PAPER

Equalisation of Time Variant Multipath Channels Using Amplitude Banded LMS Algorithms

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SUMMARY For the purpose of equalisation of rapidly time variant multipath channels, we derive a novel adaptive algorithm, the amplitude banded LMS (ABLMS), which implements a nonlinear adaptation based on a coefficient matrix. Then we develop the ABLMS algorithm as the adaptation procedure for a linear transversal equaliser (LTE) and a decision feedback equaliser (DFE) where a parallel adaptation scheme is deployed. Computer simulations demonstrate that with a small increase of computational complexity, the ABLMS based parallel equalisers provide a significant improvement related to the conventional LMS DFE and the LMS LTE in the case of a second order Markov communication channel model.

key words: *time variant multipath channel, linear transversal equaliser, LMS algorithm, amplitude banded technique*

1. Introduction

Data transmission over a number of communications channels is restricted by the nonideal characteristics of the channels, such as rapid time variation and multipath dispersion. This is typical on high-frequency (HF) channels and mobile radio channels. In general, adaptive equalisation techniques [1] are used to achieve high speed digital communications. However, rapidly time variant characteristics can adversely affect the adaptive equaliser performance, and as a result impair the efficiency of communication. Therefore, it is desired to develop an adaptive equaliser which robustly operates in time variant environments.

A linear transversal equaliser (LTE) and a decision feedback equaliser (DFE) are commonly used for communications channel equalisation. The DFE has a similar computational complexity with the LTE, but often provides better performance especially when the channel has spectral zeros. On time variant multipath channels, spectral zeros frequently occur. This seems to be the main reason why the use of the DFE has been proposed on troposcatter and HF channels [2]–[5]. In an early work, Monsen deployed the least mean square (LMS) algorithm to adapt the coefficients of the DFE [2]. However, it was later observed that rapidly time variant channels such as the HF channel limit the performance of the LMS DFE [3]. In [3]–[5], it was shown that the DFE with coefficients adapted by the recursive least square (RLS) algorithm provides better performance than the LMS DFE. This advantage was, however, obtained at the expense of much increased computation. It is well known that the computation of the RLS algorithm is, for each iteration, proportional to M^2, where M is the length of the filter, while that of the LMS algorithm is proportional to M [7]. Even if fast algorithms for the RLS are deployed, they still require extensive computation, which are at least four times larger than the computation of the LMS algorithms [9].

On the other hand, for the purpose of time variant channel estimation, Clark et al. [10] and McLaughlin et al. [11] insisted that the LMS algorithm is more suitable than the RLS algorithm in the filter structure of a transversal filter due to its cost-effectiveness. Furthermore, recently, it was shown that in the case of Markov communication channel models, the LMS and RLS transversal equalisers behave roughly equivalently [12], [13]. These reports make a motivation to deploy the LMS algorithm in this paper as the adaptation procedure for the equalisers in time variant environments.

Although the use of the DFE is often preferred to that of the LTE, it is necessary to note the fact that the superiority of the DFE is not guaranteed if the decision device involved in the structure of the DFE does not output correct sequences (In [6], under the condition without decision errors, the superiority of the DFE has been shown). The occurrence of decision errors in the DFE is obviously true in rapid time variant environments. This means that in such environments the error propagation [8] in the feedback section of the DFE leads to poor performance. The decision error problem of the DFE is obviously dependent on the characteristics of the channel and on the adaptive algorithm to be deployed. Therefore, in this paper, the use of the LTE is investigated as well as that of the DFE.

This paper proposes a novel technique for adaptive equalisers, the amplitude banded technique [14], to cope with time variant multipath channels. The amplitude banded technique implements a non-linear adaptation for the equaliser coefficients. The basic idea of the amplitude banded technique is that if some degree of freedom in a coefficient vector is permitted and if the coefficients to be updated are selected in a control-

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lable fashion which is associated with the time variant channel, the adaptation may work to effectively alleviate the time variation of the channel. The amplitude banded technique prepares a coefficient matrix to introduce this degree of freedom. On the other hand, based on the fact that one of the main distortions apparent in a communication system is amplitude distortion, we consider that the amplitude of the received sequence is directly associated with the time variation of the channel. Therefore, in the amplitude banded technique, the amplitude information is deployed to select the coefficients to be updated. Based on the amplitude level of the received sequence, the equaliser coefficients are, for each iteration, selected from the elements of the coefficient matrix, and then updated.

In this paper, combined with the amplitude banded technique, the LMS algorithm is improved and the amplitude banded LMS (ABLMS) algorithm is derived. The ABLMS algorithm is further developed as the adaptation procedure for both the LTE and the DFE, in which a parallel adaptation with the LMS algorithm is implemented. The ABLMS adaptation to be implemented in the parallel fashion induces and retains rapid tracking against time variant multipath channels, with a small increase of computation. We demonstrate the performance of the ABLMS-LMS parallel equalisers by computer simulations. The results show that the novel equalisers provide better performance than the conventional LMS LTE and LMS DFE. It is also shown how the ABLMS algorithm is affected by the selection of the filter structures of LTE and DFE.

2. Adaptation Procedure

This paper deals exclusively with the training mode of channel equalisation. Therefore, the results provided are ideal ones to be achieved by the proposed equalisers, as a periodic combination of the training mode and tracking mode is usually required in a practical system, where some deteriorations of performance could be expected.

2.1 Channel Model

The channel is assumed to be a discrete-time finite impulse response channel corrupted by additive noise. Thus if u_k is the transmitted sequence, the output of the channel is a noise-corrupted sequence x_k given by

$$x_k = \sum_{i=0}^{L-1} h_i(k) u_{k-i} + n_k \tag{1}$$

where $h_o(k), h_1(k), ..., h_{L-1}(k)$ is the channel impulse response and n_k is a Gaussian white noise uncorrelated with u_k.

2.2 ABLMS Algorithm

For the standard LMS algorithm (normalised version), the tap coefficient vector $\mathbf{c}(k)$ is updated by the following equation:

$$\mathbf{c}(k+1) = \mathbf{c}(k) + \frac{\mu}{\beta + \mathbf{x}(k)^T \mathbf{x}(k)} \mathbf{x}(k) \epsilon_k \tag{2}$$

where $\mathbf{x}(k)$ is the input vector, ϵ_k is the output error sequence, and μ and β are constant parameters to control the convergence. When $\mathbf{c}(k)$ and $\mathbf{x}(k)$ are given by $\mathbf{c}(k) = (c_0(k), c_1(k), ..., c_{M-1}(k))^T$ and $\mathbf{x}(k) = (x_k, x_{k-1}, ..., x_{k-M+1})^T$, respectively, Eq. (2) provides the adaptation procedure for an M length LTE.

For the amplitude banded algorithm to be proposed here, in the case of an LTE, a Q by M coefficient matrix $\mathbf{C_a}(k)$ is prepared, elements of which are given by $c_{ij}(k), i = 1, 2, ..., Q, j = 1, 2, ..., M$. The $\mathbf{C_a}(k)$ is initialized at $k = 0$ where all the elements are set to zero. For the adaptation of the algorithm, the elements of $\mathbf{C_a}(k)$ are updated based on the operation of switching the elements to be updated. Specifically, among the Q by M elements of $\mathbf{C_a}(k)$, only M elements, $c_{q(j)j}(k), j = 1, 2, ..., M$, are selected for each iteration and a coefficient vector is set as $\mathbf{c_a}(k) = (c_{q(1)1}(k), c_{q(2)2}(k), ..., c_{q(M)M}(k))^T$ where $q(j)$ is an integer and determined based on the amplitude level of each element x_{k-j+1} of the input vector $\mathbf{x}(k)$ for $j = 1, 2, ..., M$ as follows:

- if $|x_{k-j+1}| \le A_{max}/Q$, then $q(j) = 1$.
- if $A_{max}/Q < |x_{k-j+1}| \le 2A_{max}/Q$, then $q(j) = 2$.
- if $2A_{max}/Q < |x_{k-j+1}| \le 3A_{max}/Q$, then $q(j) = 3$.
- .
- .
- if $(Q-1)A_{max}/Q < |x_{k-j+1}|$, then $q(j) = Q$.

The A_{max} denotes the maximum amplitude of the received sequence and Q corresponds to a division number to classify the level of the amplitude of the received sequence. The output of the filter whose coefficient vector is $\mathbf{c_a}(k)$ is obtained by convolution between $\mathbf{c_a}(k)$ and $\mathbf{x}(k)$. Thus the coefficient vector is also updated by the LMS algorithm (2), but where $\mathbf{c}(k)$ should be replaced by $\mathbf{c_a}(k)$. This algorithm provides the ABLMS algorithm for an M length LTE.

The maximum amplitude of the received sequence, A_{max}, should be measured from the received sequence before the equaliser is implemented. Accurate estimation of A_{max} is desired, but slightly inaccurate estimation may be also acceptable. This is because the range of the amplitude corresponding to $q(j) = Q$ is not severely restricted, and occurs with the lowest probability compared with the other range cases.

As an example, consider that the equaliser length is $M = 5$ and the input vector is given by

$$\mathbf{x}(k) = [0.3, -0.15, 0.65, -0.5, 0.1]^T. \tag{3}$$

The A_{max} and Q are assumed to be 0.8 and 4, respectively, here. In this case, a 4 by 5 coefficient matrix

$$\mathbf{C_a}(k) = \begin{bmatrix} c_{11}(k) & c_{12}(k) & c_{13}(k) & c_{14}(k) & c_{15}(k) \\ c_{21}(k) & c_{22}(k) & c_{23}(k) & c_{24}(k) & c_{25}(k) \\ c_{31}(k) & c_{32}(k) & c_{33}(k) & c_{34}(k) & c_{35}(k) \\ c_{41}(k) & c_{42}(k) & c_{43}(k) & c_{44}(k) & c_{45}(k) \end{bmatrix} \tag{4}$$

is prepared. Since each element of $\mathbf{x}(k)$ produces $q(1) = 2$, $q(2) = 1$, $q(3) = 4$, $q(4) = 3$, and $q(5) = 1$, the coefficient vector $\mathbf{c_a}(k)$ becomes

$$\mathbf{c_a}(k) = [c_{21}(k), c_{12}(k), c_{43}(k), c_{34}(k), c_{15}(k)]^T. \tag{5}$$

This vector is updated by the LMS algorithm, and then the updated coefficients $c_{21}(k+1)$, $c_{12}(k+1)$, $c_{43}(k+1)$, $c_{34}(k+1)$, $c_{15}(k+1)$ are inserted into the coefficient matrix $\mathbf{C_a}(k+1)$. For the next iteration, a coefficient vector is again built up based on the elements of the input vector, and then updated by the LMS algorithm. In such a fashion, all the elements of $\mathbf{C_a}(k)$ are adapted as the algorithm is iterated, because the input sequence is randomly distributed on time variant multipath channels.

3. Properties of the ABLMS Algorithm

3.1 Analysis

Adaptive equalisation in rapidly time variant environments has never been understood analytically so far. In this paper, however, we set out to analyse the behaviour of the proposed ABLMS algorithm by resorting to an indication of the degree of time variation.

As suggested by Macchi [15], in a rapidly time variant case, the degree of time variation is dominated by variation noise which is a filter output noise caused by the time variation. The variation noise is described by

$$t_k = \mathbf{T}^T(k-1)\mathbf{x}(k) \tag{6}$$

where $\mathbf{T}(k-1)$ is a vector to indicate the time increments of the optimal filter coefficient (in our case, the optimal filter is an equaliser whose coefficients have time changing). If the optimal coefficient vector of an LTE involving the LMS algorithm is described by $\tilde{\mathbf{c}}(k)$ at time k, then $\mathbf{T}(k-1)$ is given by

$$\mathbf{T}_{LMS}(k-1) = \tilde{\mathbf{c}}(k-1) - \tilde{\mathbf{c}}(k). \tag{7}$$

In this case, the variation noise of the LTE involving the LMS algorithm is given by

$$t_k^{LMS} = \mathbf{T}_{LMS}^T(k-1)\mathbf{x}(k). \tag{8}$$

For the LTE involving the ABLMS algorithm, the time increments of the optimal coefficient is given by

$$\mathbf{T}_{AB}(k-1) = \tilde{\mathbf{c}}_\mathbf{a}(k-1) - \tilde{\mathbf{c}}_\mathbf{a}(k). \tag{9}$$

And, the variation noise is given by

$$t_k^{AB} = \mathbf{T}_{AB}^T(k-1)\mathbf{x}(k). \tag{10}$$

The ABLMS algorithm is based on equi-partitioning the range of the equaliser input into Q ranges, and on updating equaliser coefficients corresponding to each range. Thus, whenever the input sequence x_k is allocated to one range among Q ranges, the coefficient corresponding to the one range is always selected and updated. This means on the contrary that if the set of $\tilde{\mathbf{c}}_\mathbf{a}(k)$ is determined and settled somehow, the range of the amplitude the input sequence has is decreased by a factor of Q for each range. Then, the variation noise of the LTE involving the ABLMS algorithm is related with that of the LTE involving the LMS algorithm as

$$E((t_k^{AB})^2) \simeq \frac{1}{Q} E((t_k^{LMS})^2) \tag{11}$$

where E denotes expectation operation. Equation (11) indicates that in essential, the ABLMS algorithm is less affected by the time variation of the channel than the LMS algorithm. As the division number Q for the amplitude banding increases, the variation in each range may approach a stationary state.

On the other hand, the relation between the coefficients to be updated for the ABLMS algorithm is very similar with that between the coefficients to be updated for an adaptive algorithm using decimated samples. In [16] and [17], it is reported that utilising all decimated samples (the decimated sample and samples discarded by the operation of decimation) for each iteration, the convergence speed of the gradient lattice and LMS algorithms is accelerated. This result is supported by the fact that the decimated samples have the same statistics. The coefficients updated by each decimated sample become statistically related. Therefore, by the use of the statistically related coefficients, the convergence speed of an adaptive filter is accelerated. For the ABLMS algorithm, the coefficients to be selected and updated for each iteration are strongly associated with the previously updated coefficients for each range, because the pattern of the channel impulse response is determined from the amplitude of the received sequence. This means that for the ABLMS algorithm, $c_{q(j)j}(k)$ and $c_{q(j)j}(k-l)$ where l is a positive integer are statistically related. By this property, although all elements of the coefficient matrix are not updated for each iteration, the convergence speed for the ABLMS algorithm will not deteriorate; rather it will be preserved adequately. Therefore, combined with the nice property being less affected by the time variation of the channel, the equaliser involving the ABLMS algorithm as a result accomplishes faster tracking in rapidly time variant environments.

3.2 Simulation Examples

We now show an example to illustrate the performance

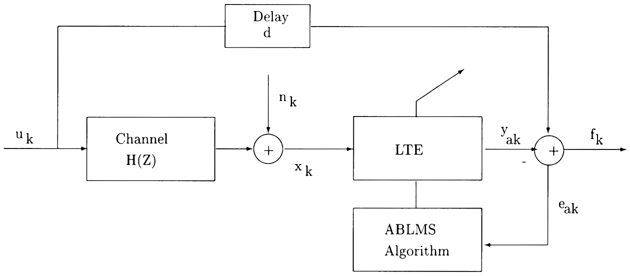

Fig. 1 Configuration of an LTE involving the ABLMS algorithm.

of the ABLMS algorithm. The ABLMS algorithm is used as the adaptation procedure for an LTE depicted in Fig. 1. The channel is assumed to be given by

$$Channel\ 1 : H_1(z) = 1 + \sin\left(\frac{2\pi}{T}k\right) z^{-1} \qquad (12)$$

where T is the period to control the rate of time variation of the channel. The input is a pseudo-random sequence with values of $+1$ or -1. This channel model may not be realistic, because the time variant coefficient is deterministic. However, by using this channel model we can investigate the capability of the equaliser only against the time variation of the channel. Usually, the performance of the equaliser may be affected by additive noise, time variation and phase characteristic of the channel. It should be noted that channel 1 is always minimum phase, resulting in that setting of no delay for the desired sequence, $d = 0$, is permitted. Thus, if the power of the additive noise is negligible, then we can investigate only the tracking performance of the equaliser against the time variation, which means pure tracking performance. We chose the parameter of the channel as $T = 3000$, and applied the ABLMS algorithm to the channel output, with the parameters of $\mu = 0.3$, $\beta = 0.05$ and $M = 6$. The division number was chosen as $Q = 2$ where $A_{max} = 2.0$ was assumed. The additive noise was -50 dB. The convergence curve, which is the average of 100 individual trials, is illustrated in Fig. 2 where the ABLMS and LMS algorithms are compared as the adaptation procedure of LTE. The LMS algorithm was used under the same conditions as those in the ABLMS algorithm. From Fig. 2, we see that the initial convergence speed of the ABLMS algorithm is inferior to that of the LMS algorithm. This is because for the ABLMS algorithm, all coefficients corresponding to each band based on the amplitude level, which are 2 by 6 coefficients in the above case, must be adjusted, while for the LMS algorithm, only coefficients of the vector, which are 6 coefficients, are adjusted. However, as suggested in [15], tracking is not a transient problem, but a steady state problem. Hence, the performance of the equaliser in time variant environments is not affected by the initial convergence time. We see that the ABLMS algorithm has the potential to provide

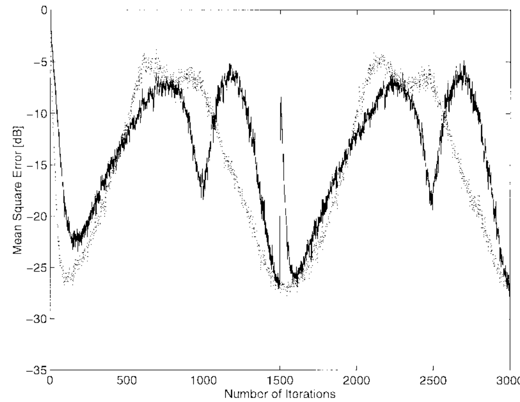

Fig. 2 Convergence of the LMS algorithm (dotted line) and ABLMS algorithm with $Q = 2$ (solid line) on channel 1.

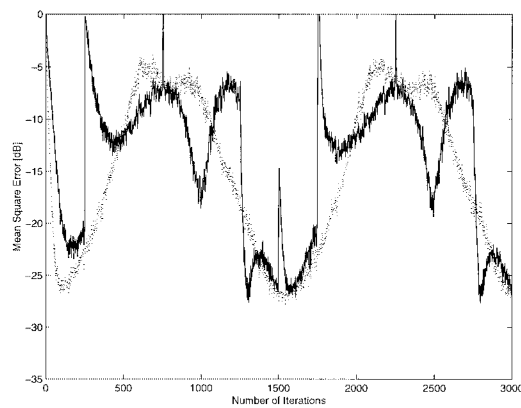

Fig. 3 Convergence of the LMS algorithm (dotted line) and ABLMS algorithm with $Q = 6$ (solid line) on channel 1.

superior tracking performance to the LMS algorithm, this is obviously visualised from $k = 500$ to 1100 and $k = 2000$ to 2600 where the ABLMS algorithm provides lower mean square errors (MSE). Because both algorithms commonly use the LMS algorithm, and in this simulation the same parameters were set, we deduce that for these durations of k, the ABLMS algorithm effectively worked against the time variation of the channel.

The ABLMS algorithm involves a nonlinear operation being selection of the coefficients to be updated for each iteration. Unfortunately, this nonlinearity may not always attain the above-mentioned superior tracking performance, as found in Fig. 2. This is because although the pattern of the channel impulse response is determined under the necessary condition from the amplitude of the received sequence, it cannot be done under the sufficient condition. Therefore, when the pattern of the equaliser coefficients selected based on the amplitude division drastically changes, sometimes the tracking performance may become worse. However, once the equaliser coefficients are adapted for the selected pattern, the ABLMS algorithm converges quickly due to less influence of the time variation on the adaptation process. As a result, the recovery time of the ABLMS algorithm is very short. These are observed from $k = 1100$ to 1500, $k = 1500$ to 1600 and $k = 2600$ to 3000 in Fig. 2. Figure 3 shows the results for the case of $Q = 6$ in the above simulation. The LMS convergence curve is common in Figs. 2 and 3. Figure 3 confirms again the superior tracking and quick recovery the ABLMS algorithm provides, although the ABLMS algorithm makes more spikes on the convergence curve. However, such undesired tracking the ABLMS algorithm provides can be easily remedied by modifying the filter structure. In Sect. 4, we will show that the ABLMS algorithm provides good performance by being aided by the standard LMS algorithm in a parallel form.

4. Filter Structure

4.1 Parallel Structure

Figure 4 illustrates the parallel structure for the ABLMS algorithm proposed in this paper. In the parallel structure, two LTEs for the ABLMS and LMS algorithms are simultaneously updated based on the error sequences e_{ak} and e_k, respectively. The comparator provides $f_k = e_{ak}$ if $(e_{ak})^2 \leq (e_k)^2$ and $f_k = e_k$ otherwise. Based on the comparator output, the combined equaliser outputs y_{ak} when $f_k = e_{ak}$, and y_k when $f_k = e_k$. Figures 5 and 6 show the convergence on channel 1 of the ABLMS-LMS parallel LTE illustrated

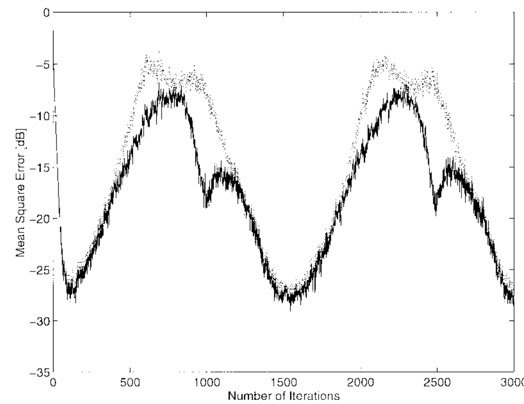

Fig. 5 Convergence of the LMS LTE (dotted line) and ABLMS-LMS LTE with $Q = 2$ (solid line) on channel 1.

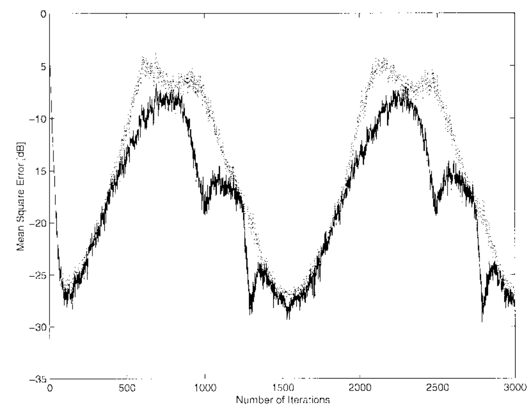

Fig. 6 Convergence of the LMS LTE (dotted line) and ABLMS-LMS LTE with $Q = 6$ (solid line) on channel 1.

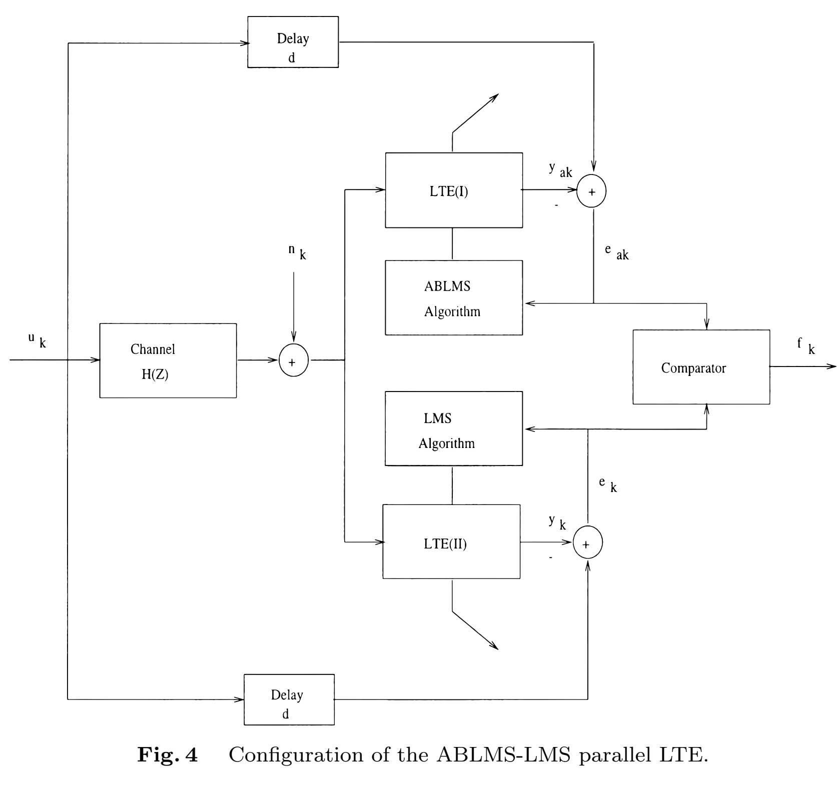

Fig. 4 Configuration of the ABLMS-LMS parallel LTE.

in Fig. 4. The simulation conditions are the same as those in Figs. 2 and 3, respectively. From Figs. 5 and 6, we see that better tracking the ABLMS provides is retained in the parallel form, and the ABLMS-LMS parallel LTE totally provides better performance than the LMS LTE. Also, from Figs. 5 and 6 it is observed that the ABLMS based LTE with $Q = 6$ provides better MSE performance than that with $Q = 2$. We refer to the configuration for the equaliser shown in Fig. 4 as the ABLMS-LMS LTE through this paper.

The ABLMS-LMS LTE would provide at least the same performance as the LMS LTE, because even if the ABLMS algorithm itself provides poorer tracking, it is always compensated by the use of the LMS algorithm. On the contrary, if the adaptation of the ABLMS algorithm effectively works, the ABLMS-LMS LTE would always provide more rapid tracking than the LMS LTE.

4.2 Extension to DFE

The ABLMS-LMS LTE can be straightforwardly extended to its DFE version. When $\mathbf{c}(k)$ and $\mathbf{x}(k)$ are replaced by $\mathbf{c}'(k) = (c_0(k), c_1(k), \ldots, c_{M_f+M_b-1}(k))^T$ and $\mathbf{x}'(k) = (x_k, x_{k-1}, \ldots, x_{k-M_f+1}, \hat{u}_{k-d-1}, \hat{u}_{k-d-2}, \ldots, \hat{u}_{k-d-M_b})^T$, respectively, Eq. (2) becomes the adaptation procedure for an $M_f + M_b$ length DFE, where $\hat{u}_{k-d}$ is an estimate of the transmitted sequence delayed by d, and M_f and M_b are the length of the feedforward and feedback filters, respectively. Thus, using $\mathbf{c_a}'(k) = (c_{q(1)1}(k), c_{q(2)2}(k), ..., c_{q(M_f+M_b)M_f+M_b}(k))^T$ and $\mathbf{x}'(k)$ instead of $\mathbf{c_a}(k)$ and $\mathbf{x}(k)$ in the ABLMS algorithm for an M length LTE, we have the ABLMS algorithm for an $M_f + M_b$ length DFE. The DFE version of the ABLMS-LMS LTE, the ABLMS-LMS DFE, is constructed in parallel with the LMS DFE, in the same fashion as the ABLMS-LMS LTE. For the ABLMS-LMS DFE, two LTEs in Fig. 4 are replaced by two DFEs. For the same reason as that in the ABLMS-LMS LTE, the ABLMS-LMS DFE would provide more rapid tracking than the LMS DFE.

4.3 Computational Complexity

In practical situations, an equaliser whose filter length is larger than a couple of tens is often required. In such cases, if the division number Q in the ABLMS algorithm is selected moderately (as clearly shown in Fig. 5, even under the setting of $Q = 2$, the ABLMS algorithm provides an improvement), then the computational complexity for switching the coefficients to be updated may be comparatively less, and the major computation may be dominated by the LMS adaptation. In this case, the computational complexity of the ABLMS algorithm approximately becomes equivalent to that of the LMS algorithm. This is obvious from that the ABLMS algorithm with setting $Q = 1$ reduces to the LMS algorithm. However, the ABLMS algorithm needs the aid of the LMS algorithm as shown in Fig. 4. Thus, the whole computational complexity required to implement the ABLMS-LMS LTE is approximately twice that required to implement the LMS LTE, but this is much less than that required to do the RLS LTE or the RLS DFE. Although several fast algorithms for the RLS adaptation have been proposed, they still require extensive computation relative to the LMS adaptation, and usually suffer from the numerically unstable problem.

The ABLMS-LMS DFE also requires a similar computational complexity to the ABLMS-LMS LTE.

4.4 Discussion

One possible equaliser structure to be created from the configuration of the ABLMS-LMS LTE in Fig. 4 is that of an LMS-LMS parallel LTE. We are interested in the LMS-LMS LTE in which a different step size is used for each LMS LTE. To investigate the performance of the LMS-LMS LTE, we used channel 1 where the parameter T was set to 3000 and the additive noise was -50 dB again. The step size of one LMS LTE was set to $\mu = 1.0$ and that of the other LMS LTE was set to $\mu = 0.5$. For both LMS LTEs, $\beta = 0.05$ and $M = 6$ were commonly set.

For the purpose of comparison, the ABLMS-LMS LTE was implemented under the same conditions where the ABLMS LTE was adapted with the step size $\mu = 1.0$ and the LMS LTE was done with $\mu = 0.5$. The division number and maximum amplitude were set to $Q = 2$ and $A_{max} = 2.0$, respectively. Furthermore, we implemented the standard RLS LTE on the same channel under the same conditions with the forgetting factor of 0.94.

Figure 7 shows the comparative results. The behaviour of the ABLMS-LMS LTE is almost the same as that of the LMS-LMS LTE at a wide range of k,

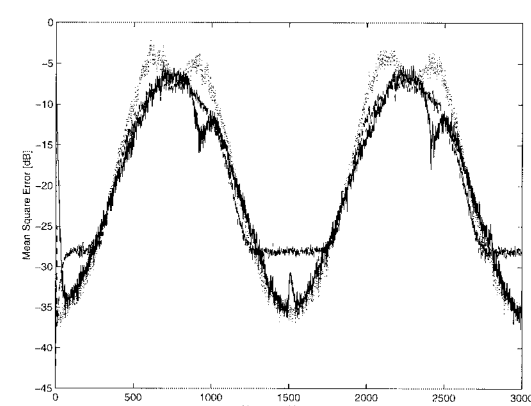

Fig. 7 Convergence of the LMS-LMS LTE (dotted line), RLS LTE (dashed line) and ABLMS-LMS LTE with $Q = 2$ (solid line) on channel 1.

but it is observed that the MSE the ABLMS-LMS LTE is lower from $k = 500$ to 1100 and $k = 2000$ to 2600. The RLS LTE behaves similarly with the ABLMS-LMS LTE from $k = 500$ to 1100 and $k = 2000$ to 2600 where the MSE the RLS LTE provides is lower than that the LMS-LMS LTE provides. However, as observed from $k = 1300$ to 1700 and $k = 2800$ to 3000, the MSE the RLS LTE provides is not always lower than that the ABLMS-LMS LTE and LMS-LMS LTE provide. Channel 1 is unequalisable at $k = 750$ and $k = 2250$ where the zero the channel has is located on the unit circle in the z plane. The ABLMS-LMS LTE providing a lower MSE near at $k = 750$ and $k = 2250$ may invoke a lower bit error rate (BER). Therefore, Fig. 7 indicates that the ABLMS algorithm has the potential to provide superior tracking performance to the conventional adaptive algorithms.

5. Performance on Multipath Channels

To further evaluate the performance of the ABLMS-LMS LTE shown in Fig. 4, we deploy a Markov communication channel model. The channel is given by

$$Channel\ 2 : H_2(z) = h_0(k) + h_1(k)z^{-1} + h_2(k)z^{-2} \tag{13}$$

where the time variant coefficients, $h_0(k)$, $h_1(k)$ and $h_2(k)$ are generated by passing a Gaussian white noise through a second order Butterworth filter which is designed with a sampling rate of 2400 sample/s. For this channel model, the channel fade rate can be quoted as the 3 dB bandwidth for the Markov process. The input sequence of both channels is a pseudo-random sequence with values of +1 or −1. Channel 2 corresponds to an HF channel model $H_3(z)$ used in [4].

Figure 8 illustrates the convergence on channel 2 with a fade rate of 2 Hz (an example of the trajectory of the generated coefficients for channel 2 is shown in Fig. 9) where the ABLMS-LMS LTE and LMS LTE are compared. The equalisers have the same filter length $M = 9$ and delay $d = 4$. The constant parameters of both equalisers have been set to $\mu = 0.5$ and $\beta = 0.05$. The additive noise is −50 dB. As will be described in the sequel, the setting of the delay in this channel model has been optimised to provide the best performance for both equalisers. Therefore, if the noise level is much lower, we can investigate the tracking performance of both equalisers against time variation. The average of 100 individual trials has been given in Fig. 8. Figure 8 shows the MSE results at the steady state, which are around −10 dB. Because the noise is −50 dB, the MSE levels for both equalisers are much higher than the noise level. Therefore, in Fig. 8 the ABLMS-LMS LTE results in a significant lower value for the steady state of the MSE, this figure indicates that the ABLMS-LMS LTE has accomplished more rapid tracking against the time variation of the channel

Figure 10 shows the BER performance of the LMS LTE, LMS DFE, ABLMS-LMS LTE and ABLMS-LMS DFE against additive noise on channel 2 with a fade rate of 2 Hz. The equalisers have the filter length $M = 9$

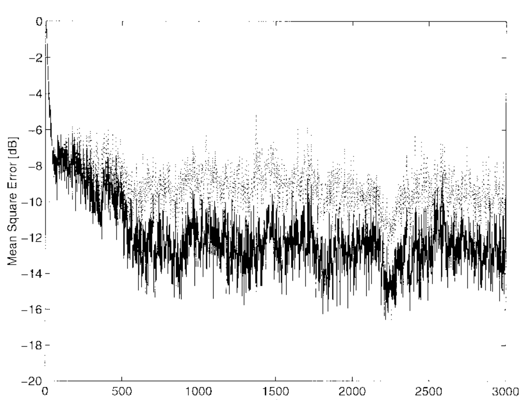

Fig. 8 Convergence of the LMS LTE (dotted line) and ABLMS-LMS LTE (solid line) on channel 2 with a fade rate of 2 Hz.

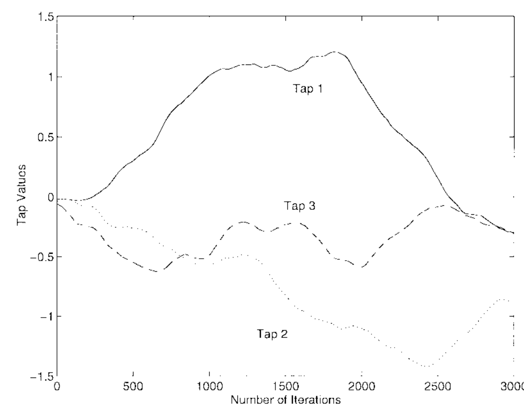

Fig. 9 An example of coefficient trajectory for channel 2. The Taps 1-3 correspond to $h_0(k)$, $h_1(k)$ and $h_2(k)$, respectively.

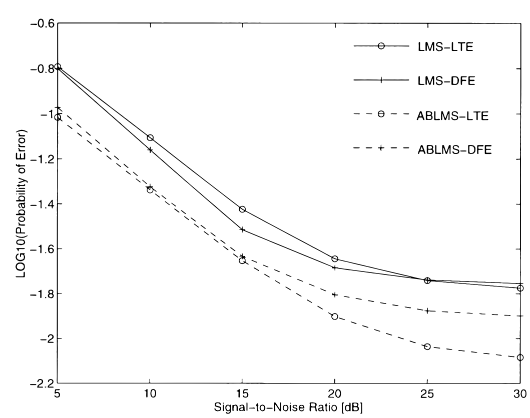

Fig. 10 Bit error rate performance against additive noise on channel 2 with a fade rate of 2 Hz. The ABLMS-LTE and ABLMS-DFE correspond to the ABLMS-LMS parallel LTE and ABLMS-LMS parallel DFE, respectively.

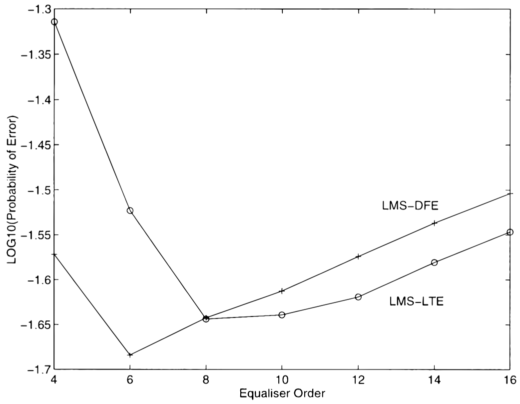

Fig. 11 Equaliser order dependency for the LMS LTE and LMS DFE on channel 2 with a fade rate of 2 Hz and a signal-to-noise ratio of 20 dB. The equaliser order corresponds to $M-1$ for the LTE and $M_f + M_b - 1$ for the DFE. For the LTE, the delay is set to $(M-1)/2$. For the DFE, M_b is fixed on $M_b = 2$.

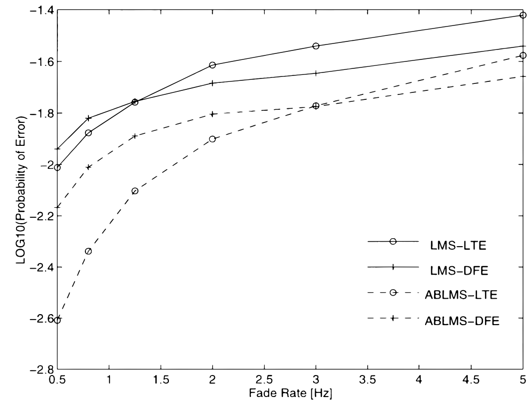

Fig. 12 Bit error rate performance against channel fade rates on channel 2 with a signal-to-noise ratio of 20 dB.

for LTEs and $M_f = 5$ and $M_b = 2$ for DFEs, and delay $d = 4$, both of which provide the best performance for the filter structure, respectively, as shown in Fig. 11. In Fig. 10, the constant parameters for both filter structures have been set to $\mu = 0.5$ and $\beta = 0.05$, respectively. These have been also optimised to achieve the best performance for each adaptive algorithm. The division number of the ABLMS algorithm has been set to $Q = 6$. It will be shown in Sect. 6 that this selection is adequate. Figure 10 reveals the robustness for each equaliser against additive noise, and clearly shows that the amplitude banded equalisers provide performance improvements. Also, Fig. 10 shows that based on the structure of the LTE rather than the DFE, the ABLMS algorithm provides better performance.

Carefully looking at Fig. 11, we notice that the DFE is more sensitive to the equaliser order than the LTE. This is because for a DFE, the effect of noise enhancement by the feedforward filter is enhanced by the feedback filter as the filter order is increased. This undesirable feature visualised for the DFE on time variant channels may motivate the use of the LTE on time variant channels, because the difference between the optimal BER achieved by the LTE and that by the DFE is slight, as shown in Fig. 11.

Figure 12 is an illustration of the BER performance against channel fade rates on channel 2 with a signal-to-noise ratio (SNR) of 20 dB where the LMS LTE, LMS DFE, ABLMS-LMS LTE and ABLMS-LMS DFE are again compared. The conditions of all the equalisers are the same as those in Fig. 10. Figure 12 shows that especially in the range of fade rates from 0.5 to 2 Hz, which are often encountered in practical situations, the ABLMS-LMS LTE significantly outperforms the ABLMS-LMS DFE as well as the LMS DFE. Also, it should be here noted that the ABLMS-LMS LTE has the potential to achieve acceptable BER less than 10^{-2}.

6. Nonuniform Division

All previous results of the ABLMS based equalisers were obtained based on a uniform division of the amplitude of the received sequence, as described in Sect. 2. In deriving the ABLMS algorithm, the use of uniform division seems to be natural due to simplicity. However, in the case of a time variant channel such as HF, it may be possible to utilise the information of distribution of the amplitude value of the received sequence in the scheme of the ABLMS adaptation. In this section, noting the fact that the distribution of instantaneous amplitude of a time variant channel output is not uniform, we set out to improve the performance of the ABLMS based equalisers. In particular, we consider the improvement of the ABLMS-LMS LTE, because the ABLMS-LMS LTE provides better performance than the ABLMS-LMS DFE in a practical range of fade rates of the channel, as shown in Sect. 5.

The idea is that if the rate of selection of coefficients from the coefficient matrix for the ABLMS adaptation is averaged, the tracking performance the ABLMS algorithm provides may be improved. This is because the distribution of the amplitude of the received sequence is approximately Gaussian, resulting in the ABLMS adaptation which at a high rate selects and updates the coefficients corresponding to a low amplitude range of the received sequence. And, basically, as the division number increases, the ABLMS-LMS LTE is inclined to provide better performance. Comparison of the convergence in Figs. 5 and 6 suggests this. Furthermore, as will be shown below, at least for the division numbers less than 20 this is true on a second order Markov communication channel. Therefore, if the same number of the amplitude division is assigned to the ABLMS adaptation, then it is better to use a narrow division in the region of small amplitude and to use a wide division in the region of large amplitude. Based on such a principle, in the improved scheme, instead

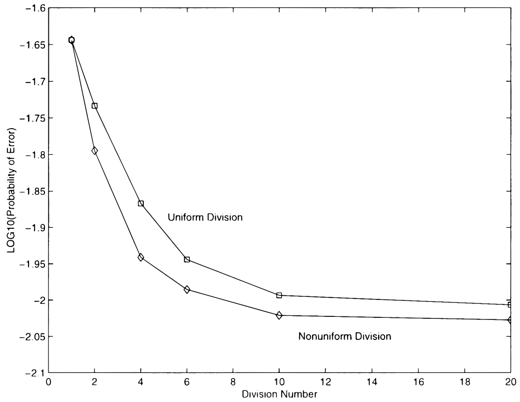

Fig. 13 Bit error rate performance of the ABLMS-LMS LTE with uniform and nonuniform divisions on channel 2 with a fade rate of 2 Hz in a signal-to-noise ratio of 20 dB.

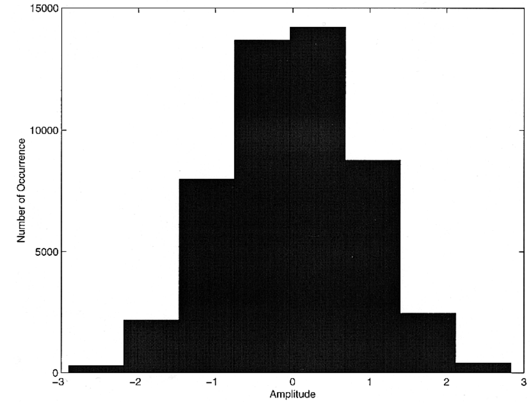

Fig. 14 Histogram in the case of uniform division for the ABLMS-LMS LTE.

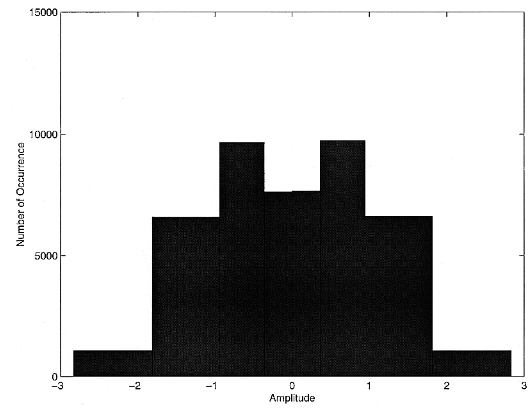

Fig. 15 Histogram in the case of nonuniform division for the ABLMS-LMS LTE.

of the uniform division used for the ABLMS algorithm to classify the amplitude of the received sequence, a nonuniform division is used as follows:

- if $|x_{k-j+1}| \leq A_{max}/S$, then $q(j) = 1$.
- if $A_{max}/S < |x_{k-j+1}| \leq 3A_{max}/S$, then $q(j) = 2$.
- if $3A_{max}/Q < |x_{k-j+1}| \leq 6A_{max}/S$, then $q(j) = 3$.
- .
- .
- if $(S - Q)A_{max}/S < |x_{k-j+1}|$, then $q(j) = Q$.

where $S = Q(Q+1)/2$. This nonuniform amplitude division, each class produced by which is increased by an equivalent distance, invokes a case in which the inclined selection of coefficients from the coefficient matrix for the ABLMS adaptation is alleviated.

Figure 13 shows how the performance is dependent on the number and form of amplitude division used in the ABLMS-LMS LTE in the case of channel 2 with a fade rate of 2 Hz and a SNR of 20 dB. Figure 13 shows that the ABLMS-LMS LTEs are inclined to decrease the BER as the division number Q increases. It is observed that the setting of $Q = 6$ for the simulations in Sect. 5 is sufficient for the ABLMS-LMS LTE providing a significant improvement related to the standard LMS equaliser (In Fig. 13, the performance of the standard LMS equaliser corresponds to the setting of $Q = 1$). On the other hand, Fig. 13 obviously shows that if the nonuniform division form is used in the ABLMS-LMS LTE, it provides better performance than the uniform division form case. This result may be verified by evaluating the histogram of the amplitude of the channel output in the uniform and nonuniform division cases. Figure 14 shows the result in the case of uniformly dividing all the amplitude of the channel output into 8 small classes. This division corresponds to $Q = 4$ for the setting of division number in the ABLMS algorithm, because the absolute values of the channel output are used in the ABLMS algorithm. Using 50,000 data samples in the case of SNR of 20 dB, the result in Fig. 14 was obtained. Figure 15 shows the result of the nonuniform division version, in which all the amplitude of the channel output were nonuniformly divided into 8 classes based on the above division rule for the improved scheme as follows:

- Class 1 = $[-A_{max}, -6A_{max}/10]$
- Class 2 = $[-6A_{max}/10, -3A_{max}/10]$
- Class 3 = $[-3A_{max}/10, -A_{max}/10]$
- Class 4 = $[-A_{max}/10, 0]$
- Class 5 = $[0, A_{max}/10]$
- Class 6 = $[A_{max}/10, 3A_{max}/10]$
- Class 7 = $[3A_{max}/10, 6A_{max}/10]$
- Class 8 = $[6A_{max}/10, A_{max}]$

This division also corresponds to $Q = 4$ in the ABLMS algorithm. Comparing Fig. 14 with Fig. 15, we notice that by using the nonuniform division, concentrated occurrence at zero amplitude and at neighbors to that appeared in the case of the uniform division is distributed.

The amount of computation is not changed by making use of the nonuniform division operation, because Q and A_{max} for the ABLMS algorithm are settled before the implementation of the equaliser regardless of the use of uniform or nonuniform division form.

7. Conclusion

The problem of equalisation of time variant multipath channels has been investigated. A novel adaptive algorithm, the ABLMS, has been derived to cope with rapid time variation. To attain good tracking, the ABLMS algorithm needs the aid of the LMS algorithm in a parallel adaptation form. Therefore, computational complexity of the ABLMS-LMS parallel equalisers are approximately twice that of the LMS equalisers, but this is much less than that of RLS equalisers. The ABLMS-LMS LTE can accomplish more rapid tracking in time variant environments than the LMS LTE, resulting in better steady state error performance. It is easy to extend the ABLMS-LMS LTE to its DFE version, but this extension does not lead to more fruitful results than the ABLMS-LMS LTE. Computer simulations have demonstrated that the ABLMS-LMS LTE provides better BER performance than the ABLMS-LMS DFE as well as LMS LTE and LMS DFE in the case of a second order Markov communication channel model. Also, it has been demonstrated that if a nonuniform division operation is used for the ABLMS adaptation, the performance of the ABLMS-LMS LTE is further improved. Future work will aim to search the optimal nonuniform division form to provide the best performance.

Acknowledgement

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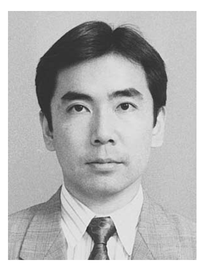

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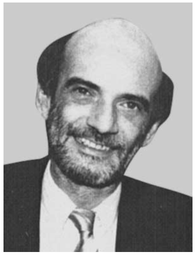

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