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Three-Form Flux with $\mathcal{N} = 2$ Supersymmetry on $\mathbf{AdS}_5 \times \mathbf{S}^5$

Madoka Nishimura[a*] and Yoshiaki Tanii[b†]

[a] *Center for Theoretical Physics*
Massachusetts Institute of Technology
Cambridge, MA 02139-4307, USA

[b] *Physics Department, Faculty of Science*
Saitama University, Saitama 338-8570, Japan

Abstract

In the context of the AdS/CFT correspondence the general form of a three-form flux perturbation to the $\mathrm{AdS}_5 \times \mathrm{S}^5$ solution in the type IIB supergravity which preserves $\mathcal{N} = 2$ supersymmetry is obtained. The arbitrary holomorphic function appearing in the $\mathcal{N} = 1$ case studied by Graña and Polchinski is restricted to a quadratic function of the coordinates transverse to the D3-branes.

* e-mail: madoka@mit.edu
† e-mail: tanii@post.saitama-u.ac.jp

1. Introduction

It was proposed that the type IIB string theory compactified on $AdS_5 \times S^5$ has a dual description by the $\mathcal{N} = 4$ super Yang-Mills theory in the large N limit [1, 2, 3]. This conjecture of the AdS/CFT correspondence has been supported by comparison of spectra, correlation functions and anomalies calculated in both of the supergravity and the Yang-Mills theory. (For a review, see ref. [4].) The AdS/CFT correspondence was also studied in various other spacetime dimensions. At first the correspondence was studied for theories with high supersymmetries such as $\mathcal{N} = 4$. To apply it to more realistic models one has to consider theories with lower supersymmtries.

One of the ways to obtain the AdS/CFT correspondence for lower supersymmetric cases is to modify supergravity solutions by adding a perturbation. In ref. [5] a perturbation of the three-form flux was added to the $AdS_5 \times S^5$, which breaks $\mathcal{N} = 4$ to $\mathcal{N} = 1$. This perturbation corresponds to fermion mass terms of the three $\mathcal{N} = 1$ chiral multiplets in the $\mathcal{N} = 4$ super Yang-Mills theory and polarizes D3 branes into 5-branes [6, 7]. Similar constructions of the AdS/CFT correspondence with lower supersymmetries were discussed in refs. [8, 9, 10, 11].

The general form of a three-form flux perturbation to the $AdS_5 \times S^5$ solution which preserves $\mathcal{N} = 1$ supersymmetry and satisfies the Bianchi identity and the linearized field equation was obtained in ref. [12]. It contains an arbitrary holomorphic function and an arbitrary harmonic function of the coordinates for the directions transverse to the D3-branes. It was argued that the holomorphic function corresponds to a superpotential in the dual super Yang-Mills theory. When the holomorphic function is quadratic in the transverse coordinates, the three-form flux coincides with that of ref. [5].

The purpose of the present paper is to obtain the general form of a three-form flux perturbation to the $AdS_5 \times S^5$ solution which preserves $\mathcal{N} = 2$ supersymmetry. We use the result of the $\mathcal{N} = 1$ case [12] and require further that the second supersymmetry is preserved. We find that the arbitrary holomorphic function in the $\mathcal{N} = 1$ case is restricted to a quadratic function of the transverse coordinates. This is a special form of the perturbation studied in ref. [5], which has one vanishing mass. It would be interesting to study a relation of our result to other works on soft breaking of $\mathcal{N} = 4$ to $\mathcal{N} = 2$ in the Coulomb branch [13, 14, 15]. In order to discuss the corresponding dual field theory and its RG flows we need to find out an exact solution with non-vanishing three-form flux. In addition, it would be also interesting to discuss the brane representations and massive vacua using S-dual transformations.

2. Unperturbed solution

The field content of the type IIB supergravity in ten dimensions [16, 17] is a metric g_{MN}, a complex Rarita-Schwinger field ψ_M, a real fourth-rank antisymmetric tensor field with a self-dual field strength F_{MNPQR}, a complex second-rank antisymmetric tensor field with a field strength G_{MNP}, a complex spinor field λ and a complex scalar filed $\tau = C + ie^{-\Phi}$. We denote ten-dimensional world indices as $M, N, \cdots = 0, 1, \cdots, 9$ and local Lorentz indices as $A, B, \cdots = 0, 1, \cdots, 9$. The fermionic fields satisfy chirality conditions $\bar{\Gamma}_{10D}\psi_M = \psi_M$, $\bar{\Gamma}_{10D}\lambda = -\lambda$, where $\bar{\Gamma}_{10D} = \Gamma^0\Gamma^1 \cdots \Gamma^9$ is the ten-dimensional chirality matrix. We choose the ten-dimensional gamma matrices Γ^A to have real components.

The field equations of this theory have a solution with the $\mathrm{AdS}_5 \times \mathrm{S}^5$ metric [18, 19]

$$g_{MN}dx^M dx^N = Z^{-\frac{1}{2}}\eta_{\mu\nu}dx^\mu dx^\nu + Z^{\frac{1}{2}}\delta_{mn}dx^m dx^n, \tag{1}$$

where $M = (\mu, m)$ ($\mu = 0, 1, 2, 3;\ m = 4, 5, \cdots, 9$), $Z = \frac{R^4}{r^4}$ and $r^2 = x^m x^n \delta_{mn}$. The constant R is a radius of AdS_5 and S^5. The fifth-rank field strength has non-vanishing components

$$\begin{aligned} F_{\mu\nu\rho\sigma m} &= \frac{1}{\kappa Z^2}\epsilon_{\mu\nu\rho\sigma}\partial_m Z, \\ F_{mnpqr} &= -\frac{Z^{\frac{1}{2}}}{\kappa}\epsilon_{mnpqrs}\partial^s Z, \end{aligned} \tag{2}$$

where κ is a coupling constant. This solution represents a supergravity configuration produced by D3-branes located at $x^m = 0$. More generally, the warp factor Z can be an arbitrary function of x^m which is harmonic except at points where D3-branes exist. We will consider the general Z but assume that the density of D3-branes vanishes for $r \to \infty$ and therefore $Z \to \frac{R^4}{r^4}$ for $r \to \infty$.

We are interested in how many supersymmetries are preserved by this solution and by a solution with a perturbation of G_{MNP} discussed later. They are found by studying vanishing of local supertransformations of the fermionic fields ψ_M and λ. The supertransformations of the fermionic fields [16, 17] in these backgrounds are

$$\begin{aligned}\delta\psi_M &= \frac{1}{\kappa} D_M \epsilon + \frac{1}{16\cdot 5!} i F_{P_1\cdots P_5}\Gamma^{P_1\cdots P_5}\Gamma_M\epsilon - \frac{1}{96} G_{NPQ}\left(\Gamma_M{}^{NPQ} - 9\delta^N_M \Gamma^{PQ}\right)\epsilon^*, \\ \delta\lambda &= \frac{1}{24} G_{MNP}\Gamma^{MNP}\epsilon, \end{aligned} \tag{3}$$

where the transformation parameter ϵ is a complex spinor satisfying the chirality condition $\bar\Gamma_{10D}\epsilon = \epsilon$. To study the supertransformations for the above backgrounds it is convenient to represent the ten-dimensional gamma matrices as

$$\begin{aligned}\Gamma^\mu &= \gamma^\mu \otimes \mathbf{1}, \\ \Gamma^m &= \bar\gamma_{4D} \otimes \gamma^m, \end{aligned} \tag{4}$$

where γ^μ and γ^m are the SO(3,1) and SO(6) gamma matrices respectively. The chirality matrices are defined as

$$\bar\gamma_{4D} = i\gamma^0\gamma^1\gamma^2\gamma^3, \qquad \bar\gamma_{6D} = i\gamma^4\gamma^5\gamma^6\gamma^7\gamma^8\gamma^9, \tag{5}$$

which are related to the ten-dimensional one as $\bar\Gamma_{10D} = -\bar\gamma_{4D}\bar\gamma_{6D}$.

The above solution (1), (2) without a perturbation has 32 supersymmetries [18, 19]. This can be seen as follows. The supertransformation $\delta\lambda$ automatically vanishes, while the vanishing of $\delta\psi_M$ requires

$$\tilde D_M \epsilon = 0, \tag{6}$$

where we have defined

$$\begin{aligned}\tilde D_\mu &= \partial_\mu - \frac{1}{8Z}\partial_m Z \gamma_\mu \gamma^m (1 + \bar\gamma_{4D}), \\ \tilde D_m &= \partial_m - \frac{1}{8Z}\partial_n Z \left(\delta^n_m \bar\gamma_{4D} - \gamma_m{}^n (1 + \bar\gamma_{4D})\right). \end{aligned} \tag{7}$$

For solutions of eq. (6) to exist the integrability condition

$$[\tilde D_M, \tilde D_N]\epsilon = 0 \tag{8}$$

must be satisfied. Using the expression (7) it is easy to show that eq. (8) is satisfied for an arbitrary ϵ. Therefore, all of 32 supersymmetries are preserved [18, 19]. From the four-dimensional field theoretical point of view in the AdS/CFT correspondence

16 of them are Poincaré supersymmetries while other 16 are conformal supersymmetries. Thus, we have $\mathcal{N}=4$ supersymmetry in four dimensions. More explicitly, the solutions of eq. (6) with the chirality $\bar{\gamma}_{4D}=-1$ have a form

$$\epsilon = Z^{-\frac{1}{8}}\eta, \tag{9}$$

where η is an arbitrary constant spinor with the chirality $\bar{\gamma}_{4D}=-1$. These solutions correspond to Poincaré supersymmetries. The solutions with the chirality $\bar{\gamma}_{4D}=+1$ depend on x^μ and correspond to conformal supersymmetries.

3. Three-form flux with $\mathcal{N}=2$ supersymmetry

By introducing a perturbation of the three-form flux G_{mnp} the $\mathcal{N}=4$ supersymmetry of the unperturbed supergravity background is broken to lower $\mathcal{N}$. In ref. [12] the conditions on G_{mnp} for unbroken $\mathcal{N}=1$ supersymmetry were studied. The supersymmetry parameter is expanded as $\epsilon=\epsilon_0+\epsilon_1+\cdots$, where ϵ_0 is a solution of eq. (6) for the unperturbed background and ϵ_1 is the first order correction due to the perturbation. Substituting it into eq. (6) ϵ_1 is determined by ϵ_0. To proceed it is convenient to define complex coordinates z^i $(i=1,2,3)$ from x^m

$$z^1=\frac{1}{\sqrt{2}}(x^4+ix^7), \quad z^2=\frac{1}{\sqrt{2}}(x^5+ix^8), \quad z^3=\frac{1}{\sqrt{2}}(x^6+ix^9). \tag{10}$$

It was required in ref. [12] that one of the four Poincaré supersymmetries $\epsilon_0 = Z^{-\frac{1}{8}}\eta$, where η is a constant spinor satisfying

$$\gamma^{\bar{1}}\eta=\gamma^{\bar{2}}\eta=\gamma^{\bar{3}}\eta=0, \tag{11}$$

is preserved. Here, $\bar{i}$ denote indices of $\bar{z}^i$, while i denote those of z^i. Using the expression $\bar{\gamma}_{6D}=(1-\gamma^1\gamma^{\bar{1}})(1-\gamma^2\gamma^{\bar{2}})(1-\gamma^3\gamma^{\bar{3}})$ it is easy to see that this ϵ_0 has the chirality $\bar{\gamma}_{4D}=-\bar{\gamma}_{6D}=-1$ appropriate for the Poincaré supersymmetry. Then, this $\mathcal{N}=1$ supersymmetry restricts the form of G_{mnp} as [12]

$$\begin{aligned}
G_{ijk} &= 0, \\
G_{ij\bar{k}} &= \frac{2}{3}\hat{\epsilon}_{\bar{k}}{}^{pq}\partial^{-2}\partial_p\partial_{[i}\phi\partial_{j]}\partial_q Z + \hat{\epsilon}_{ij}{}^{\bar{l}}\partial_{\bar{k}}\partial_{\bar{l}}\psi, \\
G_{i\bar{j}\bar{k}} &= \frac{1}{12}\hat{\epsilon}_{\bar{j}\bar{k}}{}^{l}\left(2\partial_i\partial_l\phi Z - \alpha\hat{\epsilon}_{il}{}^{\bar{k}}\partial_{\bar{k}}Z - 4\partial_{[i}\phi\partial_{l]}Z\right), \\
G_{\bar{i}\bar{j}\bar{k}} &= \frac{1}{6}\hat{\epsilon}_{\bar{i}\bar{j}\bar{k}}\delta^{l\bar{l}}\partial_l\phi\partial_{\bar{l}}Z,
\end{aligned} \tag{12}$$

where $\phi(z^1, z^2, z^3)$ is an arbitrary holomorphic function, α is an arbitrary constant and ψ is an arbitrary harmonic function.* In eq. (12) $\hat{\epsilon}_{ij}{}^{\bar{k}}$ and $\hat{\epsilon}_{\bar{i}\bar{j}}{}^{k}$ are totally antisymmetric in their indices and take constant values 0, ±1 regardless of index positions, and $\partial^2 = 2\delta^{i\bar{i}}\partial_i\partial_{\bar{i}}$ is the Laplacian. The three-form flux (12) also satisfies the Bianchi identity as well as the linearized field equation.

We shall obtain conditions on G_{mnp} for unbroken $\mathcal{N} = 2$ supersymmetry. We require that in addition to $\epsilon_0 = Z^{-\frac{1}{8}}\eta$ the second supersymmetry with the parameter

$$\epsilon_0 = Z^{-\frac{1}{8}}\gamma^1\gamma^2\eta \tag{13}$$

is also preserved. This ϵ_0 satisfies

$$\gamma^1\epsilon_0 = \gamma^2\epsilon_0 = \gamma^{\bar{3}}\epsilon_0 = 0 \tag{14}$$

and has the chirality $\bar{\gamma}_{4D} = -1$. Comparing eqs. (11) and (14) it is easy to see that the conditions for the second supersymmetry are obtained from eq. (12) by the replacements

$$1 \leftrightarrow \bar{1}, \quad 2 \leftrightarrow \bar{2}, \quad \alpha \to \alpha', \quad \phi(z^1, z^2, z^3) \to \phi'(\bar{z}^1, \bar{z}^2, z^3), \quad \psi \to \psi' \tag{15}$$

for new α', ϕ' and ψ'.

We now require that the expression (12) and that with the replacements (15) are compatible each other. Let us first consider G_{123}. From the expression (12) we have $G_{123} = 0$. From the other expression we have $G_{123} = \frac{1}{6}\partial_3^2\phi' Z$, which is obtained from $G_{\bar{1}\bar{2}3}$ in eq. (12) by the replacements (15). Thus we obtain a condition

$$G_{123}: \quad \partial_3^2\phi' = 0. \tag{16}$$

Similarly, we obtain conditions

$$\begin{aligned}
G_{2\bar{2}1} + G_{3\bar{3}1}: &\quad \partial_{\bar{2}}\partial_3\phi' = 0, \\
G_{1\bar{1}2} + G_{3\bar{3}2}: &\quad \partial_{\bar{1}}\partial_3\phi' = 0, \\
G_{\bar{1}\bar{2}3}: &\quad \partial_3^2\phi = 0, \\
G_{\bar{2}2\bar{1}} + G_{3\bar{3}\bar{1}}: &\quad \partial_2\partial_3\phi = 0, \\
G_{\bar{1}1\bar{2}} + G_{3\bar{3}\bar{2}}: &\quad \partial_1\partial_3\phi = 0, \\
G_{1\bar{2}\bar{3}}: &\quad \partial_1^2\phi = \partial_{\bar{2}}^2\phi', \\
G_{\bar{1}2\bar{3}}: &\quad \partial_2^2\phi = \partial_{\bar{1}}^2\phi'.
\end{aligned} \tag{17}$$

* In ref. [12] the constant α is required to vanish by the Bianchi identity. However, we do not agree with this result and leave α non-vanishing.

The component $G_{1\bar{1}3} + G_{2\bar{2}3}$ vanishes in both of the two expressions and gives no condition. These conditions fix the forms of ϕ and ϕ' as

$$
\begin{aligned}
\phi &= m_1(z^1)^2 + m_2(z^2)^2 + 2az^1z^2 + b_1z^1 + b_2z^2 + b_3z^3, \\
\phi' &= m_2(\bar{z}^1)^2 + m_1(\bar{z}^2)^2 + 2a'\bar{z}^1\bar{z}^2 + b_1'\bar{z}^1 + b_2'\bar{z}^2 + b_3'z^3,
\end{aligned} \tag{18}
$$

where m_1, m_2, a, a', b_i and b_i' are arbitrary constants. We further obtain conditions

$$
\begin{aligned}
G_{\bar{1}23}: &\quad \partial_{\bar{1}}^2\psi = \partial_2^2\psi', \\
G_{1\bar{2}3}: &\quad \partial_{\bar{2}}^2\psi = \partial_1^2\psi', \\
G_{31\bar{1}}: &\quad \partial_{\bar{1}}\partial_{\bar{2}}\psi = -\partial_1\partial_2\psi', \qquad a = -a'.
\end{aligned} \tag{19}
$$

By a linear transformation $z^i \to U^i{}_j z^j$ $(i, j = 1, 2)$ with a unitary matrix U we can set $a = -a' = 0$.

So far we have not used a particular form of Z. We now examine the remaining conditions first by using the asymptotic form $Z \sim \frac{R^4}{r^4}$ for $r \to \infty$ to fix the coefficients in eq. (18) and α, α'. We then check that the conditions are satisfied also for $r < \infty$. From the equation for $G_{1\bar{1}\bar{3}}$ we obtain

$$
\begin{aligned}
G_{1\bar{1}\bar{3}}: \quad & -\frac{1}{6}\partial_1\partial_2\phi Z + \frac{1}{12}\left(\alpha\partial_{\bar{3}}Z + 2\partial_1\phi\partial_2 Z - 2\partial_2\phi\partial_1 Z\right) \\
&= \frac{1}{6}\partial_{\bar{1}}\partial_{\bar{2}}\phi' Z - \frac{1}{12}\left(\alpha'\partial_{\bar{3}}Z + 2\partial_{\bar{1}}\phi'\partial_{\bar{2}}Z - 2\partial_{\bar{2}}\phi'\partial_{\bar{1}}Z\right).
\end{aligned} \tag{20}
$$

The equation for $G_{2\bar{2}\bar{3}}$ gives the same condition. Substituting the asymptotic form $Z \sim \frac{R^4}{r^4}$ and eq. (18) into eq. (20) we find $\alpha' = -\alpha$ and $b_1 = b_2 = b_1' = b_2' = 0$. The remaining conditions become

$$
\begin{aligned}
G_{1\bar{1}\bar{2}}: &\quad \partial_1\partial_{\bar{3}}\psi' = \frac{1}{12}(\alpha\partial_{\bar{2}} + 2b_3\partial_1)Z, \\
G_{3\bar{3}\bar{1}}: &\quad \partial_2\partial_{\bar{3}}\psi' = -\frac{1}{12}(\alpha\partial_{\bar{1}} - 2b_3\partial_2)Z, \\
G_{\bar{1}\bar{2}\bar{3}}: &\quad \partial_{\bar{3}}^2\psi' = \frac{1}{6}b_3\partial_{\bar{3}}Z, \\
G_{23\bar{3}}: &\quad \partial_{\bar{1}}\partial_{\bar{3}}\psi = -\frac{1}{12}(\alpha\partial_2 - 2b_3'\partial_{\bar{1}})Z, \\
G_{12\bar{2}}: &\quad \partial_{\bar{2}}\partial_{\bar{3}}\psi = \frac{1}{12}(\alpha\partial_1 + 2b_3'\partial_{\bar{2}})Z, \\
G_{12\bar{3}}: &\quad \partial_{\bar{3}}^2\psi = \frac{1}{6}b_3'\partial_3 Z.
\end{aligned} \tag{21}
$$

Comparing the equation obtained by applying $\partial_{\bar{3}}$ to the first equation in eq. (21) and that obtained by applying ∂_1 to the third equation we find $\alpha = 0$. Then, eq. (21) determines ψ, ψ' as

$$\begin{aligned} \partial_{\bar{3}}\psi &= \frac{1}{6}b'_3 Z + f(z^1, z^2, z^3), \\ \partial_{\bar{3}}\psi' &= \frac{1}{6}b_3 Z + f'(\bar{z}^1, \bar{z}^2, z^3), \end{aligned} \tag{22}$$

where f and f' are arbitrary functions of each variables. Substituting eq. (22) into the $\bar{z}^3$ derivative of eq. (19) and using the asymptotic form $Z \sim \frac{R^4}{r^4}$ we obtain $b_3 = b'_3 = 0$.

As a result of these analyses at asymptotic region $r \sim \infty$ we obtain

$$\begin{aligned} \phi &= m_1(z^1)^2 + m_2(z^2)^2, \\ \phi' &= m_2(\bar{z}^1)^2 + m_1(\bar{z}^2)^2. \end{aligned} \tag{23}$$

We have to check that eqs. (19), (20) and (21) are satisfied even for $r < \infty$. Substituting eq. (23) into eq. (21) we find that their right-hand sides vanish. The general solution of these equations are

$$\begin{aligned} \psi &= f(z^1, z^2, z^3)\bar{z}^3 + g(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3), \\ \psi' &= f'(\bar{z}^1, \bar{z}^2, z^3)\bar{z}^3 + g'(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3), \end{aligned} \tag{24}$$

where f, f', g and g' are arbitrary functions of each variables. The conditions in eq. (19) then require

$$\partial_{\bar{1}}^2 g = \partial_2^2 g', \qquad \partial_{\bar{2}}^2 g = \partial_1^2 g', \qquad \partial_{\bar{1}}\partial_{\bar{2}} g = -\partial_1\partial_2 g'. \tag{25}$$

The conditions that ψ and ψ' in eq. (24) are harmonic are

$$\begin{aligned} \partial^2 g(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3) &= -\partial_3 f(z^1, z^2, z^3), \\ \partial^2 g'(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3) &= -\partial_3 f'(\bar{z}^1, \bar{z}^2, z^3). \end{aligned} \tag{26}$$

The functions f and f' do not appear in G_{mnp} as one can see by substituting eq. (24) into eq. (12). We only need to consider g and g'. Eq. (26) means that $\partial^2 g$ and $\partial^2 g'$ are independent of $\bar{z}^1, \bar{z}^2$ and z^1, z^2 respectively. These conditions are automatically satisfied when g and g' satisfy eq. (25). The functions g and g' need not be harmonic. Finally, we have to consider eq. (20). Substituting eq. (23) into eq. (20) we obtain

$$\left(m_1 z^1\partial_2 - m_2 z^2\partial_1 + m_2\bar{z}^1\partial_{\bar{2}} - m_1\bar{z}^2\partial_{\bar{1}}\right) Z = 0. \tag{27}$$

This means that Z is invariant under SO(2) rotation of $(\sqrt{m_1}\, z^1, \sqrt{m_2}\, z^2)$ and $(\sqrt{m_2}\, \bar{z}^1, \sqrt{m_1}\, \bar{z}^2)$. Therefore, Z must be a function of SO(2) invariant variables $r^2 = 2\,(z^1\bar{z}^1 + z^2\bar{z}^2)$, $m_1(z^1)^2 + m_2(z^2)^2$, $m_2(\bar{z}^1)^2 + m_1(\bar{z}^2)^2$ and $m_1 z^1\bar{z}^2 - m_2 z^2\bar{z}^1$.

Let us summarize the result. The general form of the three-form flux G_{mnp} which preserves the $\mathcal{N} = 2$ supersymmetry at the first order of the perturbation is given by eq. (12) with $\alpha = 0$, ϕ in eq. (23) and ψ replaced by $g(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3)$ satisfying eq. (25) for some function $g'(z^1, \bar{z}^1, z^2, \bar{z}^2, z^3)$. Thus, ϕ, which is an arbitrary holomorphic function in the $\mathcal{N} = 1$ case [12], is severely restricted to a quadratic function in the $\mathcal{N} = 2$ case. Such $\mathcal{N} = 2$ preserving perturbation is possible only when the warp factor Z satisfies eq. (27).

In our analysis at the first order of the perturbation we did not need the condition $m_1 = m_2$ to obtain the $\mathcal{N} = 2$ supersymmetry. At higher orders [20] we would need the condition $m_1 = m_2$ since these parameters correspond to masses of two $\mathcal{N} = 1$ chiral multiplets, which should be combined into an $\mathcal{N} = 2$ hypermultiplet. This is indeed the case in the field theory side. To see this let us consider two $\mathcal{N} = 1$ chiral supermultiplets (A_1, ψ_1) and (A_2, ψ_2), where A_1, A_2 are complex scalar fields and ψ_1, ψ_2 are Weyl spinor fields, with the action

$$\begin{aligned} S = \int d^4x \Big[& -\partial_\mu A_1^* \partial^\mu A_1 - \partial_\mu A_2^* \partial^\mu A_2 - i\psi_1 \sigma^\mu \partial_\mu \bar{\psi}_1 - i\psi_2 \sigma^\mu \partial_\mu \bar{\psi}_2 \\ & - m_1^2 A_1^* A_1 - m_2^2 A_2^* A_2 - \frac{1}{2} m_1 \left(\psi_1\psi_1 + \bar{\psi}_1\bar{\psi}_1\right) - \frac{1}{2} m_2 \left(\psi_2\psi_2 + \bar{\psi}_2\bar{\psi}_2\right)\Big]. \end{aligned} \qquad (28)$$

Here we have used the two-component spinor notation in ref. [21]. S is invariant under the $\mathcal{N} = 1$ supertransformation

$$\delta A_i = \sqrt{2}\epsilon\psi_i, \qquad \delta\psi_i = \sqrt{2} i \sigma^\mu \bar{\epsilon} \partial_\mu A_i - \sqrt{2} m_i \epsilon A_i^* \qquad (i = 1, 2). \qquad (29)$$

The exact $N = 2$ supersymmetry of course requires $m_1 = m_2$. However, even for $m_1 \neq m_2$, it is also invariant under the second supertransformation

$$\begin{aligned} \delta A_1 &= \sqrt{2}\epsilon\psi_2, & \delta\psi_1 &= \sqrt{2} i \sigma^\mu \bar{\epsilon} \partial_\mu A_2 - \sqrt{2} m_1 \epsilon A_2^*, \\ \delta A_2 &= -\sqrt{2}\epsilon\psi_1, & \delta\psi_2 &= -\sqrt{2} i \sigma^\mu \bar{\epsilon} \partial_\mu A_1 + \sqrt{2} m_2 \epsilon A_1^* \end{aligned} \qquad (30)$$

at the first order in m_1, m_2. Thus, the condition $m_1 = m_2$ is needed only in quadratic and higher order terms for the $\mathcal{N} = 2$ supersymmetry.

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