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Microwave Characteristics of High-T_c Superconductors by Improved Three-Fluid Model

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SUMMARY In order to explain the temperature and frequency characteristics of high-T_c superconductors, a new model is proposed, which will be called the improved three-fluid model, where the momentum relaxation time τ is assumed to depend on temperature in the superconducting and normal states, respectively, although τ has been assumed to be independent of temperature for the conventional three-fluid model. According to this model, the complex conductivity $\dot{\sigma}=\sigma_1-j\sigma_2$ and the surface impedance $Z_s=R_s+jX_s$, where R_s is the surface resistance and X_s is the surface reactance, are expressed as a function of temperature. The temperature dependences of Z_s and $\dot{\sigma}$ for a YBCO bulk estimated using this model agree very well with ones measured by the dielectric-loaded cavity method in room to cryogenic temperature. In particular, a peak of σ_1 observed just below the critical temperature T_c in experiments, appeared in the calculated results based on this model. This phenomenon has been already known in the BCS theory. Thus, it is verified that this model is useful to explain the microwave characteristics of high-T_c superconductors in room to cryogenic temperature. On the other hand, the residual normal electron density $n_{res}=4.254\times10^{23}\ \mathrm{m}^{-3}$ and the total electron density $n_t=7.308\times10^{24}\ \mathrm{m}^{-3}$ are obtained by calculation. The ratio $n_{res}/n_t=0.058$ can be used as figure of merit to evaluate material quality of high-T_c superconductors; thus it means that there is 5.8% nonpairing electron in this YBCO bulk.

key words: *high-T_c superconductor, microwave, dielectric resonator, complex conductivity, momentum relaxation time, improved three-fluid model*

1. Introduction

For microwave applications of high-T_c superconductors, it is important to investigate their surface impedance $Z_s=R_s+jX_s$, where R_s is the surface resistance and X_s is the surface reactance, and their complex conductivity $\dot{\sigma}=\sigma_1-j\sigma_2$. The conventional BCS theory [1], [2] and two-fluid model [3] are useful to describe microwave characteristics of low-T_c metallic superconductors. However, these theories are not valid to describe behavior of high-T_c superconductors in cryogenic temperature range, because the residual surface resistance R_{sres} is much greater than one of low-T_c metallic superconductors [4]. To overcome this difficulty, authors proposed the so-called three-fluid model [5], where a concept of nonpairing residual normal electron density n_{res} was introduced into the conventional two-fluid model on the assumption that the momentum relaxation time τ is independent of temperature T. As a result, it was found that the calculated values of $\dot{\sigma}$ and Z_s by this model are in good agreement with measured ones [5]–[7] for a YBCO bulk in a temperature range much lower than critical temperature T_c, although this model does not explain a peak of σ_1 appearing just below T_c, which is expected from BCS theory and is observed experimentally for such samples as YBCO film [8] and bulk [7], and a BPSCCO bulk [7]. This peak of σ_1 can not be also explained by the so-called "enhanced" two-fluid model proposed by Vendik [9], who assumes that $1/\tau$ is proportional to T both in the superconducting and normal states.

Recently, Romero and others [10] have reported that the measured $1/\tau$ values of high-T_c superconductors are proportional to T in the normal state and decrease rapidly with the decrease of T below T_c. According to this experimental fact, a new model, which will be called the improved three-fluid model, is proposed to explain the microwave characteristics of high-T_c superconductors in room to cryogenic temperature, particularly the peak of σ_1 observed just below T_c. In this paper, at first, $\dot{\sigma}$ and Z_s are expressed as a function of T. Then the validity of this model is confirmed by comparison of calculation with measurement for a YBCO bulk. Finally, we discuss a method to evaluate material quality using the ratio n_{res}/n_t, where n_t is the total electron density.

2. Improved Three-Fluid Model

2.1 Temperature Dependence of $\dot{\sigma}$ in Superconducting State

In the conventional two-fluid model [3], as is well known, it is assumed that total electron density n which is given as addition of super electron density n_s and normal electron density n_n, that is, $n=n_s+n_n$, is independent of T and the temperature dependences of n_s and n_n are given by

$$\frac{n_s}{n}=1-\left(\frac{T}{T_c}\right)^4 \tag{1}$$

$$\frac{n_n}{n}=\left(\frac{T}{T_c}\right)^4 \tag{2}$$

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where T_c is the critical temperature.

In the three-fluid model proposed by authors [5], residual normal electron density n_{res} which is assumed to be independent of T, is added to n: that is,

$$n_t = n + n_{res} = n_s + n_n + n_{res} \tag{3}$$

where n_t is the total electron density which is also independent of T. In this case, the complex conductivity $\dot{\sigma} = \sigma_1 - j\sigma_2$ is given by [5]

$$\sigma_1 = \frac{n_n e^2 \tau}{m(\omega^2\tau^2+1)} + \frac{n_{res} e^2 \tau}{m(\omega^2\tau^2+1)} \tag{4}$$

$$\sigma_2 = \frac{n_s e^2}{m\omega} + \frac{n_n e^2 \tau^2 \omega}{m(\omega^2\tau^2+1)} + \frac{n_{res} e^2 \tau^2 \omega}{m(\omega^2\tau^2+1)} \tag{5}$$

where $e = -1.6022 \times 10^{-19}$ C is the electron charge, $m = 9.1096 \times 10^{-31}$ kg is the mass of electron, ω is the angular frequency, τ is the momentum relaxation time, which has been assumed to be independent of T in the three- and two-fluid models.

In the improved three-fluid model proposed here, τ in the superconducting state is assumed as

$$\tau = \tau_c\left(\frac{T}{T_c}\right)^\alpha + \tau_0\left\{1 - \left(\frac{T}{T_c}\right)^\alpha\right\} \quad (T \leqq T_c) \tag{6}$$

where τ_c is τ at T_c K, τ_0 ($>\tau_c$) is τ at 0 K, and α is a constant. These three values should be adjusted so that the temperature dependence of R_s calculated using the improved three-fluid model based on Eq.(6) is fitted best to the measured results of R_s. Then, in order to determine the temperature dependences of σ_1 and σ_2, formulas for n, n_{res}, τ_c, and τ_0 as a function of Z_s are derived below.

Defining σ_{10} and σ_{20} by σ_1 and σ_2 at 0 K, respectively, we obtain from Eqs.(1)-(6)

$$\sigma_{10} = \frac{n_{res} e^2 \tau_0}{m(\omega^2\tau_0^2+1)} \tag{7}$$

$$\sigma_{20} = \frac{ne^2}{m\omega} + \frac{n_{res} e^2 \tau_0^2 \omega}{m(\omega^2\tau_0^2+1)} \tag{8}$$

Also, defining σ_{1c} by σ_1 at T_c K, we obtain from Eqs. (1)-(4) and (6)

$$\sigma_{1c} = \frac{n_t e^2 \tau_c}{m(\omega^2\tau_c^2+1)} \approx \frac{n_t e^2 \tau_c}{m} \tag{9}$$

Defining A by

$$\frac{\tau_c}{\tau_0} = A \tag{10}$$

we can derive the following relations:

$$\tau_0 = \frac{\sigma_{1c} - \sigma_{10}A}{\sigma_{20}\omega A} \tag{11}$$

from Eqs.(3) and (7)-(10),

$$\tau_c = \frac{\sigma_{1c} - \sigma_{10}A}{\sigma_{20}\omega} \tag{12}$$

from Eqs.(10) and (11),

$$n_t = \frac{\sigma_{1c}\sigma_{20} m\omega}{e^2(\sigma_{1c} - \sigma_{10}A)} \tag{13}$$

from Eqs.(9) and (12),

$$n_{res} = \frac{\sigma_{10} m\omega\{(\sigma_{1c} - \sigma_{10}A)^2 + \sigma_{20}^2 A^2\}}{e^2 \sigma_{20} A(\sigma_{1c} - \sigma_{10}A)} \tag{14}$$

from Eqs.(7) and (11), and

$$n = \frac{m\omega}{e^2(\sigma_{1c} - \sigma_{10}A)} \cdot \left[\sigma_{1c}\sigma_{20} - \frac{\sigma_{10}\{(\sigma_{1c} - \sigma_{10}A)^2 + \sigma_{20}^2 A^2\}}{\sigma_{20}A}\right] \tag{15}$$

from Eqs.(3), (13) and (14).

Actually, the values of σ_{10}, σ_{20}, and σ_{1c} given by Eqs.(7)-(9) are determined from the measured values of R_s and X_s [5]: that is,

$$\sigma_{10} = \frac{2\omega\mu R_{s0} X_{s0}}{(R_{s0}^2 + X_{s0}^2)^2} \tag{16}$$

$$\sigma_{20} = \frac{\omega\mu(X_{s0}^2 - R_{s0}^2)}{(R_{s0}^2 + X_{s0}^2)^2} \tag{17}$$

$$\sigma_{1c} = \frac{\omega\mu}{2R_{sn}^2} \tag{18}$$

where R_{s0} and X_{s0} are R_s and X_s at 0 K, R_{sn} is R_s at T_c K, and $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m.

Therefore, if the values of A and α are determined, the temperature dependences of σ_1 and σ_2 in the superconducting state can be calculated from the equations described above.

2.2 Temperature Dependence of $\dot{\sigma}$ in the Normal state

In the normal state, considering that the condition $\omega^2\tau^2 \ll 1$ hold in the microwave range, we obtain the following relations from Eqs.(1)-(5):

$$\sigma_1 = \frac{n_t e^2 \tau}{m}, \quad \sigma_2 = 0 \tag{19}$$

According to a familiar relation that τ is proportional to $1/T$ in the normal state [10], we assume the temperature dependence of τ to be

$$\tau = \tau_c - B\left(1 - \frac{T_c}{T}\right) \tag{20}$$

where B is a constant which should be adjusted so that the temperature dependence of R_s calculated using the improved three-fluid model based on Eq.(20) is fitted best to the measured results of R_s. Thus, if the value of B is determined, the temperature dependences of σ_1 and σ_2 in the normal state can be calculated from Eqs. (12), (13), and (16)-(20).

2. 3 Temperature Dependence of Z_s

The temperature dependences of R_s and X_s can be calculated from the temperature dependences of σ_1 and σ_2 as follows [5]:

$$R_s=\sqrt{\frac{\omega\mu(\sqrt{\sigma_1^2+\sigma_2^2}-\sigma^2)}{2(\sigma_1^2+\sigma_2^2)}} \tag{21}$$

$$X_s=\sqrt{\frac{\omega\mu(\sqrt{\sigma_1^2+\sigma_2^2}+\sigma_2)}{2(\sigma_1^2+\sigma_2^2)}} \tag{22}$$

3. Measured and Calculated Results

3. 1 Surface Impedance Z_s

Figure 1 shows a structure of a dielectric-loaded cavity used for measurement of Z_s, which is constructed from a copper plated brass cylinder, a copper plated brass plate, a YBCO bulk having diameter d_0=28 mm and T_c=95 K, and a BMT ceramic rod (Murata Mfg. Co. Ltd.) having relative permittivity ε_r=24.5 at room temperature. Figure 2(a) shows the measured results of R_s for the YBCO bulk at 9.7 GHz [7]. Extrapolating R_s curve until 0 K, we obtain R_{s0}=0.0022 Ω. Also, we obtain R_{sn}=0.3955 Ω at T_c=95 K. Figure 2(b) shows the measured results of X_s for the YBCO bulk after considering the surface roughness [7] 2 μm of the YBCO bulk. Extrapolating X_s curve until 0 K, we obtain X_{s0}=0.1520 Ω. Thus, from the R_{s0}, R_{sn}, X_{s0}, and Eqs.(16)-(18), we obtain $\sigma_{10}=9.578\times10^4$ S/m, $\sigma_{20}=3.308\times10^6$ S/m, and $\sigma_{1c}=2.445\times10^5$ S/m. The values of α, A, and B are determined so that the temperature dependence of R_s calculated using the improved three-fluid model based on Eqs.(6), (10), and (20), which is indicated in Fig. 2(a) by "3-fluid (τ)," is fitted best to the measured results of R_s. As a result, authors obtain $\alpha=1$, $A=0.057$, and $B=1\times10^{-12}$. Thus, from these values and Eqs.(11)-(15), we obtain $\tau_0=2.083\times10^{-11}$ s, $\tau_c=1.187\times10^{-12}$ s, $n_t=7.308\times10^{24}$ m^{-3}, $n_{res}=4.254\times10^{23}$ m^{-3}, and $n=6.882\times10^{24}$ m^{-3} which corresponds to London's penetration depth λ_L= 2 μm. Also, the calculated results of X_s for the improved three-fluid model is indicated in Fig. 2(b) by "3-fluid (τ)." Furthermore, ones for the conventional two- and three-fluid models are indicated by "2-fluid" and "3-fluid," respectively. As a result, the calculated results of X_s for the improved three-, two- and, three-fluid models agree with the measured ones in the range of $T<T_c$. Furthermore, the improved three-fluid model enables us to explain the temperature dependence of X_s in the range of $T>T_c$ which can not be explained by the conventional two- and three-fluid models.

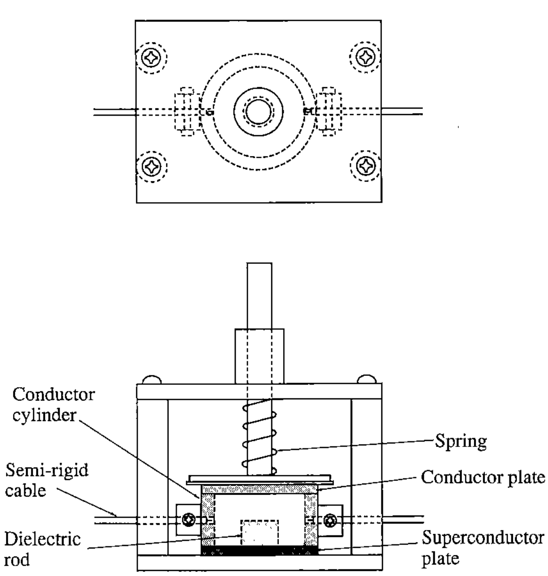
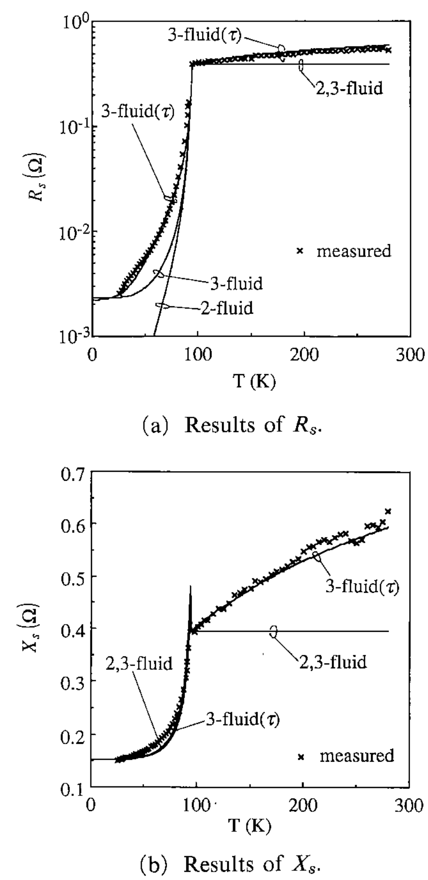


Fig. 1 Structure of dielectric-loaded cavity used for measurement of Z_s.

3. 2 Complex Conductivity $\dot{\sigma}$

The measured results of σ_1 and σ_2 for the YBCO bulk

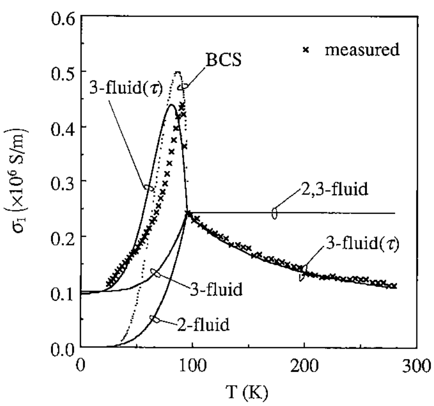


(a) Results of R_s.

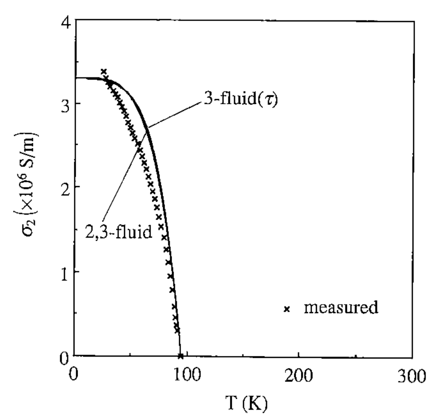
(b) Results of X_s.

Fig. 2 Measured results of R_s and X_s and calculated ones by improved three-fluid, two-fluid, and three-fluid models for a YBCO bulk.

are shown in Figs. 3(a) and (b), respectively. In the σ_1 results, a peak is observed at 90 K, which is expected from BCS theory. The calculated results of σ_1 and σ_2 for the improved three-fluid model are indicated in Figs. 3(a) and (b) by "3-fluid (τ)". Also, ones for the conventional two- and three-fluid models are indicated by "2-fluid" and "3-fluid", respectively. Furthermore, calculated ones of σ_1 for BCS theory using the energy gap $\Delta=3.2k_BT_c\sqrt{1-T/T_c}$ is indicated by "BCS". The conventional two- and three-fluid models can not explain the measured peak of σ_1. Also, the calculated result of σ_1 for BCS theory agree with the measured one in the range of 50 K$< T < T_c$, but does not agree with measured one in the range of $T<50$ K. On the other hand, the calculated results of σ_1 for the improved three-fluid model agree well with measured one in the range of $T<T_c$; particularly the peak observed just below T_c. Thus, the improved three-fluid model enables us to explain the peak of σ_1 which can not be explained by the conventional two-, three-, and "enhanced" two-fluid models, and authors conclude that the peak of σ_1 is appeared because increasing τ surpasses decreasing n_n in just below T_c. Furthermore, the improved three-fluid model enables us to explain the temperature dependence of σ_1 in the range of $T>T_c$ which can not be explained by the conventional two- and three-fluid models. Also, the calculated results of σ_2 for the improved three-fluid model are almost the same as ones for the conventional two- and three-fluid models, and these results agree with the measured ones in the range of $T<T_c$.



(a) Results of σ_1.



(b) Results of σ_2.

Fig. 3 Measured results of σ_1 and σ_2 and calculated ones by improved three-fluid, two-fluid, and three-fluid models for a YBCO bulk.

3.3 Electron Densities n_s, n_n, n_{res}, and n_t

Figure 4 shows the calculated results of the n_s, n_n, n_{res}, and n_t for the YBCO bulk by the improved three-fluid model from Eqs. (1), (2), (13), and (14). It is found that the ratio n_{res}/n_t is 0.058 and there is 5.8% non-pairing electron in this YBCO bulk. We can use the ratio n_{res}/n_t as figure of merit to evaluate material quality of high-T_c superconductors.

3.4 Momentum Relaxation Time τ

Figure 5 shows the calculated results of $1/\tau$ for the YBCO bulk by the improved three-fluid model from Eqs. (6) and (20). $1/\tau$ is proportional to T in the

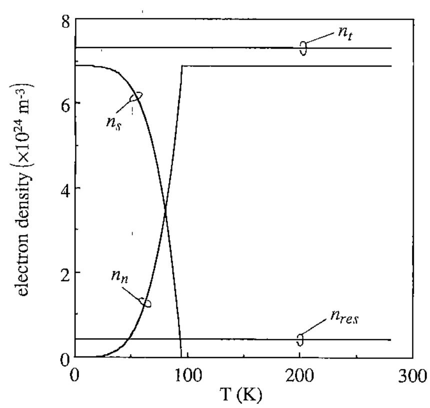


Fig. 4 Calculated results of n_s, n_n, n_{res}, and n_t for a YBCO bulk by improved three-fluid model.

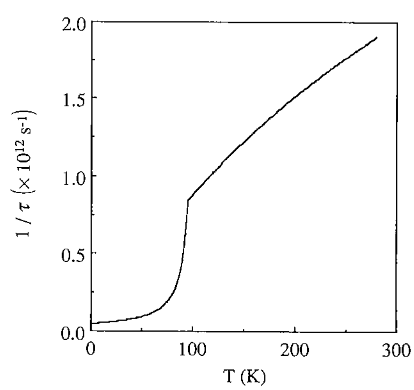


Fig. 5 Calculated results of $1/\tau$ for a YBCO bulk by improved three-fluid model.

normal state and decrease rapidly with the decrease of T below T_c. Considering the temperature dependence of τ shown in this figure, we can explain the microwave characteristics in room to cryogenic temperature, particularly a peak of σ_1 observed below T_c.

4. Conclusion

It was verified that the improved three-fluid model proposed in this paper is useful to explain the microwave characteristics in room to cryogenic temperature, and particularly a peak of σ_1 observed just below T_c which can not be explained by the conventional two-, three-, and "enhanced" two-fluid models model. If the values of n_s, n_n, n_{res}, and τ do not have the frequency dependence, we can calculate easily the frequency and temperature dependances of R_s by using the improved three-fluid model. It is expected that these calculated values are useful to design the microwave devices.

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