# Microscopic Description of Systematic Doublet Bands in $A \sim 130$ Region 

Naotaka Yoshinaga ${ }^{1}$ and Koji Higashiyama ${ }^{2}$<br>${ }^{1}$ Department of Physics, Saitama University, Saitama City 338-8570, Japan<br>${ }^{2}$ Department of Physics, University of Tokyo, Hongo, Tokyo 113-0033, Japan<br>E-mail: yoshinaga@phy.saitama-u.ac.jp


#### Abstract

The properties of the yrast and yrare states in the mass $A \sim 130$ region are studied by a full microscopic theoretical framework of the pair-truncated shell model. This approach for energy levels and electromagnetic transition rates in ${ }^{134} \mathrm{La}$ gives good agreement with experiment. The analysis of the wave functions reveals new band structure, which results from chopsticks configurations of two angular momenta of the unpaired neutron and the unpaired proton, weakly coupled with the quadrupole collective excitations of the even-even core.


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## 1. Introduction

The study of yrast and yrare states in doubly-odd nuclei has recently been one of the most interesting subjects in nuclear physics. A large number of experimental data $[1,2,3,4,5,6,7,8]$ have been accumulated in mass $A \sim 130$ region, showing that the yrast and yrare states with the $\nu h_{11 / 2} \otimes \pi h_{11 / 2}$ configuration form $\Delta I=1$ doublet bands which are nearly degenerate in energy. They are built on the single particle states of a valence neutron and a proton in the same unique-parity orbital $0 h_{11 / 2}$. These $\Delta I=1$ doublet bands had been interpreted as a manifestation of "chirality" in the meaning of the angular momentum coupling, which was predicted by Frauendorf and Meng [9]. In their picture, the chiral mechanism is explained as follows. When three angular momenta of the even-even core, the unpaired neutron and the unpaired proton are perpendicular to each other, they can form either a left-handed or a right-handed geometrical configuration. These configurations are energetically equivalent, and two degenerate bands are constructed as linear combinations of these. The chiral structure of the $\Delta I=1$ doublet bands was investigated theoretically in the framework of the threedimensional tilted axis cranking model $[1,2,8,9,10]$, the particle-rotor model (PRM) $[4,11]$ and the phenomenological core-particle-hole coupling model [6, 7]. Despite a large number of theoretical studies, they are insufficient to obtain accurate quantitative results for both energy levels and electromagnetic properties of these bands, simultaneously.

In this paper, we present a new interpretation of the yrast and yrare states with the $\nu h_{11 / 2} \otimes \pi h_{11 / 2}$ configuration for the $A \sim 130$ doubly-odd nuclei in the context of a pair-truncated shell model (PTSM) [12]. Through analysis of their structure, it turns out that the level scheme of $\Delta I=2 E 2$ bands arises from different angular momentum configurations of an unpaired neutron and an unpaired proton, weakly coupled with the quadrupole collective excitations of the even-even core.

## 2. Theoretical Framework

In the simplest version of the PTSM, the shell model basis is restricted to the $S D$ subspace where angular momenta zero $(S)$ and two $(D)$ collective pairs are used as the building blocks of the model. Thus the many-body wave function of the evennucleon system is represented by the basis states $\left|S^{n_{s}} D^{n_{d}} I \eta\right\rangle$, where $I$ is the total angular momentum and $\eta$ is any other index needed to completely label the nuclear state. Here the angular momentum coupling is exactly carried out. Then the manybody wave function of the odd-nucleon system among like nucleons is expressed as $\left|j S^{n_{s}} D^{n_{d}} I \eta\right\rangle$. A basis state of any doubly-odd nucleus with total spin $I$ is written as a product of the above state in neutron space and that in proton space as $|\Phi(I \eta)\rangle=$ $\left[\left|j_{\nu} S_{\nu}^{\bar{n}_{s}} D_{\nu}^{\bar{n}_{d}} I_{\nu} \eta_{\nu}\right\rangle \otimes\left|j_{\pi} S_{\pi}^{n_{s}} D_{\pi}^{n_{d}} I_{\pi} \eta_{\pi}\right\rangle\right]^{(I)}$, where $\bar{N}_{\nu}=2 \bar{n}_{s}+2 \bar{n}_{d}+1$ and $N_{\pi}=2 n_{s}+2 n_{d}+1$ are numbers of valence neutron holes and proton particles, respectively. In this study, valence neutrons are treated as holes, and valence protons, as particles.

The effective Hamiltonian employed in the present calculation consists of the single particle energies, and the monopole and quadrupole pairing plus quadrupole-quadrupole interactions. The detailed prescriptions for the PTSM have been given in Refs. [13, 14] in addition to the strengths of the interactions.

## 3. Numerical Results of Doubly-Odd Nuclei

In Fig. 1, the experimental energy spectrum based on the $\nu h_{11 / 2} \otimes \pi h_{11 / 2}$ configuration is compared with the PTSM calculation. For the yrast states, energy levels are almost perfectly reproduced, except that in our calculation the $8_{1}^{+}$state is predicted in between the $9_{1}^{+}$and $10_{1}^{+}$states. Also for the yrare states, our theoretical result provides a successful description of the energy levels, though only four levels are observed experimentally.

In Fig. 2(a), theoretical ratios $B(M 1 ; I \rightarrow I-1) / B(E 2 ; I \rightarrow I-2)$ for the yrast states are compared with experiment. The effective charges and gyromagnetic ratios are taken as follows: $e_{\nu}=-1.2 e, e_{\pi}=2.2 e, g_{\ell \nu}=0.00, g_{\ell \pi}=1.00, g_{s \nu}=-2.68$ and $g_{s \pi}=3.91$. The large-amplitude staggering of the $B(M 1) / B(E 2)$ ratios is in excellent agreement with experimental data, except for the $16_{1}^{+}$state. In Fig. 2(b), the theoretical $B(E 2 ; I \rightarrow I-2)$ values between yrast states and between yrare states are shown as functions of spin $I$. The behavior of $E 2$ transitions is similar for both the even-spin yrast states $(I \geq 12)$ and the odd-spin yrast states $(I \geq 13)$. The strong $E 2$ transitions


Figure 1. Comparison of energy spectrum in experiment (expt.) with those of the PTSM (PTSM). The experimental data are taken from Ref. [5].


Figure 2. (a) Comparison of the calculated $B(M 1) / B(E 2)$ ratios for the yrast states with experiment. Experimental data are taken from Ref. [5]. (b) $B(E 2)$ values calculated in the PTSM. (c) $B(M 1)$ values calculated in the PTSM.
with spins greater than 12 indicate that the odd-spin and the even-spin yrast states respectively form two $\Delta I=2$ bands starting from the bandhead states of $11_{1}^{+}$and $10_{1}^{+}$. The $B(E 2)$ values between the yrare $\Delta I=2$ states are smaller than those between the yrast $\Delta I=2$ states. Nevertheless, since the yrare states are linked by the strong $E 2$ transitions between the $\Delta I=2$ states, quadrupole collectivity plays an important role in describing the even-spin and the odd-spin yrare states for $I>11$. Concerning the interband transitions between the yrast and yrare states with spins greater than 11 (not shown in the figure), the calculated $B(E 2)$ values are smaller than the value $0.02 \mathrm{e}^{2} \mathrm{~b}^{2}$, except for the $B\left(E 2 ; 11_{1}^{+} \rightarrow 9_{2}^{+}\right)=0.0266 \mathrm{e}^{2} \mathrm{~b}^{2}$ and $B\left(E 2 ; 11_{2}^{+} \rightarrow 9_{1}^{+}\right)=0.131 \mathrm{e}^{2} \mathrm{~b}^{2}$ values. From analysis of the $B(E 2)$ values, we conclude that the following members form five $\Delta I=2 E 2$ bands each starting from the first member as the bandhead state (see Fig. 3): (1) $11_{1}^{+}, 13_{1}^{+}, 15_{1}^{+}, 17_{1}^{+}$, (2) $10_{1}^{+}, 12_{1}^{+}, 14_{1}^{+}, 16_{1}^{+}$, (3) $9_{1}^{+}, 11_{2}^{+}, 13_{2}^{+}, 15_{2}^{+}$, (4) $8_{1}^{+}, 10_{2}^{+}, 12_{4}^{+}$, and (5) $12_{2}^{+}, 14_{2}^{+}, 16_{2}^{+}$.

The calculated $B(M 1 ; I \rightarrow I-1)$ values of the yrast and the yrare states are


Figure 3. Partial level scheme of ${ }^{134} \mathrm{La}$ suggested by the PTSM calculation. The arrows indicate $E 2$ transitions $\left(B(E 2) \geq 0.02 \mathrm{e}^{2} \mathrm{~b}^{2}\right)$, and the dotted arrows denote $M 1$ transitions $\left(B(M 1) \geq 0.40 \mu_{N}^{2}\right)$. The numerals on the right side of the $E 2$ transitions denote the $B(E 2)$ values (in $10^{-2} \mathrm{e}^{2} \mathrm{~b}^{2}$ ), and those beneath the $M 1$ transitions denote the $B(M 1)$ values (in $\left.\mu_{N}^{2}\right)$.
shown as functions of spin $I$ in Fig. 2(c). Concerning the yrast states, the $B(M 1)$ values $(I \geq 11)$ are large for the transitions from odd spin to even spin, and small for the transitions from even spin to odd spin. On the contrary, for both cases $B(M 1)$ values are found to be small for the yrare states $(I \geq 12)$. This fact implies that the structure of the yrare band differs from that of the yrast band. The strong $M 1$ transitions ( $I \geq 11$ ) connect the odd-spin yrast states $(I)$ to the even-spin yrast states $(I-1)$, and connect those states $(I-1)$ to the odd spin states $(I-2)$. These large $B(M 1)$ values indicate that the $\Delta I=1 M 1$ bands are composed of the following four level sequences: (a) $8_{1}^{+}$, $9_{1}^{+}, 10_{1}^{+}, 11_{1}^{+}$, (b) $10_{2}^{+}, 11_{2}^{+}, 12_{1}^{+}, 13_{1}^{+}$, (c) $13_{2}^{+}, 14_{1}^{+}, 15_{1}^{+}$, and (d) $15_{2}^{+}, 16_{1}^{+}, 17_{1}^{+}$. The partial level scheme of ${ }^{134} \mathrm{La}$ constructed from the theoretical results of the M1 and E2 transition rates is shown in Fig. 3. Our model gives five $\Delta I=2 E 2$ bands. The states within four $\Delta I=2 E 2$ bands with the bandhead states of $8_{1}^{+}, 9_{1}^{+}, 10_{1}^{+}$and $11_{2}^{+}$are connected by the strong $E 2$ transitions to the same members of the $\Delta I=2 E 2$ bands, and by the strong $M 1$ transitions to the states in the neighboring $\Delta I=2 E 2$ bands. The structure of the $\Delta I=2 E 2$ band with the bandhead state of $12_{2}^{+}$is quite different from those of the other $\Delta I=2 E 2$ bands, since these states in the former band are not connected by the strong $M 1$ transitions to any member of the other $\Delta I=2 E 2$ bands.

In search of the microscopic origin of the magnetic transitions, the reduced matrix elements of $M 1$ operators are analyzed. Figure 4 shows the comparison of three kinds of the absolute $M 1$ reduced matrix elements. It is seen that the main contribution of the reduced matrix elements of $M 1$ operators comes from the $0 h_{11 / 2}$ orbitals. To pin down


Figure 4. Three kinds of absolute reduced matrix elements of $M 1$ operators: the total reduced matrix elements $[T(M 1)]$, contributions only from the $0 h_{11 / 2}$ orbitals $\left[T_{11 / 2}(M 1)\right]$, and their absolute differences $\left[T_{\text {oth }}(M 1)\right]$.


Figure 5. Band scheme predicted by the PTSM calculation in the weak coupling limit.
their detailed microscopic origin, we consider a two-nucleon system of one neutron and one proton both in the same $0 h_{11 / 2}$ orbital. From simple geometrical considerations, the $8^{+}$state of the two-nucleon system is built by perpendicular coupling of two angular momenta of the neutron and proton, while the $11^{+}$state is built by parallel coupling. By comparing the results for this two-nucleon system with those of actual ${ }^{134} \mathrm{La}$ calculations, the odd-spin yrast states $(I)(I=11,13,15,17)$, the even-spin yrast states $(I-1)$ and the odd spin states $(I-2)$ have the configurations of the unpaired nucleons with angular momentum 11, 10 and 9 , respectively.

The configurations of two angular momenta of the unpaired nucleons with angular momentum $0 \sim 11$ are called chopsticks configurations hereafter. Since quadrupole collectivity plays an important role in describing the $\Delta I=2 E 2$ bands, the main structure of these $\Delta I=2 E 2$ bands is interpreted as arising from a weak coupling of the chopsticks configurations with the quadrupole collective motion of the even-even core. In Fig. 3, schematic illustrations of the chopsticks configuration are presented below each $\Delta I=2 E 2$ band.

Our new interpretation leads to a schematic illustration of the band structure shown in Fig. 5, which is expected to occur in the weak coupling limit of the chopsticks configurations with the core excitations. The bandhead states of the $\Delta I=2 E 2$ bands are built on the unpaired nucleons both in the $0 h_{11 / 2}$ orbitals, coupled with the eveneven core of angular momentum 0 . The spin of the bandhead states corresponds to one of the possible chopsticks configurations with angular momentum $0 \sim 11$, whose schematic illustrations are shown below for each $\Delta I=2$ band in Fig. 5. In the actual calculations the PTSM provides four $\Delta I=2 E 2$ bands with the bandhead states of 81 , $9_{1}^{+}, 10_{1}^{+}$and $11_{1}^{+}$.

## 4. Summary

To conclude, we have applied the PTSM to the structure study of the yrast and yrare states with the $\nu h_{11 / 2} \otimes \pi h_{11 / 2}$ configuration in ${ }^{134} \mathrm{La}$. The calculation reproduces the experimental energy levels and electromagnetic transition rates, especially the staggering of the $B(M 1) / B(E 2)$ ratios. Through analysis of their structure, it is found that the main structure of the yrast and yrare states is described in terms of a weak coupling of the chopsticks configurations, which represent two angular momenta of the unpaired neutron and the unpaired proton, to the multi-phonon excitations of the even-even core. The detailed results are presented in Ref. [19] and a forthcoming paper [14].

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