

A Supersymmetric SO(10) Grand Unified Theory with an Intermediate Symmetry

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We show a concrete construction of a supersymmetric SO(10) Grand Unified Theory with an intermediate symmetry $SU(2)_L \otimes SU(2)_R \otimes SU(3)_C \otimes U(1)_{B-L}$. The intermediate symmetry breaks down to the standard model symmetry at an intermediate scale 10^{10-12} GeV. By the breakdown of the intermediate symmetry right-handed neutrinos acquire mass of the intermediate scale through a *renormalizable* Yukawa coupling.¹⁾

§1. Introduction

When we construct a Grand Unified Theory(GUT) based on SO(10),²⁾ in general, we have singlet fermions under the Standard Model(SM) -what we call right-handed neutrino. Then it is the question what scale is the right-handed neutrino mass($\equiv M_{\nu_R}$).

One of the plausible answer is that $M_{\nu_R} \sim 10^{10-12}$ GeV,³⁾ which we will call the intermediate scale. The experimental data on the deficit of the solar neutrino can be explained by the Mikheyev-Smirnov-Wolfenstein(MSW) solution.⁴⁾ According to one of the MSW solutions, the mass of the muon neutrino seems to be $m_{\nu_\mu} \simeq 10^{-3}$ eV. Such a small mass can be led by the seesaw mechanism:⁵⁾ A muon neutrino can acquire a mass of $O(10^{-3})$ eV if the Majorana mass of the right-handed muon neutrino is about 10^{12} GeV.

Under the SM right-handed neutrinos can have Majorana masses because they are singlet, so the scale of the right-handed neutrinos M_{ν_R} is expected to be a scale below which the SM is realized. It is natural to consider that a certain symmetry breaks down to the SM symmetry at the intermediate scale.

In the Minimal Supersymmetric Standard Model(MSSM), however, the GUT symmetry SO(10) cannot be the symmetry which breaks down at the intermediate scale, because we know that the present experimental values of gauge couplings are successfully unified at a unification scale $M_G \simeq 10^{16}$ GeV.⁶⁾

Thus we are led to a possibility that a certain group breaks down to the SM group at the intermediate scale at which right-handed neutrinos gain mass through a *renormalizable* coupling.

In the previous work⁷⁾ it was shown that we have a possibility to construct a Supersymmetric(SUSY) SO(10) GUT with an intermediate symmetry $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes SU(3)_C$ ($\equiv G_{2231}$)^{**)} which breaks down to the SM group at an intermediate scale $M_{\nu_R} \sim 10^{10-12}$ GeV where right-handed neutrinos gain mass.

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^{**)} We use a notation $G_{lmn\dots}$ to represent $SU(l) \otimes SU(m) \otimes SU(n)\dots$. If $l = 1$, it means U(1).

The purpose of this paper is to show an explicit form of a superpotential for a SUSY SO(10) GUT whose symmetry breaks down to G_{2231} at a GUT scale M_G and G_{2231} breaks down to the SM symmetry at the intermediate scale M_{ν_R} .

In §2 we see the outline of the scenario by recaptulating the previous work⁷⁾ briefly. Then we see the explicit model for a SUSY SO(10) GUT with an intermediate symmetry in §3. Finally, in §4 summary and discussion are given.

§2. Scenario

The outline of the scenario is graphically depicted in Fig. 1.

This is a graph expressing the running of the gauge couplings. At a certain scale $M_G \sim 10^{16}$ GeV, SO(10) breaks down to G_{2231} . G_{2231} breaks down to the SM symmetry G_{231} at the intermediate scale M_{ν_R} .

In the region between M_G and M_{ν_R} , the intermediate region, we have extra fields in addition to the ordinary matter, lepton, quark and Higgs doublets. Because we require that right-handed neutrinos gain mass when G_{2231} breaks down to G_{231} we have to introduce $(1,3,1,-6) + \text{h.c.}^*$ multiplet in the intermediate region.

Introducing only this multiplet in the intermediate region destroys the gauge unification. To achieve gauge unification we have to introduce other multiplets.

For example, if we have the following multiplets in the intermediate region in addition to leptons (including right-handed neutrinos) and quarks

$$\begin{array}{llll}
 (1, 3, 1, -6) & 1 & (1, 3, 1, 6) & 1 \quad \text{responsible for } \nu_R \text{ mass} \\
 (2, 2, 1, 0) & 2 & & \text{ordinary Higgs doublets} \\
 (2, 1, 3, -1) & 1 & (2, 1, \bar{3}, 1) & 1 \\
 (2, 1, 1, 3) & 1 & (2, 1, 1, -3) & 1 \\
 (1, 3, 1, 0) & 1 & & \\
 (1, 1, 8, 0) & 1 & &
 \end{array} \quad , \quad (2.1)$$

gauge couplings are unified at $M_G \sim 10^{16}$ GeV. In the list, for example, $(1,3,1,-6)$ 1 stands for that the representation of the matter under G_{2231} is $(1,3,1,-6)$ and its number is one. In this case the unified coupling is calculated to be about $1/18$. **)

*) $(1,3,1,-6)$ stands for the representation under G_{2231} . We adopt the normalization for $U(1)_{B-L}$, $T_4^{15} = \text{diag}(1, 1, 1, -3)$.

**) This is not the case of Fig. 1.

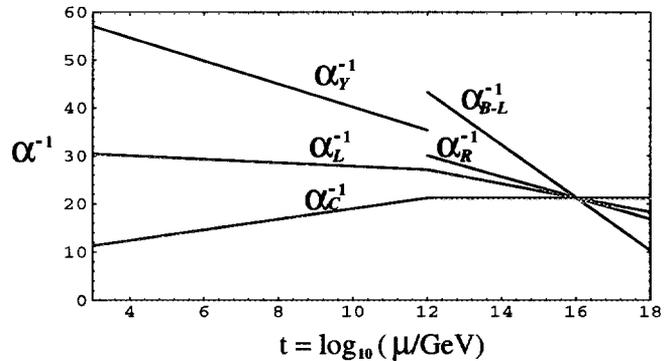


Fig. 1. Evolution of coupling constants cited from Ref. 7). The calculation is based on one-loop approximation. M_R is taken to be 10^{12} GeV.

$(1,3,1,-6) + \text{h.c}$ is a Higgs field responsible for the right-handed neutrino mass. One combination of $(2,2,1,0)$ becomes so called Higgs doublets after G_{2231} breaks down to G_{231} . Others are introduced to achieve gauge unification.

Below M_{ν_R} the ordinary matter of MSSM survive.

Though there are many other combination of multiplets which realize gauge unification we will use this matter content for the intermediate region.

§3. Analysis

3.1. Model

To have the multiplets (2·1) in the intermediate region we introduce the following SO(10) multiplets.

	SO(10)	G_{2231}	
H	10	$(2, 2, 1, 0), \dots$	
A	45	$(1, 3, 1, 0), (1, 1, 8, 0), \dots$	
Φ	126	$(1, 3, 1, -6), (2, 2, 1, 0), \dots$	
$\bar{\Phi}$	$\overline{126}$	$(1, 3, 1, 6), (2, 2, 1, 0), \dots$	(3·1)
Δ	210	$(1, 3, 1, 0), (1, 1, 8, 0), \dots$	
$\Psi_{i=1\sim 4}$	16	$(2, 1, 3, -1), (2, 1, 1, 3)$, quarks/leptons	
$\bar{\Psi}$	$\overline{16}$	$(2, 1, \bar{3}, 1), (2, 1, 1, -3), \dots$	

In this list numbers in the column of SO(10) means SO(10) representation. In the last column we show what representation in (2·1) is contained in the corresponding SO(10) multiplet.

By the requirement that the right-handed neutrinos get mass through a renormalizable coupling, we introduce 126 and $\overline{126}$. Also they breaks G_{2231} to G_{231} . As a candidate of ordinary Higgs doublets 10 is introduced. There are other candidates for ordinary Higgs doublets in 126 and $\overline{126}$. Then the ordinary Higgs doublets will be a mixture of these three. To break SO(10) to the SM group via G_{2231} , namely to have the intermediate symmetry G_{2231} , we use 45 and 210.

With these multiplets the superpotential for the theory is written as follows.

$$W = W_{\text{mass}} + W_{\text{int}} + W_{\psi}. \tag{3·2}$$

W_{mass} consists of the most general bilinear terms:

$$W_{\text{mass}} = \frac{1}{2}M_H H^2 + M_{\Phi} \bar{\Phi} \Phi + \frac{1}{2}M_{\Delta} \Delta^2 + \frac{1}{2}M_A A^2 + M_{\Psi} \bar{\Psi} \Psi_4. \tag{3·3}$$

We define only Ψ_4 has a mass term with $\bar{\Psi}$, because by a redefinition of Ψ_4 , namely by a rotation among $\Psi_{i=1-4}$, it is possible that only the new Ψ_4 has a mass term with $\bar{\Psi}$.

We require all mass parameters are $O(M_U)$ because M_U is the natural order for them.

W_{int} has the most general interaction terms without 16 and $\overline{16}$:

$$\begin{aligned}
 W_{\text{int}} = & Y_{H\Phi\Delta}H\Phi\Delta + Y_{H\bar{\Phi}\Delta}H\bar{\Phi}\Delta + \frac{1}{3!}Y_{\Delta}\Delta^3 + Y_{\Phi\Delta}\bar{\Phi}\Delta\Phi + Y_{\Phi A}\bar{\Phi}A\Phi \\
 & + \frac{1}{2}Y_{\Delta A^2}A^2\Delta + \frac{1}{2}Y_{\Delta^2 A}A\Delta^2.
 \end{aligned}
 \tag{3.4}$$

We require all Yukawa couplings are at most $O(1)$.*)

Finally, W_{Ψ} represents the most general interaction terms with 16 and $\bar{16}$.

$$\begin{aligned}
 W_{\Psi} = & \sum_{i=3}^4 Y_{\Psi\Delta i}\bar{\Psi}\Delta\Psi_i + \sum_{i=2}^4 Y_{\Psi A i}\bar{\Psi}A\Psi_i + \sum_{ij} y_{ij}\Psi_i\Psi_j\bar{\Phi} + y'\bar{\Psi}\bar{\Psi}\Phi \\
 & + \sum_{ij} \tilde{y}_{ij}\Psi_i\Psi_jH + \tilde{y}'\bar{\Psi}\bar{\Psi}H.
 \end{aligned}
 \tag{3.5}$$

By the same reason that only Ψ_4 has a mass term with $\bar{\Psi}$, only $\Psi_{3,4}$ have couplings with Δ and only $\Psi_{2,3,4}$ have couplings with A .

What we will do is to give a relation between couplings appearing in the superpotential which realize the scenario in terms of the vacuum expectation values.

3.2. Vacuum expectation value(VEV)

There are many component which can have VEV.**)

Field	:	Component	LittleGroup	
A	:	α	G_{2231}	
	:	β	G_{241}	
Φ	:	ϕ	$SU(5)$	
$\bar{\Phi}$:	$\bar{\phi}$	$SU(5)$	(3.6)
Δ	:	a	G_{224}	
		b	G_{2231}	
		c	G_{2311}	

For example, 45 has two component which can have VEV. We denote a G_{2231} singlet component, that is, the component which can break $SO(10)$ down to G_{2231} by α . The other component is expressed by β which is a singlet under G_{241} . While α can have a VEV of $O(M_G)$, the VEV of β can be at most $O(M_{\nu_R})$ since it is not G_{2231} singlet. The others are understood in the same way. In 210 there are two component which can have VEVs of $O(M_G)$ and one component whose VEV is at most $O(M_{\nu_R})$ and so on. Here after α, β , and so on mean the VEV of the corresponding component.

In our scenario, above GUT scale we have the multiplets (3.1). At the GUT scale M_G , G_{2231} singlet components of these matters α, a and b acquire VEV of $O(M_G)$ by which $SO(10)$ breaks down to G_{2231} . After $SO(10)$ breaking almost of all components contained in the matter (3.1) besides the multiplets listed in (2.1) and quarks/leptons

*) More exactly, as an expansion parameter for the perturbation we require they are at most $O(1)$. As an expansion parameter for the perturbation they appear with multiplied by a certain overall factor. We do not touch the detail here.

**) We assume the VEVs of 16 and $\bar{16}$ are 0.

gain the mass of $O(M_G)$ and hence decouple from the spectrum. Then we have the multiplets (2.1) and quarks/leptons in the intermediate region. At the intermediate scale M_{ν_R} SM singlet components acquire VEVs and G_{2231} breaks down to G_{231} . At that time matters besides the ordinary matter gain the mass of $O(M_{\nu_R})$ and below M_{ν_R} MSSM is realized. Thus in our model the right-handed neutrino mass is naturally understood to be $O(M_{\nu_R} \sim 10^{10\sim 12} \text{ GeV})$.

3.3. Result

From now on we express the relation between couplings appearing in the superpotential (3.2) in terms of VEVs like the case of SUSY SU(5) GUT.

The relation is derived from the potential minimum condition and the mass matrix for the matter. The potential minimum condition is read off from the D-flat and F-flat condition in SUSY theory.

From D-flat condition we have

$$|\phi|^2 - |\bar{\phi}|^2 = 0, \tag{3.7}$$

that is the VEV of 126 equals that of $\bar{126}$.

The F-flat condition is given as follows:

$$C \begin{pmatrix} M_\Delta \\ M_A \\ Y_\Delta \\ Y_{\Delta A^2} \\ Y_{\Delta^2 A} \end{pmatrix} = - \begin{pmatrix} 1/(10\sqrt{6})Y_{\Phi\Delta} \\ 1/(10\sqrt{2})Y_{\Phi\Delta} \\ 1/10Y_{\Phi\Delta} \\ \sqrt{6}/10Y_{\Phi A} \\ 1/5Y_{\Phi A} \end{pmatrix} \phi\bar{\phi}, \tag{3.8}$$

$$M_\phi = -Y_{\Phi A} \left(\frac{\sqrt{6}\alpha}{10} + \frac{\beta}{5} \right) - Y_{\Phi\Delta} \left(\frac{a}{10\sqrt{6}} + \frac{b}{10\sqrt{2}} + \frac{c}{10} \right), \tag{3.9}$$

where C is a matrix consist of VEVs,

$$C = \begin{pmatrix} a, & 0, & \frac{1}{12\sqrt{6}}c^2, & -\frac{1}{2\sqrt{6}}\beta^2, & 24\sqrt{2}iab \\ b, & 0, & \frac{1}{18\sqrt{2}}b^2 + \frac{1}{18\sqrt{2}}c^2, & -\frac{1}{3\sqrt{2}}\alpha^2, & 24\sqrt{2}ia\alpha + 24\sqrt{2}i\beta c \\ c, & 0, & \frac{1}{6\sqrt{6}}ac + \frac{1}{9\sqrt{2}}bc, & -\frac{1}{\sqrt{6}}\alpha\beta, & 16\sqrt{6}iac + 24\sqrt{2}ib\beta \\ 0, & \alpha, & 0, & -\frac{\sqrt{2}}{3}\alpha b - \frac{1}{\sqrt{6}}\beta c, & 24\sqrt{2}iab + 8\sqrt{6}ic^2 \\ 0, & \beta, & 0, & -\frac{1}{\sqrt{6}}\alpha c - \frac{1}{\sqrt{6}}a\beta, & 24\sqrt{2}ibc \end{pmatrix}. \tag{3.10}$$

Then couplings, in general satisfy the above relation.

In the scenario considered here there are also constraints from the fact that the multiplets (2.1) survive at the intermediate region. Those constraints give the relation between not only the couplings but also the VEVs.

For example, from the constraints that (1,3,1,0) and (1,1,8,0) survive at the intermediate region we have

$$a = -0.987293b + a_1\epsilon, \tag{3.11}$$

$$\beta = -1.83185\frac{\alpha}{b}c + \beta_2\epsilon^2, \tag{3.12}$$

where ϵ is the ratio $\frac{M_{\nu R}}{M_G}$ and expected to be $10^{-4\sim-6}$. b, α, a_1 and β_2 are free parameters of $O(M_G)$. Because β and c are not G_{2231} singlet their order of magnitude is $O(M_{\nu R})$.

By substituting the above relation into (3.8) and (3.9) we get the exact relation between couplings. We can explicitly calculate that all couplings are $O(M_G)$ and $O(1)$ with respect to mass parameters and Yukawa couplings.

We have other relations by considering the mass matrix of $(2,2,1,0)$, $(2,1,1,-3)$ + h.c and so on. Thus we can construct a SUSY SO(10) GUT with an intermediate symmetry.

§4. Summary and discussion

We have constructed a SUSY SO(10) GUT with an Intermediate Scale.

Because there are many candidates for matter content in the intermediate region ⁷⁾ there must be many other GUTs similar with what considered here.

Then We can understand the right-handed neutrino mass as a reflection of a symmetry breaking. The right-handed neutrino mass may be naturally understood by the scenario.

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