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Phenomenological Description of Microwave Characteristics of Low-T_c Superconductor by Three-Fluid Model

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SUMMARY It is shown that a three-fluid model, which was successfully introduced to explain microwave characteristics of high-T_c superconductors phenomenologically, is suit also to explain those of low-T_c superconductors. In this model, the two contributions of a residual normal electron, in addition to a super and a normal electron in the two-fluid model, and of the temperature (T) dependence of momentum relaxation time τ for the two normal electrons are taken into account. Measured results of the T dependence of surface resistance R_s for a Nb film with critical temperature T_c=9.2 K agree very well with an R_s curve calculated using the present model, where a residual surface resistance at T=0 K, R_{s0}, and the T dependence of τ were determined using the surface reactance at 0 K X_{s0}=37.6 mΩ calculated using the BCS theory to fit a calculated R_s curve with the measured values as a function of T. Furthermore, microwave characteristics predicted from the BCS theory cannot be explained phenomenologically using the conventional two-fluid model. This difficulty can be solved by using an improved two-fluid model, called the two-fluid (τ) model, where the T dependence of τ is taken into account. Finally the frequency dependence of R_s calculated for the Nb film is $f^{1.9}$ for the BCS theory and $f^{2.0}$ for the three-fluid (τ) model on the assumption of the frequency independence of τ.

key words: *low-T_c superconductors, microwave, surface impedance, conductivity*

1. Introduction

For microwave applications of superconductors, it is important to investigate microwave behavior of their surface impedance $Z_s=R_s+jX$, where R_s is the surface resistance and X_s is the surface reactance, and their complex conductivity $\sigma=\sigma_1-j\sigma_2$. Conventional BCS theory [1], [2] and two-fiuid model [3] have been commonly used to explain the microwave characteristics of metallic low-T_c superconductors, although residual loss due to acoustic loss is suggested to play an important role in real materials below the temperature $T=0.33T_c$ [4], where T_c is the critical temperature. In addition we cannot explain the peak of σ_1 appearing just below T_c, which is predicted from calculations based on the BCS theory, using the two-fluid model.

On the other hand, a three-fluid model proposed by the authors [5]-[6], which is designated as "three-fluid (τ) model," was successfully introduced to explain microwave characteristics of high-T_c superconductors phenomenologically. In this model, two contributions are taken into account: one is of a residual normal electron in addition to a super and a normal electron in the conventinal two-fiuid model, and the other is that of the T dependence of momentum relaxation time τ for both these normal electrons.

In this paper, the validity of the three-fluid (τ) model is discussed to explain microwave characteristics of low-T_c superconductors. Furthermore, an improved two-fluid model, which is designated as "two-fluid (τ) model" and corresponds to the case of $R_{s0}=0$ in the three-fluid (τ) model or to a two-fluid model with τ dependent on temperature, is introduced to explain a peak of σ_1 appearing just below T_c, which is predicted from the BCS theory but cannot be explained by the conventional two-fluid model. The behaviors of the three-fluid (τ), two-fluid (τ), and two-fluid models and BCS theory are compared with measured results of the T dependence of R_s for a Nb film with T_c=9.2 K [10]. Finally, the frequency dependence of R_s for the Nb film is discussed from results evaluated using these models.

2. Calculation of Z_s and σ

2.1 The Three-Fluid (τ) Model

In the two-fluid model [3], as is well known, it is assumed that total electron density n, which is given as the sum of superelectron density n_s and normal electron density n_n, that is, $n=n_s+n_n$, is independent of T and the T dependences of n_s and n_n are given by

$$n_s=n\left[1-\left(\frac{T}{T_c}\right)^4\right] \qquad n_s=n\left[1-\left(\frac{T}{T_c}\right)^4\right]. \tag{1}$$

In the three-fluid (τ) model [8], residual normal electron density n_{res}, which is assumed to be independent of T, is added to n of the two-fluid model; that is,

$$n_t=n_s+n_n+n_{res}=n+n_{res}. \tag{2}$$

where n_t is the total electron density which is also independent of T. In addition, the T dependence of

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momentum relaxation time τ for n_n and n_{res} are taken into account. For high-T_c superconductors, the following equation, which was derived by Imai and Kobayashi [8] on the basis of measured results presented by Romero, et al. [11], was used:

$$\tau=\tau_c\left(\frac{T}{T_c}\right)^{\alpha}+\tau_0\left\{1-\left(\frac{T}{T_c}\right)^{\alpha}\right\} \quad (T\leq T_c), \tag{3}$$

where τ_c is τ at T_c K, $\tau_0(>\tau_c)$ is τ at 0 K, and α is a constant. Equation (3) is assumed to be valid also for low-T_c superconductors. The process of calculating Z_s and σ by the present model is described elsewhere [8] and reviewed briefly below. First, R_{sn} and R_{s0}, which are R_s values at T_c and 0 K, respectively, are determined from the result of the T dependence of R_s measured at f_0 for a superconductor with T_c K. Second, X_{s0}, which is the X_s value at 0 K, is taken to be equal to one calculated using the BCS theory, as discussed later. Third, fitting parameters $A=\tau_c/\tau_0$ and α in Eq. (3) are determined so as to fit a R_s curve, calculated using the model, with the measured R_s values. Fourth, the values of n_{res}, n, τ_0, and τ_0 are calculated according to Ref. [8]. Finally complex conductivity $\sigma=\sigma_1-j\sigma_2$ is given by

$$\sigma_1=\frac{(n_n+n_{res})\,e^2\tau}{m(\omega^2\tau^2+1)} \tag{4}$$

$$\sigma_2=\frac{n_s e^2}{m\omega}+\frac{(n_n+n_{res})\,e^2\tau^2\omega}{m(\omega^2\tau^2+1)}, \tag{5}$$

where $e=-1.6022\times10^{-19}$ C is the electron charge, $m=9.1096\times10^{-31}$ kg is the mass of an electron, and ω is the angular frequency. Then the relationship between Z_s and σ is given by

$$Z_s=\sqrt{\frac{j\omega\mu_0}{\sigma}}, \tag{6}$$

where $\mu_0=4\pi\times10^{-7}$ H/m is the permeability.

2.2 The Two-Fluid Model and the Two-Fluid (τ) Model

The two-fluid [3] and two-fluid (τ) models correspond to the case of $n_{res}=0$ for the three-fluid model described above; thus Eqs. (4) and (5) yield

$$\sigma_1=\frac{n_n e^2\tau}{m(\omega^2\tau^2+1)} \tag{7}$$

$$\sigma_2=\frac{n_s e^2}{m\omega}+\frac{n_n e^2\tau^2\omega}{m(\omega^2\tau^2+1)}, \tag{8}$$

where τ is independent of T for the two-fluid model and is given by Eq. (3) also for the two-fluid (τ) model. Then R_s and X_s are calculated from Eqs. (6), (7) and (8).

2.3 The BCS Theory

For the BCS theory σ_1 and σ_2 are given by [1], [2]

$$\frac{\sigma_1}{\sigma_n}=\frac{2}{\hbar\omega}\int_{\Delta}^{\infty}[f(E)-f(E+\hbar\omega)]g(E)\,dE + \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{-\Delta}[1-2f(E+\hbar\omega)]g(E)\,dE \tag{9}$$

$$\frac{\sigma_2}{\sigma_n}=\frac{1}{\hbar\omega}\int_{\substack{\Delta-\hbar\omega(\hbar\omega<2\Delta)\\-\Delta(\hbar\omega>2\Delta)}}^{\Delta}[1-2f(E+\hbar\omega)] \times\frac{E^2+\Delta^2+\hbar\omega E}{\sqrt{\Delta^2-E^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}}dE. \tag{10}$$

where

$$f(E)=\frac{1}{1+\exp(E/K_BT)} \tag{11}$$

$$g(E)=\frac{E^2+\Delta^2+\hbar\omega E}{\sqrt{E^2-\Delta^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}} \tag{12}$$

$$\sigma_n=\frac{\omega\mu_0}{R_{sn}^2}. \tag{13}$$

In the above, $\hbar=h/2\pi=1.0546\times10^{-34}$ Js is the Plank's constant, $K_B=1.3806\times10^{-23}$ J/K is the Boltzmann's constant, and $f(E)$ is the Fermi function. The energy gap Δ is given by

$$\frac{1}{N(0)V}=\int_0^{\hbar\omega_c}\frac{\tanh\frac{1}{2}\beta\sqrt{\zeta^2+\Delta^2}}{\sqrt{\zeta^2+\Delta^2}}d\xi, \tag{14}$$

where $N(0)$ is the density of states of one spin in energy at the Fermi surface, V is the attractive phonon interaction dominate of a constant average [1] and β is given by

$$\beta=\frac{1}{K_BT}. \tag{15}$$

Since Δ becomes 0 for $T=T_c$, Eq. (14) is expressed as

$$\frac{1}{N(0)V}=\int_0^{\beta_c\hbar\omega_c/2}\frac{\tanh x}{x}dx, \tag{16}$$

where β_c and ω_c are values of β and ω at T_c K. In actual numerical calculation, first the value of $1/N(0)V$ independent of T is calculated from Eq. (16), second the Δ values are calculated from Eqs. (14) and (15) as a function of T, third σ_1 and σ_2 are calculated from Eqs. (9) to (13), and finally R_s and X_s are calculated from Eq. (6).

3. Measured and Calculated Results for a Nb Film

3.1 R_s-Curve Fitting of Two-Fluid Model into BCS Theory

First, the T dependence of Z_s according to the BCS theory was calculated from Eqs. (6) and (9) to (16) using $R_{sn}=X_{sn}=0.17\ \Omega$ so as to fit the R_s values measured at 27.45 GHz for a Nb film with $T_c=9.2$ K [10]. In a similar manner, that in the two-fluid model was calculated from Eqs. (6), (7) and (8) using $\tau=3.79\times10^{-13}$ sec and $n=3.51\times10^{26}\ \mathrm{m}^{-3}$, which are independent of T. These results are shown in Fig. 1 (a).

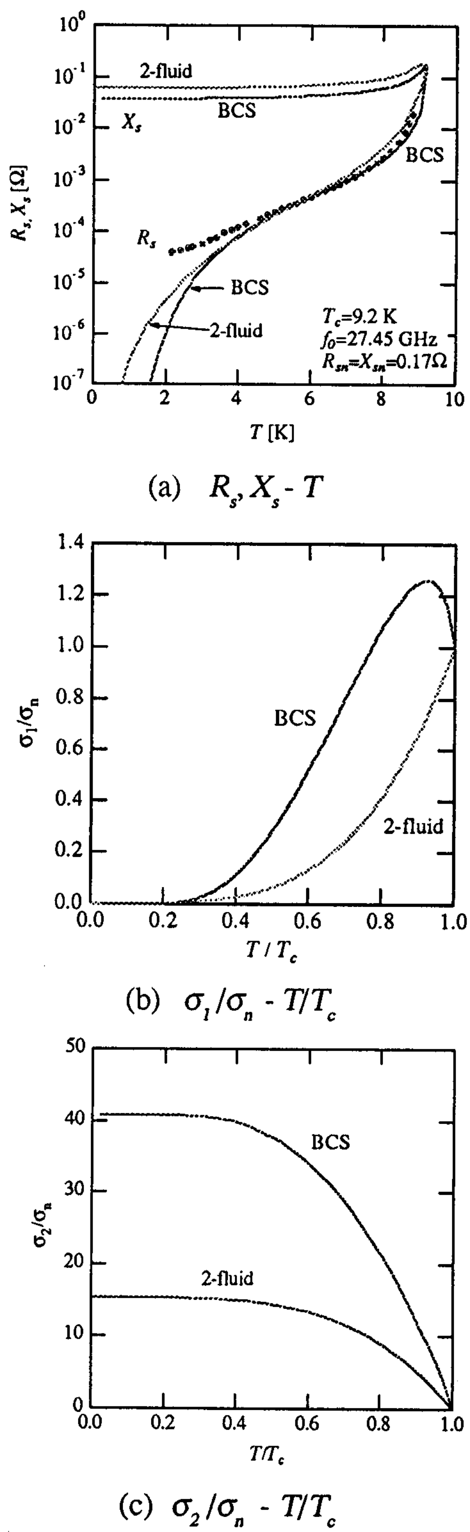


Fig. 1 Measured results of R_s for the Nb film, calculated results using the BCS theory and the two-fluid model in which the R_s curve is fitted with the measured R_s values.

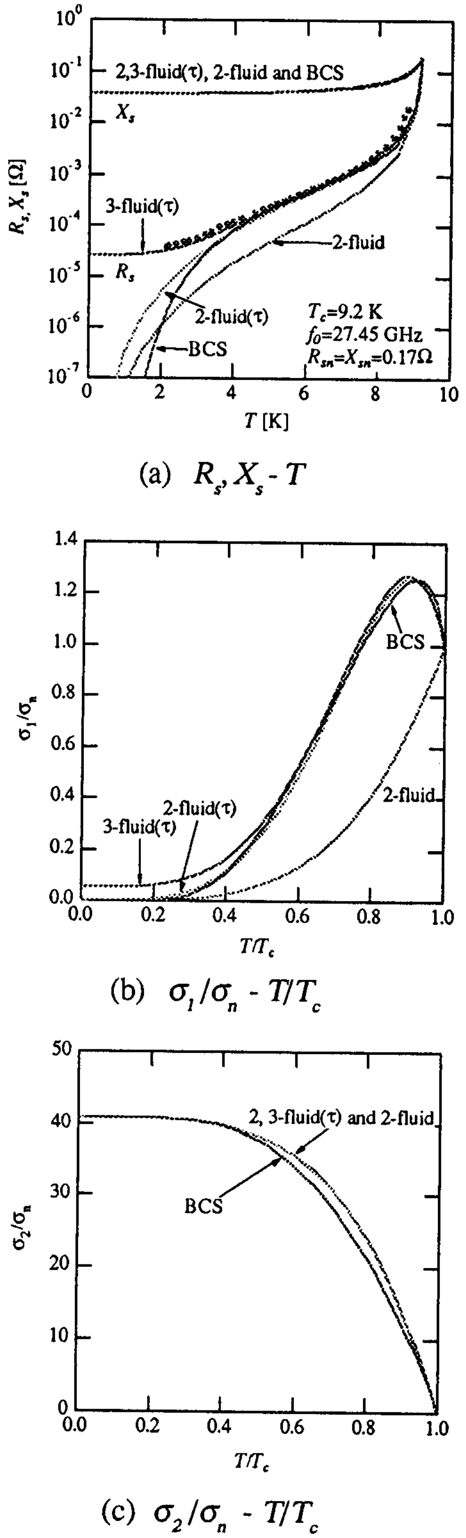


Fig. 2 Measured results of R_s for the Nb film and Z_s and σ calculated using the four models when $X_{s0}=37.6\ \mathrm{m}\Omega$ calculated from the BCS theory was used for other models.

These two calculated R_s curves agree well with the measured values in the temperature range from 4 to 9.2 K. On the other hand, the X_s values for the two-fluid model are different from those for the BCS theory. This causes the discrepancy that values of London penetration depth λ_L are different between these two models, since $\lambda_L = X_{s0}/\omega\mu_0$. Furthermore, results of σ_1/σ_n and σ_2/σ_n calculated from the R_s and X_s values for the two models are shown in Figs. 1(b) and (c). As seen in Fig. 1(b), a peak of σ_1 appears just below T_c for the BCS theory but such a peak does not appear for the two-fluid model. In Fig. 1(c) the σ_2 curve calculated for the two-fluid model does not coincide with one for the BCS theory. As a result we cannot phenomenologically explain microwave characteristics predicted from the BCS theory using the two-fluid model.

3.2 X_s-Curve Fitting of Three models with BCS Theory

Another approach was attempted, in which the X_{s0} values for the three models, the two-fluid, two-fluid (τ) and three-fluid (τ) models, were taken to be equal to $X_{s0}=37.6$ mΩ calculated by the BCS theory. The calculated results of Z_s are shown in Fig. 2(a), together with the measured R_s values and calculated Z_s curves for the BCS theory given in Fig. 1(a). For the two-fluid (τ) model the R_s curve corresponds to the case of $n_{res}=0$ in the three-fluid (τ) model. For the two-fluid model the R_s curve corresponds to the case of the constant value of $\tau=1.42\times10^{-13}$ sec at $T=T_c$ in the two-fluid (τ) model. Then the results of σ_1/σ_n and σ_2/σ_n calculated from Fig. 2(a) for these four models are shown in Figs. 2(b) and (c). The temperature dependence of τ used in the calculations is shown in Fig. 3, and the parameters used in the calculations are summarized in Table 1.

As shown in Fig. 2(a), the four X_s curves coincide very well for these four models. This corresponds to a good fit of four σ_2/σ_n curves in Fig. 2(c), since $\sigma_2=\omega\mu_0/X_s^2$. This means that these four models have the same values of penetration depth λ which is given by $\lambda = X_s/\omega\mu_0$.

Also for this case, we cannot phenomenologically explain microwave characteristics predicted form the BCS theory using the two-fluid model, because of the distinct difference between the R_s curve of the two-fluid model and the measured values.

The R_s curve of the three-fluid (τ) model coincides well with the measured R_s values at temperatures below 9.2 K. It is found from this result that the three-fluid (τ) model, which was valid in the case of the high-T_c superconductors [5]-[9], is also valid in the case of the low-T_c superconductor. For this Nb film the ratio of residual normal electron density to total electron density is $n_{res}/n_t=0.011$, as shown in Table 1, which is about one-tenth that for YBCO bulk [8].

As shown in Fig. 2, the curves of the two-fluid (τ) model agree well with those of the BCS theory in the temperature range from 4 to 9.2 K. The introduction of the T dependence of τ into the conventional two-fluid model is essential to explain the microwave characteristics using the BCS theory phenomenologically.

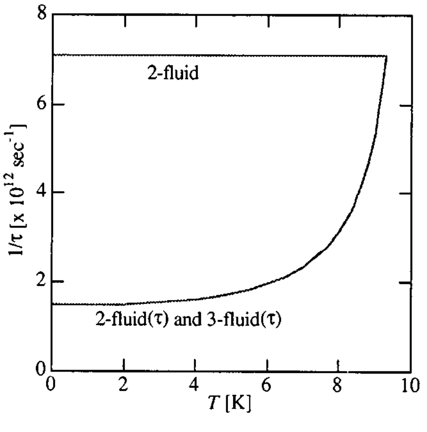


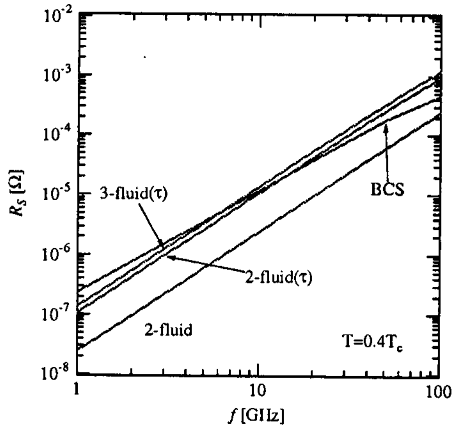
Fig. 3 Calculated results of the temperature dependence of $1/\tau$ used in the three-fluid (τ), two-fluid (τ) and two-fluid models.

4. Frequency Dependence of R_s

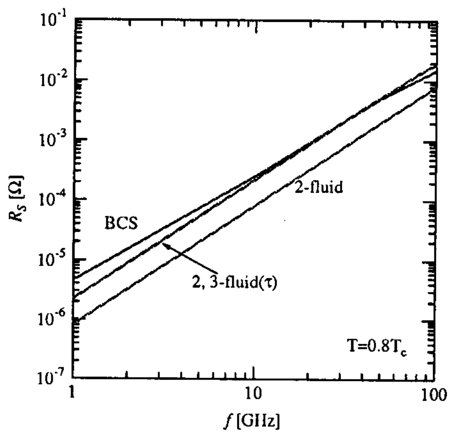
The T dependences of the Z_s values of the Nb film at 27.45 GHz were discussed above. We discuss the frequency dependences of R_s of this Nb film below, assuming that there is no dependence of τ on frequency. The frequency dependences of R_s for the four models were calculated from the equations given above. The results are shown in Fig. 4(a) for $T=0.4T_c$ and in Fig. 4(b) for $T=0.8T_c$. It was found from Fig. 4 that frequency dependences of R_s for the Nb film are f_2 for the three-fluid (τ), two-fluid (τ) and two-fluid models and $f^{1.9}$ for the BCS theory. Results calculated in a similar manner for only the three-fluid

Table 1 Parameters used in calculation for the Nb film using the four models.

	T_c(K)	f_0(GHz)	R_{sn}(Ω)	R_{s0}(mΩ)	X_{s0}(mΩ)	n_{res}(m^{-3})	n_t(m^{-3})	τ_0(sec)	τ_c(sec)	α	n_{res}/n_t	τ_0/τ_c
3-fluid(τ)	9.2	27.45	0.17	0.024	37.6	1.07×10^{25}	9.48×10^{26}	6.70×10^{-13}	1.42×10^{-13}	2.7	0.011	4.72
2-fluid(τ)				----		0	9.38×10^{26}	6.70×10^{-13}	1.42×10^{-13}	2.7	0	4.72
2-fluid				----		0	9.38×10^{26}	1.42×10^{-13}	1.42×10^{-13}	---	0	1
BCS				----		----	----	----	----	---	----	----



(a) R_s - f at T=0.4T_c K



(b) R_s - f at T=0.8T_c K

Fig. 4 Calculated results of the frequency dependence of R_s using the four models.

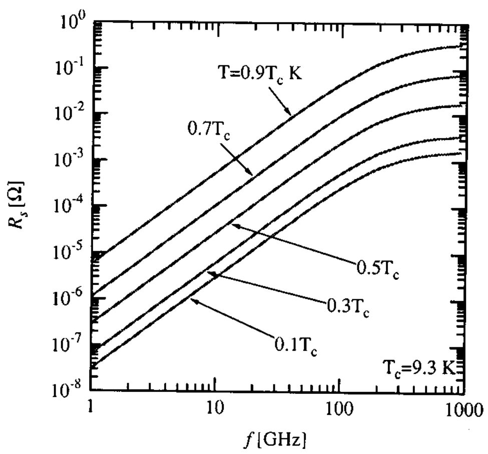


Fig. 5 Calculated results of the frequency dependence of R_s using the three-fluid (τ) model.

(τ) model in the frequency range from 1 to 1000 GHz are shown in Fig. 5. The f^2 dependences of R_s are constant independent of T below 70 GHz, and the influence of the term $\omega\tau$ in Eqs. (4) and (5) appears for the frequency dependences of R_s over 70 GHz.

5. Conclusion

Conclusions are summarized as follows.

(1) The three-fluid (τ) model is useful for phenomenological description of the T dependence of the measured R_s values of a Nb film at temperatures below T_c.

(2) Microwave characteristics predicted from the BCS theory cannot be explained phenomenologically by the conventional two-fluid model, where the T dependence of τ is not taken into account. This difficulty can be solved by using the two-fluid (τ) model, where the T dependence of τ is taken into account.

(3) The frequency dependence of R_s calculated for a Nb film with T_c=9.2 K using the BCS theory is $f^{1.9}$ below 40 GHz. On the other hand, the results calculated by the three-fluid (τ) model on the assumption of the frequency independence of τ is $f^{2.0}$ below 70 GHz.

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References

[1] J. Bardeen, L. N. Cooper, and J. S. Schrieffe, "Theory of superconductivity," Phys. Rev., vol. 108, no. 5, pp. 1175–1204, 1957.
[2] D. C. Mattis and J. Bardeen, "Theory of the anomalous skin effect in normal and superconducting materials," Phys. Rev., vol. 111, no. 2, pp. 412–417, 1958.
[3] T. V. Duzer and C. W. Turner, "Principles of superconductive devices and circuits," Chapter 3, Elsevier North Holland, Inc., 1981.
[4] W. H. Hartwig, "Superconducting resonators and devices," Proc. IEEE, vol. 77, no. 1, pp. 58–70, June 1993.
[5] Y. Kobayashi and T. Imai, "Phenomenological description of conduction mechanism of high-T_c superconductors by three-fluid model," IEICE Trans. Electron., vol. E78-C, no. 5, pp. 498–502, 1995.
[6] T. Imai, T. Sakakibara, and Y. Kobayashi "Microwave characteristics of high-T_c superconductors by improved three-fluid model," IEICE Trans., vol. E74, no. 7, pp. 1986–1992, 1991.
[7] Y. Kobayashi, T. Imai, and T. Sakakibara, "Phenomenological description of "coherence peak" of high-T_c superconductors by improved three-fluid model," Proc. of 23rd EuMC, pp. 596–599, 1993.
[8] T. Imai and Y. Kobayashi, "Phenomenological description of temperature and frequency dependence surface resistance of high-T_c superconductors by improved three-fluid model," IEICE Trans. Electron., vol. E78-C, no. 5, pp. 498–502, 1995.
[9] Y. Kobayashi, "Microwave applications of high-temperature superconductor devices," Proc. of 26th EuMC, pp. 823–829, 1996.
[10] C. Wilker, Z. - Y. Shen, V. X. Nguyen, and M. S. Brenner,

"A sapphire resonator for microwave characterization of superconducting thin films," IEEE Trans. Appl. Superconductivity, vol. 3, no. 1, pp. 1457-1460, 1993.

[11] D. B. Romero, C. D. Porter, D. B. Tanner, L. Forro, D. Mandrus, L. Mihaly, G. L. Carr, and G. P. Williams, "Quasiparticle damping in $Bi_2Sr_2CaCu_2O_8$ and $Bi_2Sr_2CuO_6$," Phys. Rev. Lett., vol. 68, no. 10, pp. 1590-1593, 1992.

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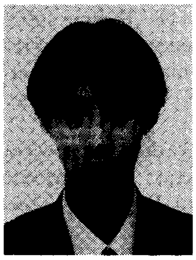

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