§] λ_R^i and D^I belong to representations **2** and **3** of SU(2) respectively while other fields are singlets. By the charge conjugation we define left-handed spinor fields as $\lambda_{Li} = C\bar{\lambda}_{Ri}^T$. We use the antisymmetric symbols ϵ^{ij} and ϵ_{ij} $(\epsilon^{12} = \epsilon_{21} = +1)$ to raise and lower SU(2) indices as $\psi^i = \epsilon^{ij}\psi_j$, $\psi_i = \epsilon_{ij}\psi^j$.

The $N = 2$ linear SUSY transformations of these component fields generated by constant spinor parameters ζ_L^i are

$$\begin{aligned}
\delta_Q \phi &= -\sqrt{2}\bar{\zeta}_R \lambda_L, \\
\delta_Q \lambda_{Li} &= -\frac{1}{2} F_{ab}\gamma^{ab}\zeta_{Li} - \sqrt{2} i \partial\!\!\!/ \phi \zeta_{Ri} + i(\zeta_L \sigma^I)_i D^I, \\
\delta_Q A_a &= -i\bar{\zeta}_L \gamma_a \lambda_L - i\bar{\zeta}_R \gamma_a \lambda_R, \\
\delta_Q D^I &= \bar{\zeta}_L \sigma^I \partial\!\!\!/ \lambda_L + \bar{\zeta}_R \sigma^I \partial\!\!\!/ \lambda_R,
\end{aligned} \tag{1}$$

where $\zeta_{Ri} = C\bar{\zeta}_{Li}^T$, $F_{ab} = \partial_a A_b - \partial_b A_a$, and σ^I are the Pauli matrices. The contractions of SU(2) indices are defined as $\bar{\zeta}_R \lambda_L = \bar{\zeta}_{Ri}\lambda_L^i$, $\bar{\zeta}_R \sigma^I \lambda_L = \bar{\zeta}_{Ri}(\sigma^I)^i{}_j \lambda_L^j$, etc. These supertransformations satisfy a closed off-shell commutator algebra

$$[\delta_Q(\zeta_1), \delta_Q(\zeta_2)] = \delta_P(v) + \delta_g(\theta), \tag{2}$$

where $\delta_P(v)$ and $\delta_g(\theta)$ are a translation and a U(1) gauge transformation with pa-

[§] Minkowski spacetime indices are denoted by $a, b, ... = 0, 1, 2, 3$, and the flat metric is $\eta^{ab} = \mathrm{diag}(+1, -1, -1, -1)$. Gamma matrices satisfy $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ and we define $\gamma^{ab} = \frac{1}{2}[\gamma^a, \gamma^b]$.

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$$\begin{aligned} v^a &= 2i(\bar{\zeta}_{1L}\gamma^a\zeta_{2L} - \bar{\zeta}_{1R}\gamma^a\zeta_{2R}), \\ \theta &= -v^a A_a + 2\sqrt{2}\bar{\zeta}_{1L}\zeta_{2R}\phi - 2\sqrt{2}\bar{\zeta}_{1R}\zeta_{2L}\phi^*. \end{aligned} \tag{3}$$

Only the gauge field A_a transforms under the U(1) gauge transformation

$$\delta_g(\theta)A_a = \partial_a\theta. \tag{4}$$

Although in our discussion on the derivation of the relation between the linear and nonlinear SUSY transformations we have not used a form of the action, it is instructive to consider a free action which is invariant under Eq. (1)

$$S_{\rm lin} = \int d^4x \left[\partial_a\phi\partial^a\phi^* - \frac{1}{4}F_{ab}^2 + i\bar{\lambda}_R\partial\!\!\!/\lambda_R + \frac{1}{2}(D^I)^2 - \frac{1}{\kappa}\xi^I D^I\right], \tag{5}$$

where κ is a constant whose dimension is (mass)$^{-2}$ and ξ^I are three arbitrary real parameters satisfying $(\xi^I)^2 = 1$. The last term proportional to κ^{-1} is an analog of the Fayet-Iliopoulos D term in the $N = 1$ theories [2]. The field equations for the auxiliary fields give $D^I = \xi^I/\kappa$ indicating a spontaneous SUSY breaking.

On the other hand, in the $N = 2$ V-A model [19] we have a nonlinear SUSY transformation law of the N-G fermion fields ψ_L^i

$$\delta_Q\psi_L^i = \frac{1}{\kappa}\zeta_L^i - i\kappa\left(\bar{\zeta}_L\gamma^a\psi_L - \bar{\zeta}_R\gamma^a\psi_R\right)\partial_a\psi_L^i, \tag{6}$$

where $\psi_{Ri} = C\bar{\psi}_{Li}^T$. This transformation satisfies off-shell the commutator algebra (2) without the U(1) gauge transformation on the right-hand side. The V-A action invariant under Eq. (6) reads

$$S_{\rm VA} = -\frac{1}{2\kappa^2}\int d^4x\ \det w, \tag{7}$$

where the 4×4 matrix w is defined by

$$w^a{}_b = \delta^a_b + \kappa^2 t^a{}_b, \qquad t^a{}_b = -i\bar{\psi}_L\gamma^a\partial_b\psi_L + i\bar{\psi}_R\gamma^a\partial_b\psi_R. \tag{8}$$

The V-A action (7) is expanded in κ as

$$\begin{aligned} S_{\rm VA} = -\frac{1}{2\kappa^2}\int d^4x \Big[1 &+ \kappa^2 t^a{}_a + \frac{1}{2}\kappa^4(t^a{}_a t^b{}_b - t^a{}_b t^b{}_a) \\ &-\frac{1}{6}\kappa^6\epsilon_{abcd}\epsilon^{efgd}t^a{}_e t^b{}_f t^c{}_g - \frac{1}{4!}\kappa^8\epsilon_{abcd}\epsilon^{efgh}t^a{}_e t^b{}_f t^c{}_g t^d{}_h\Big]. \end{aligned} \tag{9}$$

We would like to obtain a SUSY invariant relation between the component fields of the $N = 2$ vector supermultiplet and the N-G fermion fields ψ^i at the leading orders of κ. It is useful to imagine a situation in which the linear SUSY is broken with the auxiliary fields having expectation values $D^I = \xi^I/\kappa$ as in the free theory (5). Then, we expect from the experience in the $N = 1$ cases [8, 9, 10] and the transformation law of the spinor fields in Eq. (1) that the relation should have a form

$$\begin{aligned} \lambda_{Li} &= i\xi^I(\psi_L\sigma^I)_i + \mathcal{O}(\kappa^2), \\ D^I &= \frac{1}{\kappa}\xi^I + \mathcal{O}(\kappa), \\ \text{(other fields)} &= \mathcal{O}(\kappa). \end{aligned} \tag{10}$$

Higher order terms are obtained such that the linear SUSY transformations (1) are reproduced by the nonlinear SUSY transformation of the N-G fermion fields (6).

After some calculations we obtain the relation between the fields in the linear theory and the N-G fermion fields as

$$\begin{aligned} \phi(\psi) &= \frac{1}{\sqrt{2}} i\kappa\xi^I\bar{\psi}_R\sigma^I\psi_L - \sqrt{2}\kappa^3\xi^I\bar{\psi}_L\gamma^a\psi_L\bar{\psi}_R\sigma^I\partial_a\psi_L \\ &\quad -\frac{\sqrt{2}}{3}\kappa^3\xi^I\bar{\psi}_R\sigma^J\psi_L\bar{\psi}_R\sigma^J\sigma^I\partial\!\!\!/\psi_R + \mathcal{O}(\kappa^5), \\ \lambda_{Li}(\psi) &= i\xi^I(\psi_L\sigma^I)_i + \kappa^2\xi^I\gamma^a\psi_{Ri}\bar{\psi}_R\sigma^I\partial_a\psi_L + \frac{1}{2}\kappa^2\xi^I\gamma^{ab}\psi_{Li}\partial_a\left(\bar{\psi}_L\sigma^I\gamma_b\psi_L\right) \\ &\quad +\frac{1}{2}\kappa^2\xi^I(\psi_L\sigma^J)_i\left(\bar{\psi}_L\sigma^J\sigma^I\partial\!\!\!/\psi_L - \bar{\psi}_R\sigma^J\sigma^I\partial\!\!\!/\psi_R\right) + \mathcal{O}(\kappa^4), \\ A_a(\psi) &= -\frac{1}{2}\kappa\xi^I\left(\bar{\psi}_L\sigma^I\gamma_a\psi_L - \bar{\psi}_R\sigma^I\gamma_a\psi_R\right) \\ &\quad +\frac{1}{4}i\kappa^3\xi^I\Big[\bar{\psi}_L\sigma^J\psi_R\bar{\psi}_R\left(2\delta^{IJ}\delta^b_a - \sigma^J\sigma^I\gamma_a\gamma^b\right)\partial_b\psi_L \\ &\quad -\frac{1}{4}\bar{\psi}_L\gamma^{cd}\psi_R\bar{\psi}_R\sigma^I\left(2\gamma_a\gamma_{cd}\gamma^b - \gamma^b\gamma_{cd}\gamma_a\right)\partial_b\psi_L + (L \leftrightarrow R)\Big] + \mathcal{O}(\kappa^5), \\ D^I(\psi) &= \frac{1}{\kappa}\xi^I - i\kappa\xi^J\left(\bar{\psi}_L\sigma^I\sigma^J\partial\!\!\!/\psi_L - \bar{\psi}_R\sigma^I\sigma^J\partial\!\!\!/\psi_R\right) \\ &\quad +\kappa^3\xi^J\Big[\bar{\psi}_L\sigma^I\psi_R\partial_a\bar{\psi}_R\sigma^J\partial^a\psi_L - \bar{\psi}_L\sigma^K\gamma^c\psi_L\Big\{i\epsilon^{IJK}\partial_c\bar{\psi}_L\partial\!\!\!/\psi_L \\ &\quad -\frac{1}{2}\partial_a\bar{\psi}_L\sigma^J\sigma^K\sigma^I\gamma_c\partial^a\psi_L + \frac{1}{4}\partial_a\bar{\psi}_L\sigma^J\sigma^I\sigma^K\gamma^a\gamma_c\partial\!\!\!/\psi_L\Big\} \\ &\quad -\frac{1}{4}\bar{\psi}_L\sigma^K\psi_R\left\{\partial_a\bar{\psi}_R\sigma^J\sigma^I\sigma^K\gamma^b\gamma^a\partial_b\psi_L - \bar{\psi}_R\left(2\delta^{IK} + \sigma^I\sigma^K\right)\sigma^J\Box\psi_L\right\} \\ &\quad +\frac{1}{16}\bar{\psi}_L\gamma^{cd}\psi_R\left\{\partial_a\bar{\psi}_R\sigma^J\sigma^I\gamma^b\gamma_{cd}\gamma^a\partial_b\psi_L + \bar{\psi}_R\sigma^I\sigma^J\gamma^b\gamma_{cd}\gamma^a\partial_a\partial_b\psi_L\right\} \end{aligned}$$

$$+(L \leftrightarrow R)\Big] + \mathcal{O}(\kappa^5). \tag{11}$$

The transformation of the N-G fermion fields (6) reproduces the transformation of the linear theory (1) except that the transformation of the gauge field $A_a(\psi)$ contains an extra U(1) gauge transformation

$$\delta_Q A_a(\psi) = -i\bar{\zeta}_L\gamma_a\lambda_L(\psi) - i\bar{\zeta}_R\gamma_a\lambda_R(\psi) + \partial_a X, \tag{12}$$

where

$$X = \frac{1}{2}i\kappa^2\xi^I\bar{\zeta}_L\left(2\delta^{IJ} - \sigma^{IJ}\right)\psi_R\bar{\psi}_R\sigma^J\psi_L + (L \leftrightarrow R). \tag{13}$$

The U(1) gauge transformation parameter X satisfies

$$\delta_Q(\zeta_1)X(\zeta_2) - \delta_Q(\zeta_2)X(\zeta_1) = -\theta, \tag{14}$$

where θ is defined in Eq. (3). Due to this extra term the commutator of two supertansformations on $A_a(\psi)$ does not contain the U(1) gauge transformation term in Eq. (2). This should be the case since the commutator on ψ does not contain the U(1) gauge transformation term. For gauge invariant quantities like F_{ab} the transformations exactly coincide with those of the linear SUSY. In principle we can continue to obtain higher order terms in the relation (11) following this approach. However, it will be more useful to use the $N=2$ superfield formalism [20] as was done in Refs. [8, 10, 16] for the $N=1$ theories.

We note that the leading terms of A_a in Eq. (11) can be written as

$$A_a = -\kappa\xi^1\bar{\chi}\gamma_5\gamma_a\varphi + i\kappa\xi^2\bar{\chi}\gamma_a\varphi - \frac{1}{2}\kappa\xi^3\left(\bar{\chi}\gamma_5\gamma_a\chi - \bar{\varphi}\gamma_5\gamma_a\varphi\right) + \mathcal{O}(\kappa^3), \tag{15}$$

where we have defined Majorana spinor fields

$$\chi = \psi_L^1 + \psi_{R1}, \qquad \varphi = \psi_L^2 + \psi_{R2}. \tag{16}$$

When $\xi^1 = \xi^3 = 0$, this shows the vector nature of the U(1) gauge field as we expected.

The relation (11) reduces to that of the $N=1$ SUSY by imposing, e.g. $\psi_L^2 = 0$. When $\xi^1 = 1$, $\xi^2 = \xi^3 = 0$, we find $\lambda_{L2} = 0$, $A_a = 0$, $D^3 = 0$ and that the relation between $(\phi, \lambda_{L1}, D^1, D^2)$ and ψ_L^1 becomes that of the $N=1$ scalar supermultiplet obtained in Ref. [9]. ¶ When $\xi^1 = \xi^2 = 0$, $\xi^3 = 1$, on the other hand, we find $\lambda_{L1} = 0$, $\phi = 0$, $D^1 = D^2 = 0$ and that the relation between (λ_{L2}, A_a, D^3) and ψ^1_L becomes that of the $N = 1$ vector supermultiplet obtained in Refs. [8, 16].

In our above discussion on the derivation of the result (11) we have not used a form of the action. However, as for the relation between the free linear SUSY action $S_{\rm lin}$ in Eq. (5) and the V-A action $S_{\rm VA}$ in Eq. (7), we have explicitly shown that $S_{\rm lin}$ indeed coincides with the V-A action $S_{\rm VA}$ at least up to and including $O(\kappa^0)$ in Eq. (9) by substituting Eq. (11) into the linear action (5). We consider this is a strong indication of the all order coincidence between the actions (5) and (7) from the experience in the $N = 1$ cases [8, 9, 10, 16].

Finally we summarize our results. In this paper we have constructed the SUSY invariant relation between the component fields of the $N = 2$ vector supermultiplet and the N-G fermion fields ψ^i_L at the leading orders of κ. We have explicitly showed that the U(1) gauge field A_a has the vector nature in terms of the N-G fermion fields in contrast to the models with the $N = 1$ SUSY [16]. The relation (11) contains three arbitrary real parameters ξ^I/κ, which can be regarded as the vacuum expectation values of the auxiliary fields D^I. When we put $\psi^2_L = 0$, the relation reduces to that of the $N = 1$ scalar supermultiplet or that of the $N = 1$ vector supermultiplet depending on the choice of the parameters ξ^I. We have also briefly discussed that the free action $S_{\rm lin}$ in Eq. (5) with the Fayet-Iliopoulos D term reduces to the V-A action $S_{\rm VA}$ in Eq. (7) at least up to and including $O(\kappa^0)$ by subsituting the relation (11) into the linear action (5). From the results in this Letter we anticipate the equivalence of the action of N-extended standard supermultiplets to the N-extended V-A action of a nonlinear SUSY, which is favorable for the SGM scenario.

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¶ $N = 1$ scalar supermultiplet is also obtained by adopting $\xi^1 = 1 = \xi^2$, $\xi^3 = 0$.

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