An SU(5) SUSY Model with R-Parity Violation and Radiatively Induced Neutrino Masses

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The radiatively induced neutrino mass matrix is investigated on the basis of an SU(5) SUSY model. In order to evade the proton decay, an ansatz based on a discrete symmetry Z_2 is assumed: although, at the unification scale, we have two types of superfields $\Psi_{L(\pm)} = \overline{5}_{L(\pm)} + 10_{L(\pm)}$, which are transformed as $\Psi_{L(\pm)} \to \pm\Psi_{L(\pm)}$ under the discrete symmetry Z_2, the particles $\Psi_{L(+)}$ are decoupled after the SU(5) symmetry is broken, so that our quarks and leptons belong to $\Psi_{L(-)}$. The R-parity-violating terms for our quarks and leptons $\Psi_{L(-)}$ are basically forbidden under the symmetry Z_2. However, we assume that mixings between members of $\Psi_{L(+)}$ and those of $\Psi_{L(-)}$ are in part caused after SU(5) is broken. As a result, the R-parity-violating interactions are in part allowed, so that the neutrino masses are radiatively generated, while the proton decay due to the R-parity violating terms is still forbidden because the term $d_R^c d_R^c u_R^c$ has $z = -1$.

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I. INTRODUCTION

The origin of the neutrino mass generation is still a mysterious problem in the unified understanding of the quarks and leptons. The Zee model [1] is one of several promising models, because it has only 3 free parameters and it can naturally lead to a large neutrino mixing [2], especially, to a bimaximal mixing [3]. However, the original Zee model is not on the framework of a grand unification theory (GUT). The most attractive idea [4] to embed the Zee model into GUTs is to identify the Zee scalar h^+ as the slepton $\widetilde{e}_R$ in an R-parity-violating supersymmetric (SUSY) model. However, usually, it is accepted that SUSY models with R-parity violation are incompatible with a GUT scenario, because the R-parity-violating interactions induce proton decay [5].

In the present paper, in order to suppress this kind of proton decay, a discrete symmetry Z_2 is introduced. The essential idea is as follows: At the unification scale $\mu = M_X$, we have two types of superfields $\Psi_{L(\pm)} = \overline{5}_{L(\pm)} + 10_{L(\pm)}$, which are transformed with ± 1 under the discrete symmetry Z_2 (we will call it "Z_2-parity" hereafter). We consider that the particles $\Psi_{L(+)}$ are decoupled after the SU(5) symmetry is broken (but Z_2 is still unbroken), so that our quarks and leptons $\overline{5}_L + 10_L$ belong to $\Psi_{L(-)}$. The R-parity violating terms are given by the combinations $\overline{5}_{(+)}\overline{5}_{(+)}10_{(+)}$, $\overline{5}_{(-)}\overline{5}_{(-)}10_{(+)}$ and $\overline{5}_{(+)}\overline{5}_{(-)}10_{(-)}$, so that they basically do not contribute to the quarks and leptons with Z_2-parity $z = -1$, because of the Z_2 symmetry. However, we assume that mixings of the members of $\Psi_{L(+)}$ with those of $\Psi_{L(-)}$ are caused in part after SU(5) is broken. As a result, the R-parity-violating interactions are in part allowed, so that the neutrino masses are radiatively generated, while the proton decay due to the R-parity-violating terms is still forbidden because the term $d_R^c d_R^c u_R^c$ is still exactly forbidden below $\mu = M_X$ in the present scheme. The details will be discussed in the next section.

The purpose of the present paper is to investigate the possible forms of the radiatively induced neutrino mass matrix under the Z_2 symmetry. In Sec. III, we will give them, including a numerical study. In Sec. IV, we will give a comment on the Higgs scalars in the present scheme. Finally, Sec. V is devoted to our conclusion.

II. Z_2 SYMMETRY AND THE PROTON DECAY

We identify the Zee scalar h^+ as the slepton $\widetilde{e}_R^+$, which is a member of SU(5) 10-plet sfermions $\widetilde{\psi}_{10}$. Then, the Zee interactions correspond to the following R-parity-violating interactions

$$\begin{aligned}
&\lambda_{ijk}(\overline{\psi}_{\overline{5}}^c)_i^A(\psi_{\overline{5}})_j^B(\widetilde{\psi}_{10})_{kAB} \\
&= \frac{1}{\sqrt{2}}\lambda_{ijk}\Big\{\varepsilon_{\alpha\beta\gamma}(\overline{d}_R)_i^\alpha(d_R^c)_j^\beta(\widetilde{u}_R^\dagger)_k^\gamma \\
&\quad -[(\overline{e}_L^c)_i(\nu_L)_j - (\overline{\nu}_L^c)_i(e_L)_j](\widetilde{e}_R^\dagger)_k \\
&\quad -[(\overline{e}_L^c)_i(d_R^c)_j^\alpha - (\overline{d}_R)_i^\alpha(e_L)_j](\widetilde{u}_L)_{k\alpha} \\
&\quad +[(\overline{\nu}_L^c)_i(d_R^c)_j^\alpha - (\overline{d}_R)_i^\alpha(\nu_L)_j](\widetilde{d}_L)_{k\alpha}\Big\} , \qquad (2.1)
\end{aligned}$$

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where $\psi^c \equiv C\overline{\psi}^T$ and the indices $(i,j,\cdots)$, $(A,B,\cdots)$ and $(\alpha,\beta,\cdots)$ are family-, SU(5)$_{GUT}$- and SU(3)$_{colour}$-indices, respectively. The coefficients λ_{ijk} are antisymmetric in i and j. On the other hand, in SUSY GUT models, if the interactions (2.1) exist, the following R-parity-violating interactions will also exist:

$$\begin{aligned}
&\lambda_{ijk}(\overline{\psi}^c_{\overline{5}})^A_i(\psi_{10})_{kAB}(\widetilde{\psi}_{\overline{5}})^B_j \ , \\
&= \frac{1}{\sqrt{2}}\lambda_{ijk}\Big\{\varepsilon_{\alpha\beta\gamma}(\overline{d}_R)^\alpha_i(\widetilde{d}^\dagger_R)^\beta_j(u^c_R)^\gamma_k \\
&\quad -[(\overline{e}^c_L)_i(\widetilde{\nu}_L)_j-(\overline{\nu}^c_L)_i(\widetilde{e}_L)_j](e^c_R)_k \\
&\quad -[(\overline{e}^c_L)_i(\widetilde{d}^\dagger_R)^\alpha_j-(\overline{d}_R)^\alpha_i(\widetilde{e}_L)_j](u_L)_{k\alpha} \\
&\quad +[(\overline{\nu}^c_L)_i(\widetilde{d}^\dagger_R)^\alpha_j-(\overline{d}_R)^\alpha_i(\widetilde{\nu}_L)_j](d_L)_{k\alpha}\Big\} \ ,
\end{aligned} \tag{2.2}$$

which contribute to the proton decay through the intermediate state $\widetilde{d}_R$. Also, the term $(\overline{d}_R)^\alpha_i(d^c_R)^\beta_j(\widetilde{u}^\dagger_R)^\gamma_k$ in the interactions (2.1) can contribute to the nucleon decay through the intermediate state $\widetilde{u}_R$. The upper limits of the coupling constants λ_{ijk} from proton decay experiments have been investigated by Smirnov and Vissani [5], and the values must be highly suppressed.

In order to forbid the contribution of the interactions (2.1) and (2.2) to the proton decay, we must consider that in the R-parity-violating interactions $\overline{5}\times\overline{5}\times 10$, the term $d^c_R d^c_R u^c_R$ is exactly forbidden, while the terms $\nu_L e_L e^c_R$ and/or $\nu_L d^c_R d_L$ are in part allowed.

For such purpose, we introduce a discrete symmetry Z$_2$, which exactly holds at every energy scale. At the unification scale $\mu = M_X$, we have two types of superfields $\Psi_{L(\pm)} = \overline{5}_{L(\pm)} + 10_{L(\pm)}$, which are transformed as $\Psi_{L(\pm)} \to \pm\Psi_{L(\pm)}$ under the discrete symmetry Z$_2$. We consider that the particles $\Psi_{L(+)}$ are basically decoupled after the SU(5) symmetry is broken, so that our quarks and leptons (and their SUSY partners) $\overline{5}_L + 10_L$ are regarded as $\Psi_{L(-)} = \overline{5}_{L(-)} + 10_{L(-)}$. The R-parity-violating terms for quarks and leptons (and their SUSY partners) are basically forbidden under the symmetry Z$_2$ below $\mu = M_X$, because the terms are composed of $\overline{5}_{L(-)}\overline{5}_{L(-)}10_{L(-)}$.

However, if we assume that mixings between the members of $\Psi_{L(+)}$ and those of $\Psi_{L(-)}$ in part take place after SU(5) is broken, R-parity-violating interactions $\Psi_{L(+)}\Psi_{L(-)}\Psi_{(-)}$ become available at the low energy $\mu = m_Z$, too. For example, we assume a mixing

$$(2,1)_{Li} = (2,1)_{L(-)i}\cos\theta^A_i + (2,1)_{L(+)i}\sin\theta^A_i \ , \tag{2.3}$$

between the $(2,1)$ components of SU(2)×SU(3) for the i-th family. (Hereafter, we will refer to the mixing (2.3) as a mixing of type A$_i$.) Then, the R-parity-violating interactions

$$\begin{aligned}
&\sin\theta^A_i\lambda_{ijk}\nu_{Li}d^c_{Rj}d_{Lk} \ , \quad \sin\theta^A_i\lambda_{ijk}e_{Li}d^c_{Rj}u_{Lk} \ , \\
&\sin\theta^A_i\cos\theta^A_j\lambda_{ijk}\nu_{Li}e_{Lj}e^c_{Rk} \ ,
\end{aligned} \tag{2.4}$$

become available from the interactions

$$\lambda_{ijk}\overline{5}_{(+)i}\overline{5}_{(-)j}10_{(-)k} \ , \tag{2.5}$$

above the unification scale $\mu = M_X$. Also, we can consider a mixing

$$(2,3)_{Lk} = (2,3)_{L(-)k}\cos\theta^B_k + (2,3)_{L(+)k}\sin\theta^B_k \ , \tag{2.6}$$

between the $(2,3)$ components of SU(2)×SU(3) for the k-th family. (Hereafter, we will refer to the mixing (2.6) as a B$_k$-type mixing.) Then, the R-parity-violating interactions

$$\sin\theta^B_k\lambda'_{ijk}\nu_{Li}d^c_{Rj}d_{Lk} \text{ and } \sin\theta^B_k\lambda'_{ijk}e_{Li}d^c_{Rj}u_{Lk} \tag{2.7}$$

become available from the interactions

$$\lambda'_{ijk}\overline{5}_{(-)i}\overline{5}_{(-)j}10_{(+)k} \ . \tag{2.8}$$

On the other hand, note that the interaction

$$d^c_R d^c_R u^c_R \tag{2.9}$$

is exactly forbidden, independently of whether the mixings (2.3) and (2.6) occur or not, because those interactions have the Z$_2$ parity $z=-1$. Therefore, the proton decay due to the R-parity-violating terms is exactly forbidden because of the absence of the term $d^c_R d^c_R u^c_R$. On the other hand, the neutrino masses are radiatively generated through the interactions $\nu_L d^c_R d_L$ and $\nu_L e_L e^c_R$ with $z=+1$. The possible forms of the radiative neutrino mass matrix will be discussed in the next section.

At present, we do not know a reasonable mechanism not only for such a mixing, but also for the decoupling of $\Psi_{L(+)}$. In order to make $\Psi_{L(+)} = \overline{5}_{L(+)} + 10_{L(+)}$ heavy, the SU(2)$_L$ symmetry must be broken, but, of course, we cannot consider a scenario in which SU(2) is broken just after SU(5) is broken. In the present paper, we give only a phenomenological selection rule: if the superfield Ψ can make a five-body SU(5) singlet operator $\Psi\Psi\Psi\Psi\Psi$ with the Z$_2$ parity $z=+1$, then the superfield Ψ can be decoupled below $\mu = M_X$. Obviously, according to this selection rule, the superfield $\overline{5}_{L(+)}$ can be decoupled below $\mu = M_X$. Similarly, the superfield $10_{L(+)}$ is decoupled below $\mu = M_X$. However, note that those operators in the SU(5) singlets are symbolically expressed in terms of SU(2)×SU(3) components as follows:

$$\left(\overline{5}_{L(+)}\right)^5 = [(2,1)_{L(+)}]^2\times[(1,3)_{L(+)}]^3 \ , \tag{2.10}$$

$$\begin{aligned}
\left(10_{L(+)}\right)^5 &= (1,1)_{L(+)}\times[(1,\overline{3})_{L(+)}]^2\times[(2,3)_{L(+)}]^2 \\
&\quad +(1,\overline{3})_{L(+)}\times[(2,3)_{L(+)}]^4 \ ,
\end{aligned} \tag{2.11}$$

and that even if the interchanges $(2,1)_{L(+)i} \leftrightarrow (2,1)_{L(-)i}$, and/or $(2,3)_{L(+)k} \leftrightarrow (2,3)_{L(-)k}$, are caused, the composite operators

$$\left(\overline{5}_L\right)^5 = [(2,1)_{L(-)}]^2\times[(1,3)_{L(+)}]^3 \ , \tag{2.12}$$

$$(10_L)^5 = (1,1)_{L(+)} \times [(1,\overline{3})_{L(+)}]^2 \times [(2,3)_{L(-)}]^2 + (1,\overline{3})_{L(+)} \times [(2,3)_{L(-)}]^4 \quad (2.13)$$

still have $z = +1$. Such interchanges are possible only for the components $(2,1)_L$ and $(2,3)_L$. As a result, only the combination

$$\overline{5}_L + 10_L = [(2,1)_{L(+)} + (1,\overline{3})_{L(-)}] + [(1,1)_{L(-)} + (2,3)_{L(-)} + (1,\overline{3})_{L(-)}] \ , \quad (2.14)$$

for the i-th family and/or

$$\overline{5}_L + 10_L = [(2,1)_{L(-)} + (1,\overline{3})_{L(-)}] + [(1,1)_{L(-)} + (2,3)_{L(+)} + (1,\overline{3})_{L(-)}] \ , \quad (2.15)$$

for the k-th family survive below $\mu = M_X$ as the quarks and leptons (and their SUSY partners).

Of course, the above selection rule cannot be justified within the framework of the minimal SUSY standard model. At present, this is only an ansatz to select which components of SU(2)×SU(3) can be interchanged.

III. RADIATIVE NEUTRINO MASSES

In a SUSY GUT scenario, there are many origins of the neutrino mass generations. For example, the sneutrinos $\widetilde{\nu}_{iL}$ can have vacuum expectation values (VEVs), and the neutrinos ν_{Li} acquire their masses thereby (for example, see Ref. [6]). Although we cannot rule out a possibility that the observed neutrino masses can be understood from such compound origins, we do not take such a point of view in the present paper, because the observed neutrino masses and mixings appear to be rather simple and characteristic. We simply assume that the radiative masses are only dominated even if there are other origins of the neutrino mass generations.

In the present scenario, the origins of the radiatively induced neutrino masses are two: one is induced by the R-parity-violating interactions $\nu_L d^c_R \widetilde{d}_L$ and $\nu_L \widetilde{d}^c_R d_L$; the other one is induced by $\nu_L e_L \widetilde{e}^c_R$ and $\nu_L \widetilde{e}_L e^c_R$. Note that there is no Zee-type diagrams due to H^+_d–$\widetilde{e}^+_R$ mixing in this scheme.

First, we discuss the down-quark loop contributions. For simplicity, we assume that the masses $\widetilde{M}_{Li}$ and $\widetilde{M}_{Ri}$ of the squarks $\widetilde{d}_{Li}$ and $\widetilde{d}_{Ri}$ are approximately constant, independently of the flavours, although we consider the flavour-dependent structure for the mass terms $\widetilde{d}^\dagger_L \widetilde{M}^2_d \widetilde{d}_R$. Then, the radiatively induced neutrino mass matrix due to the A-type mixing is given by

$$(M_\nu)_{ij} = m_0 \lambda_{ikm} \lambda_{jln} (M^\dagger_d)_{kn} (\widetilde{M}^{2\dagger}_d)_{lm} + (i \leftrightarrow j) \ , \quad (3.1)$$

where sine-factors have been drooped for simplicity, M_ν is defined by $\overline{\nu}_L M_\nu \nu^c_L$, and the coupling constants λ_{ijk} are redefined by

$$\lambda_{ijk} \left[\overline{\nu}_{Li} d_{Rj} \widetilde{d}^\dagger_{Lk} + \overline{\nu}_{Li} \widetilde{d}_{Rj} d^c_{Lk} - (\nu_L \leftrightarrow d^c_R) \right] \ . \quad (3.2)$$

Here, we have changed the definition of λ_{ijk} from that in (2.1) as $\lambda_{ijk} \to \lambda^*_{ijk}$ for the convenience of the expression of M_ν defined by $\overline{\nu}_L M_\nu \nu^c_L$. In the present paper, the unitary matrix U_ν used to diagonalize the Majorana mass matrix M_ν is defined as $U^\dagger_\nu M_\nu U^*_\nu = D_\nu$. Then, the so-called Maki–Nakagawa–Sakata–Pontecorvo [7] matrix (we will simply call it the "lepton mixing matrix") $U \equiv U_{MNSP}$ is given by $U = U^{e\dagger}_L U_\nu$. Usually, it is considered that the matrix form of $\widetilde{M}^2_d$ is proportional to the form M_d. Then, the neutrino mass matrix (3.1) becomes, in a more concise form:

$$(M_\nu)_{ij} = m_0 \lambda_{ikm} \lambda_{jln} (M^\dagger_d)_{kn} (M^\dagger_d)_{lm} \ , \quad (3.3)$$

where we have redefined the common factor m_0 from that in (3.1). Of course, exactly speaking, for the A_i mixings with $\sin\theta^A_i$-factors, we should read the expression (3.3) as

$$(M_\nu)_{ij} = m_0 s^A_i s^A_j \lambda_{ikm} \lambda_{jln} (M^\dagger_d)_{kn} (M^\dagger_d)_{lm} \ , \quad (3.4)$$

where $s^A_i = \sin\theta^A_i$ defined in (2.3). When we consider the B_k mixings, we read the expression (3.3) as

$$(M_\nu)_{ij} = m_0 s^B_m s^B_n \lambda'_{ikm} \lambda'_{jln} (M^\dagger_d)_{kn} (M^\dagger_d)_{lm} \ , \quad (3.5)$$

where $s^B_k = \sin\theta^B_k$ defined in (2.6). For mixed-type mixings of A_i and B_k, we read (3.3) as

$$(M_\nu)_{ij} = m_0 (s^A_i c^B_m \lambda_{ikm} + c^A_i s^B_m \lambda'_{ikm}) (s^A_j c^B_n \lambda_{jln} + c^A_j s^B_n \lambda'_{jln}) (M^\dagger_d)_{kn} (M^\dagger_d)_{lm} \ , \quad (3.6)$$

where $c^A_i = \cos\theta^A_i$ and $c^B_k = \cos\theta^B_k$.

The contributions from the charged lepton loops are essentially the same as (3.3), except for the absence of the B-type mixing and the replacement $M_d \to M^T_e$. For simplicity, we will continue the investigation for the case of the down-quark loop contributions.

For the phenomenological study of the mass matrix (3.3), it is convenient to take the basis on which the down-quark mass matrix M_d is diagonal:

$$U^{d\dagger}_L M_d U^d_R = D_d \equiv \mathrm{diag}(m^d_1, m^d_2, m^d_3) \ . \quad (3.7)$$

We consider that, on the basis with $M_d = D_d$, the charged lepton mass matrix M_e is also approximately diagonal, $U^{e\dagger}_L M_e U_R \simeq D_e = \mathrm{diag}(m^e_1, m^e_2, m^e_3)$, so that the unitary matrix U_ν approximately gives the lepton mixing matrix $U = U^{e\dagger}_L U_\nu$. Then, we can express (3.3) as

$$\begin{aligned}
(M_\nu)_{11} &= (m^d_3)^2 (\lambda_{133})^2 + (m^d_2)^2 (\lambda_{122})^2 + 2 m^d_3 m^d_2 \lambda_{123} \lambda_{132} \ , \\
(M_\nu)_{22} &= (m^d_3)^2 (\lambda_{233})^2 + (m^d_1)^2 (\lambda_{211})^2 + 2 m^d_3 m^d_1 \lambda_{213} \lambda_{231} \ , \\
(M_\nu)_{33} &= (m^d_2)^2 (\lambda_{322})^2 + (m^d_1)^2 (\lambda_{311})^2 + 2 m^d_2 m^d_1 \lambda_{312} \lambda_{321} \ , \\
(M_\nu)_{12} &= (m^d_3)^2 \lambda_{133} \lambda_{233} + m^d_3 m^d_2 \lambda_{123} \lambda_{232} \\
&\quad + m^d_3 m^d_1 \lambda_{131} \lambda_{213} + m^d_2 m^d_1 \lambda_{121} \lambda_{212} \ ,
\end{aligned}$$

$$
\begin{aligned}
(M_\nu)_{13} &= (m_2^d)^2\lambda_{122}\lambda_{322} + m_3^d m_2^d \lambda_{132}\lambda_{323} \\
&\quad + m_3^d m_1^d \lambda_{131}\lambda_{313} + m_2^d m_1^d \lambda_{121}\lambda_{312} \ , \\
(M_\nu)_{23} &= (m_1^d)^2\lambda_{211}\lambda_{311} + m_3^d m_2^d \lambda_{232}\lambda_{323} \\
&\quad + m_3^d m_1^d \lambda_{231}\lambda_{313} + m_2^d m_1^d \lambda_{212}\lambda_{321} \ ,
\end{aligned}
\tag{3.8}
$$

where, for simplicity, we have dropped the common factor m_0. In order to give the best-fit values for the observed neutrino data [8,9]

$$
R \equiv \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \simeq \frac{5.0\times 10^{-5}\mathrm{eV}^2}{2.5\times 10^{-3}\mathrm{eV}^2} = 2.0\times 10^{-2} \ , \tag{3.9}
$$

$$
\begin{aligned}
&\sin^2 2\theta_{solar} = \sin^2 2\theta_{12} = 0.76 \ , \\
&[\tan^2\theta_{solar} = 0.34] \ ,
\end{aligned}
\tag{3.10}
$$

$$
\sin^2 2\theta_{atm} = \sin^2 2\theta_{23} = 1.0 \ , \tag{3.11}
$$

we must seek a parameter set that gives $(M_\nu)_{22} \simeq (M_\nu)_{33}$ and $(M_\nu)_{12} \sim (M_\nu)_{13}$ for the expression (3.8). When we consider the A_i-type mixings, we obtain [10]

$$
M_\nu \propto \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} \ , \tag{3.12}
$$

for $\varepsilon = s_1^A/s_2^A = s_1^A/s_3^A$ and $\lambda_{2j3}/\lambda_{3j2} \simeq m_2^d/m_3^d$. Generally, when we consider only A_i-type mixings, the solutions are highly dependent on the fine-tuning among the coefficients λ_{ijk}, and, besides, since the mass matrix is too near to a rank-1 matrix, it is difficult to give the solution a small but sizeable value of R (it leads to an extremely small value of R). Even if we take the charged lepton loop contributions into consideration, the situation is not improved unless we assume the special parametrization for λ_{ijk} ($\lambda_{2j3}/\lambda_{3j2} \simeq m_2^d/m_3^d$ and so on).

Next, we consider the case of the B_k-type mixings. When we consider a B_1 mixing, we obtain a simple mass matrix form

$$
M_\nu = (s_1^B)^2 (m_1^d)^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & (\lambda'_{211})^2 & \lambda'_{211}\lambda'_{311} \\ 0 & \lambda'_{211}\lambda'_{311} & (\lambda'_{311})^2 \end{pmatrix} \ , \tag{3.13}
$$

because the case gives the relations $\lambda'_{ij2}s_2^B = \lambda'_{ij3}s_3^B = 0$. It is natural to consider that $\lambda'_{211} \simeq \lambda'_{311}$ since those come from the same interactions (2.8) [not from (2.5)]. Therefore, the mass matrix (3.13) can give a maximal mixing between ν_μ and ν_τ. A small additional term will reasonably give a bimaximal mixing.

For example, we consider a mixed-type, A_3 and B_1, mixing. In this case, the λ_{ijk} in (3.3) must be replaced according to Table I. It is natural to consider that $\lambda_{ijk} \simeq \mathrm{const} \equiv \lambda$ and $\lambda'_{ijk} \simeq \mathrm{const} \equiv \lambda'$. Then, the mass matrix M_ν is reduced to the form

$$
M_\nu \simeq (s_1^B)^2 (m_1^d) \begin{pmatrix} 0 & 0 & \varepsilon(r_3+r_2) \\ 0 & 1 & 1+\varepsilon r_3 \\ \varepsilon(r_3+r_2) & 1+\varepsilon r_3 & (1+\varepsilon r_2)^2 \end{pmatrix} \ , \tag{3.14}
$$

where $\varepsilon = s_3^A\lambda/s_1^B\lambda' \ll 1$, $r_2 = m_2^d/m_1^d$ and $r_3 = m_3^d/m_1^d$, and we have assumed $c_3^A \simeq 1$. This mass matrix can give a reasonable value of R together with a nearly bimaximal mixing. For example, for the parameter value $\varepsilon = 0.000374$, we obtain the following numerical results:

$$
m_1^\nu = 0.0822 m_0, \quad m_2^\nu = -0.3276 m_0, \quad m_3^\nu = 2.2604 m_0, \tag{3.15}
$$

$$
U = \begin{pmatrix} 0.8726 & -0.4822 & 0.0776 \\ -0.3925 & -0.5978 & 0.6990 \\ 0.2907 & 0.6404 & 0.7109 \end{pmatrix} \ , \tag{3.16}
$$

i.e.

$$
R = 0.0201 \ , \tag{3.17}
$$

$$
\sin^2 2\theta_{12} \equiv 4U_{11}^2 U_{12}^2 = 0.708 \ , \tag{3.18}
$$

$$
\sin^2 2\theta_{23} \equiv 4U_{23}^2 U_{33}^2 = 0.988 \ . \tag{3.19}
$$

These values are in good agreement with the best fit values (3.9)–(3.11) for the observed neutrino data. Although it is difficult in the original Zee model to give a sizeable deviation of $\sin^2 2\theta_{12}$ from 1 [11] (it must be $\sin^2 2\theta_{12} = 1.0$), the present model can give a reasonable deviation from $\sin^2 2\theta_{12} = 1.0$. The result

$$
U_{13}^2 = 0.00602 \tag{3.20}
$$

is also consistent with the present experimental upper limit

$$
|U_{13}|^2 < 0.03 \ , \tag{3.21}
$$

from the CHOOZ collaboration [12].

IV. HIGGS SECTORS

In the present model, the quark and charged lepton mass matrices are generated by the VEVs of the Higgs scalars with $z = +1$. Therefore, even if $\mathrm{SU(2)}_L$ is broken later, the Z_2 symmetry still exactly holds.

Let us show the mass matrices M_f for the case of the mixed-type mixing A_3 and B_1 as an example of the explicit forms of M_f:

$$
M_e = \begin{pmatrix} c_1 & b_1 & a_1 \\ c_2 & b_2 & a_2 \\ c_1^A c_3 & c_1^A b_3 & c_1^A a_3 \end{pmatrix} \ , \tag{4.1}
$$

$$M_d = \begin{pmatrix} c_1^B c_1 & c_1^B c_2 & c_1^B c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{pmatrix} , \tag{4.2}$$

$$M_u = \begin{pmatrix} c_1^B c_1' & c_1^B c_2' & c_1^B c_3' \\ b_1' & b_2' & b_3' \\ a_1' & a_2' & a_3' \end{pmatrix} . \tag{4.3}$$

Here, M_f have been defined by $\overline{f}_L M_f f_R$ $(f = u, d, e)$. Note that the mass matrix M_d has a form different from M_e^T because of the mixing factors. Usually, if we consider one type of Higgs scalar of SU(5) 5-plet ($\overline{5}$-plet), it is difficult to obtain realistic mass matrices M_f $(f = u, d, e)$. Therefore, the present model has a possibility to improve this problem. However, whether we can give reasonable mass matrix forms of M_f or not is a future task to us.

Now, we would like to give a comment on the Higgs sectors. In Sec. II, we have assumed that although the superfield $\overline{5}_{L(+)}$ is decoupled below $\mu = M_X$, the components $(2,1)_{(+)}$ of $\overline{5}_{L(+)}$ can contribute to low energy phenomena through the mixing (2.3). If we consider the Higgs fields $\overline{H}_{d(+)}$ and $\overline{H}_{d(-)}$ with SU(5) $\overline{5}$-plet, and if we assume a situation similar to the matter fields $\overline{5}_L$, then we obtain

$$\overline{H}_d = (2,1)_{(+)} + (1,\overline{3})_{(-)} , \tag{4.4}$$

where we have assumed a perfect interchange between $(2,1)_{(-)}$ and $(2,1)_{(+)}$, not a mixing. Note that in this scheme, the Higgs scalar component $(3,1)_{(-)}$ cannot couple to the fermions $\overline{d}_R u_R^c$ with $z = +1$ independently of the mixings A_i and B_k, so that the scalar $(3,1)_{(-)}$ cannot contribute to the proton decay and it need not be super-heavy. (Although it couples to the fermions $(\overline{\nu}_L^c d_L - \overline{e}_L^c u_L)$, these interactions cannot contribute to the proton decay.) We can consider a similar mechanism for the Higgs fields H_u with SU(5) 5-plet. The interactions of $(1,3)_{(-)}$ with $\overline{u}_L^c d_L$ and $\overline{e}_R u_R^c$ are absent, so that the scalar $(1,3)_{(-)}$ need not be super-heavy. The so-called μ-terms are composed of $H_{u(+)}\overline{H}_{d(+)}$ and $H_{u(-)}\overline{H}_{d(-)}$.

V. CONCLUSION

In conclusion, we have investigated possible forms of radiatively induced neutrino mass matrix under an ansatz [(2.3) and (2.6)] within the framework of the SU(5) SUSY model. We have assumed two types of matter fields $\Psi_L = \overline{5}_L + 10_L$, $\Psi_{(\pm)}$, which are transformed as $\Psi_{(\pm)} \to \pm\Psi_{(\pm)}$ under the Z_2 symmetry. We have assumed that the Z_2 symmetry exactly holds, even if SU(5) is broken. The essential ansatz is in the mixings (2.3) and (2.6). Although the origin of the mixings (2.3) and (2.6) is still an open question, if we admit this ansatz, we can obtain very interesting and simple neutrino mass matrix form, which can give satisfactory numerical results for the observed neutrino data. How we can justify the ansatz is a future task for us.

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i	$k=1$	$k=2$	$k=3$
1	$s_1^B\lambda'_{1j1}$	0	0
2	$s_1^B\lambda'_{2j1}$	0	0
3	$c_3^A s_1^B\lambda'_{3j1}$ $+s_3^A c_1^B\lambda_{3j1}$	$s_3^A\lambda_{3j2}$	$s_3^A\lambda_{3j3}$

TABLE I. Rule of the replacement λ_{ijk} in the mass matrix (3.3) for the case of A_3 and B_1 mixings.