LETTER *Special Section on Spread Spectrum Techniques and Applications*

Analysis of BER Performance of the Spread Spectrum Communication System with Constrained Spreading Code

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SUMMARY In this paper, the bit error rate (BER) performance of the Spread Spectrum communication system with Constrained Spreading Codes (SS-CSC) is analyzed. The BER of the SS-CSC system is the same as that of the Bi-orthogonal system. Moreover, the frequency utilization efficiency of the SS-CSC system is better than that of the Bi-orthogonal system when $K \leq 10$ and $N = 3$.

key words: *spread spectrum communication, M-ary/SS system, bi-orthogonal system*

1. Introduction

M-ary Spread Spectrum (M-ary/SS) communication systems and Bi-orthogonal systems have been attracting increasing interest in recent years [1]–[6]. The bit error rate (BER) performance of these systems can approach Shannon's Limit when the number of the spreading codes increases in Additive White Gaussian Noise (AWGN) environments, but these systems have two serious problems; (1) these are high-complexity systems that should prepare a number of correlators equal to the number of spreading codes; (2) the synchronization of these systems is difficult because they have several spreading codes.

We reported that a Spread Spectrum communication system with Constrained Spreading Code (SS-CSC) [4]–[6] can be a solution to the first of these problems. The performance of the SS-CSC system was evaluated by computer simulation in [5], [6].

In this paper, we analyze BER performance on the SS-CSC system, and compare the performance of the SS-CSC system with that of the Bi-orthogonal system.

2. Principle of the SS-CSC System

The structure of the SS-CSC system is shown in Fig. 1 [5].

In the transmitter, source data are converted into $\log_2(2^{K+N})$ bit data by a Data Converter, and the converted data are distributed into two classes (K[bit] data and N[bit] data). Here K is the length of message bit to select one of the M spreading codes and N is the length of message bit to modulate the selected spreading code. Next, one of the M spreading codes is selected by K[bit] data at a Spreading Code Selector. (In this study, orthogonal sequences are employed.) Lastly, the selected spreading code is modulated by the polarity of N[bit] data.

In the receiver, K[bit] data is demodulated by estimation of the transmitted spreading code, and N[bit] data is determined by the polarity of the estimated spreading code.

The SS-CSC system corresponds to DS/SS systems when $K = 0$, and is equivalent to the Bi-orthogonal system when $N = 1$.

3. Theoretical Analysis

An analysis of the BER of the SS-CSC system is presented [4]. BER analysis of the SS-CSC system is an expansion of that of the Bi-orthogonal system [1]. Two kinds of errors can occur in a SS-CSC signal set. The

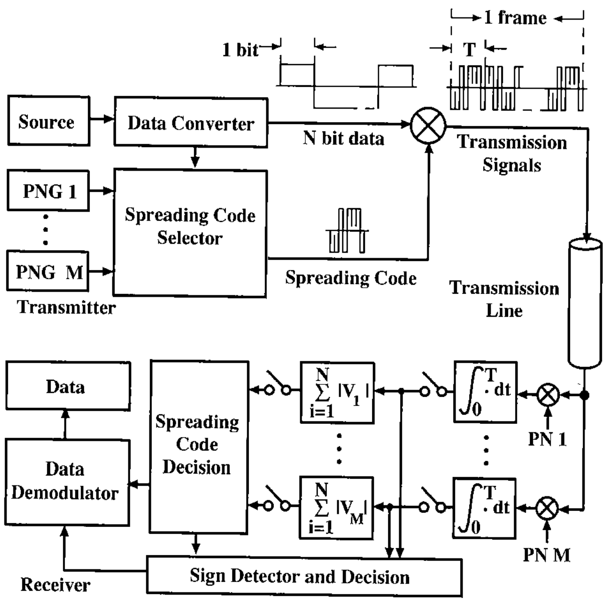

Fig. 1 Structure of the SS-CSC system.

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error of the first kind is an estimation error of the transmitted spreading code; that is, P_K. The error of the second kind, which occurs when $s_1(t)$ is transmitted and $-s_1(t)$ is selected, is a decision error of polarity; that is, P_N.

Firstly, we consider the estimation error of the transmitted spreading code; that is, P_K. The output signal q_i of the ith multiplier in the receiver, given that spreading code $s_j(t)$ is transmitted, is represented by

$$q_i = \int_0^T s_j(t)s_i(t)dt + \int_0^T n(t)s_i(t)dt, \tag{1}$$

where T is a period of the spreading code, $n(t)$ is AWGN, and $s_i(t)$ is the ith spreading code at the receiver. Therefore, the output of correlators is given by

$$x = |q_1| * |q_2| * \cdots * |q_L|, \tag{2}$$

where $|q_1|$ is the absolute value of q_1 and $|q_1| * |q_2|$ is the convolution integral of $|q_1|$ and $|q_2|$. The covariance matrix Q of the correlator outputs is given by

$$Q = \left(\frac{K+N}{2}N_0E_b\right)I, \tag{3}$$

where E_b is the transmitted signal energy per message bit, N_0 is the power spectrum density (PSD) of Gaussian noise, and matrix I is a unit matrix. Therefore, when the signal $s_1(t)$ is transmitted, the joint probability density function of the correlator output x is represented by

$$p(x|s_1) = \frac{1}{(2\pi)^{2^{K-1}}|Q|^{\frac{1}{2}}}\exp[-\frac{1}{2}(x-\bar{x}) \cdot Q^{-1}(x-\bar{x})^T], \tag{4}$$

where $\bar{x}$ is the average of x, $|Q|$ is the determinant of the matrix Q, Q^{-1} is the inverse of Q, and $(x-\bar{x})^T$ is the transpose of $(x-\bar{x})$. Therefore, the probability P_c of a correct estimation is given by

$$P_c = \int_{-\infty}^{\infty}\int_{-\infty}^{x_1}\cdots\int_{-\infty}^{x_1} p(x_1, x_2, \cdots, |s_1)dx = \int_{-\infty}^{\infty} p(x_1|s_1)\left[\int_{-\infty}^{x_1} p(x_{\substack{j\\j\neq1}}|s_1)dx_{\substack{j\\j\neq1}}\right]^{2^K-1} dx_1. \tag{5}$$

Thus, the error probability P_K of selecting any one of the 2^K-1 spreading codes that are orthogonal to the transmitted spreading code is given by

$$P_K = 1 - P_c. \tag{6}$$

Secondly, we consider the decision error of the polarity of the estimated spreading code, that is, P_N. If the estimation of the transmitted spreading code is correct, $P_N = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{(K+N)E_b}{LN_0}}\right)$. The conditional bit error probability, given that the error of the first kind has occurred, is $1/2$. Therefore, the bit error probability of the second kind, P_N, is given by

$$P_N = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{(K+N)E_b}{LN_0}}\right)(1-P_K)+\frac{1}{2}P_K, \tag{7}$$

where $L = \begin{cases} N & (N \geq 1) \\ 1 & (N = 0) \end{cases}$.

Finally, for Eqs. (6) and (7), the BER of the SS-CSC system is

$$\begin{aligned} BER &= \frac{K}{N+K}\left(\frac{2^{K-1}}{2^K-1}\right)P_K + \frac{N}{N+K}P_N \\ &= \frac{K}{N+K}\left(\frac{2^{K-1}}{2^K-1}\right)P_K \\ &\quad + \frac{N}{N+K}\left\{\frac{1}{2}\text{efrc}\left(\sqrt{\frac{(K+N)E_b}{LN_0}}\right) \cdot (1-P_K) + \frac{1}{2}P_K\right\}. \end{aligned} \tag{8}$$

4. Performance Evaluation

In this section, we evaluate the BER properties and the frequency utilization effciency properties by theoretical analysis. We assume an AWGN environment and complete synchronization.

Figure 2 shows BER properties versus length of N[bit] when $E_b/N_0 = 5$[dB] and $K = 3$. The mean BER performance reaches a minimum value when $N = 3$. P_K decreases with increasing N. P_N is smallest when $N = 3$ because the errors are dominated by the first term of Eq. (7) when $N \geq 3$ and the errors are dominated by the second term of Eq. (7) when $N < 3$. Therefore we take 3 for N in the following discussion.

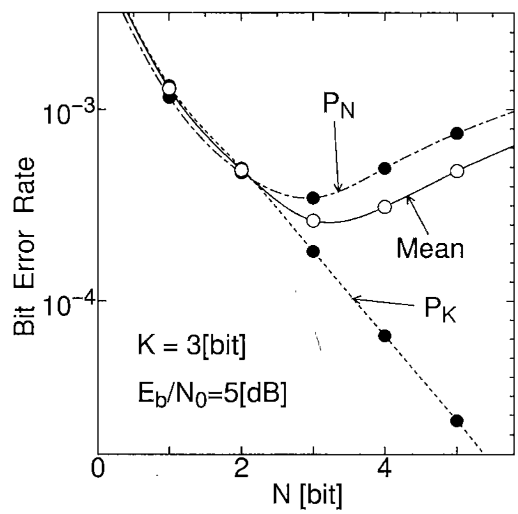

Fig. 2 BER versus the length of N[bit] data.

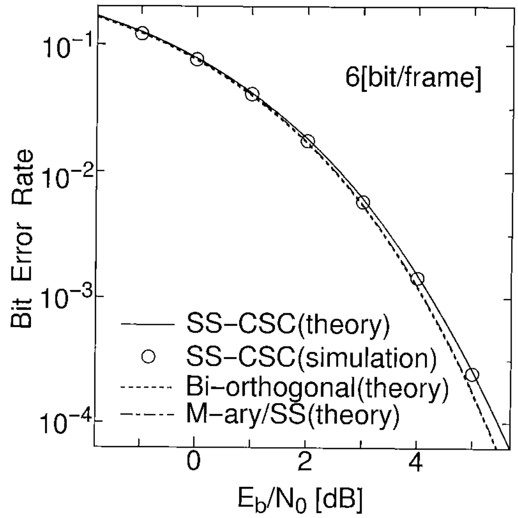

Fig. 3 Mean BER properties of the SS-CSC system and conventional systems at 6[bit/frame].

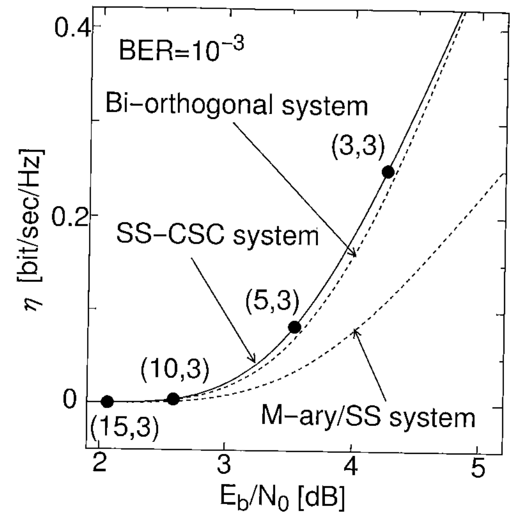

Fig. 4 Frequency utilization efficiency of the SS-CSC system when $N=3$ and $BER=10^{-3}$.

Figure 3 shows mean BER properties determined by theoretical analysis and computer simulation when 6[bit/frame] ($K=3$, $N=3$). The theoretical values are in good agreement with the simulation values. The BER property of the SS-CSC system is much the same as that of the Bi-orthogonal system.

A comparison of the frequency utilization efficiency for the SS-CSC system, the M-ary/SS system and the Bi-orthogonal system is shown in Fig. 4 when $BER=10^{-3}$. It is found that the frequency utilization efficiency of the SS-CSC system is better than that of the Bi-orthogonal system when $K \leq 10$.

5. Conclusions

The analysis of the BER of the SS-CSC system has been presented. The performance of the SS-CSC system was compared that of the Bi-orthogonal system. The results are as follows.

- The BER of the SS-CSC system is lowest when $N=3$. The BER property of the SS-CSC system is much the same as that of the Bi-orthogonal system.
- The theoretical values correspond with the results of computer simulation.
- The frequency utilization efficiency of the SS-CSC system is better than that of the Bi-orthogonal system when $K \leq 10$.

Future problems are examination of the synchronization performance and examination of Code Division Multiple Access (CDMA) capability.

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