

LETTER

An ARMA Prefiltering Approach to Adaptive Equalization

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SUMMARY In this paper, we propose an adaptive IIR equalizer based on prefiltering techniques. The proposed equalizer has a cascade structure of an ARMA prefilter and an adaptive FIR equalizer. The ARMA prefilter is designed based on the transfer function estimated by the gradient-type instrumental variable algorithm. Simulation results are shown to confirm the performance of the proposed adaptive IIR equalizer.

key words: *prefilter, instrumental variable algorithm, adaptive IIR equalizer*

1. Introduction

Much of the recent work in the area of adaptive equalization has been devoted to the objective of achieving rapid convergence [1]–[5], [14], [15]. Transversal or finite impulse response (FIR) filters have been commonly used for the adaptive equalizers in practice due to the simplicity of their structures. This type of equalizers typically uses the so-called LMS algorithm developed by Widrow and Hoff [6]. The LMS algorithm, however, has a rather severe disadvantage that the convergence degrades as the channel distortion increases.

Since all-zero nature of the adaptive FIR equalizers frequently results in degraded performance, infinite impulse response (IIR) equalization is attractive. IIR filters have poles as well as zeros. This type of equalizers can result in a significant reduction of the equalizer length [7].

Recently, system identification techniques have been discussed in the context of adaptive filtering [8], [9]. In [9], it is shown that the recursive algorithm for the instrumental variable method, the recursive instrumental variable (RIV) algorithm, is a powerful candidate for adaptive IIR filtering algorithms. The RIV algorithm gives consistent parameter estimates for colored noise, and the estimates cannot be stuck in local minima. On the under-parameterized case, the RIV algorithm also provides meaningful approximation models. These properties seem preferable for the purpose

of identification of a communication channel. However, computational complexity of the RIV algorithm is very high, because the RIV algorithm basically has the same formula as the recursive least squares (RLS) algorithm. In general, complicated calculations are not preferable in communication systems, so the RIV algorithm is desired to be modified into its more reduced-complexity version. In this paper, a gradient-type instrumental variable (GIV) algorithm is derived by replacing the correlation matrix in the RIV algorithm with a scaled matrix.

Main contribution of this paper is to propose an adaptive IIR equalization scheme using a channel estimation technique. The proposed adaptive equalizer contains a pole-zero or autoregressive moving average (ARMA) filter which is designed based on the channel estimates obtained by the GIV algorithm. The ARMA filter is cascaded with an adaptive FIR equalizer. That is, the ARMA filter behaves as a prefilter to the cascaded adaptive FIR equalizer. Using the ARMA prefilter, we can reduce the effects of severe distortion of the data acquired at the receiver. The output of the ARMA filter has a small degree of correlations and as a result, the start-up time wasted on the cascaded adaptive FIR equalizer can be significantly shortened. Despite of its fast convergence, the proposed adaptive equalization algorithm needs no computational complexity such as inverse matrix calculations required in the RLS algorithm. Also, its stability is guaranteed by a simple operation.

Performance of the proposed adaptive IIR equalizer will be favorably compared with that of the commonly used adaptive FIR equalizer. The proposed adaptive equalizer achieves faster convergence and more reduced mean squared error (MSE) even under the condition that the channel distortion is rather severe. It is pointed out that the ARMA prefiltering approach to channel equalization is more efficient than the AR filtering approach previously proposed by the authors [10].

Section 2 is devoted to the derivation of the GIV algorithm. In Sect. 3, the structure and adaptation algorithm for the proposed adaptive IIR equalizer are described. Section 4 shows some simulation based experimental results. Conclusions are drawn in Sect. 5.

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2. System Modeling

We consider a dynamical system described by

$$y(n) = G(z^{-1})u(n) + v(n) \quad (1)$$

where $y(n)$ is the output at time n , $u(n)$ is the input and $v(n)$ is additive noise. $G(z^{-1})$ is a stable filter and z^{-1} denotes the backward shift operator such that $z^{-1}u(n) = u(n-1)$, etc. Assume that the system is modeled as

$$y(n) = \frac{B(z^{-1})}{A(z^{-1})}u(n) + \varepsilon(n) \quad (2)$$

where $\varepsilon(n)$ is the output error. The polynomial $A(z^{-1})$ and $B(z^{-1})$ are represented by

$$A(z^{-1}) = 1 + a_1z^{-1} + \dots + a_nz^{-na} \quad (3)$$

$$B(z^{-1}) = b_0 + b_1z^{-1} + \dots + b_nz^{-nb}. \quad (4)$$

The vector $\theta = [a_1 \dots a_{na} b_0 b_1 \dots b_{nb}]^T$ is the parameter vector to be estimated from the available data.

The RIV algorithm is a powerful tool to estimate the parameter vector θ [9]. The algorithm is described as follows.

$$\hat{\theta}(n) = \hat{\theta}(n-1) + K(n)\varepsilon(n) \quad (5)$$

$$\varepsilon(n) = y(n) - \varphi^T(n)\theta(n-1) \quad (6)$$

$$P(n) = P(n-1) - \frac{P(n-1)z(n)\varphi^T(n)P(n-1)}{1 + \varphi^T(n)P(n-1)z(n)} \quad (7)$$

$$K(n) = P(n)z(n) \quad (8)$$

where

$$\varphi(n) = [-y(n-1) - y(n-2) \dots - y(n-na)u(n)u(n-1) \dots u(n-nb)]^T \quad (9)$$

In (5), $\hat{\theta}(n)$ denotes the estimate of θ based on the data up to and including time n . Distinguishing $\varepsilon(n)$ in (2), we call $\varepsilon(n)$ in (6) the equation error.

The vector $z(n)$ is the instrumental variable vector and can be chosen in several ways. The popular two examples are

$$z_1(n) = [-\hat{x}(n-1) - \hat{x}(n-2) \dots - \hat{x}(n-na)u(n)u(n-1) \dots u(n-nb)]^T \quad (10)$$

and

$$z_2(n) = [u(n)u(n-1) \dots u(n-na-nb)]^T. \quad (11)$$

Comparison of using (10) and (11) in the RIV algorithm

is made in [16].

Although the RIV algorithm gives good performance, it suffers from its implementation which is similar with that of the RLS algorithm. The major problems are its high computational complexity and numerical instability. For adaptive filtering applications, simpler algorithms are preferable in many cases. Thus, we derive a simple version of the RIV algorithm, which is the GIV algorithm.

The derivation of the GIV algorithm is not so difficult. This is because it has been suggested in [9] that the general recursive identification algorithm can reduce to its gradient version by matrix manipulations. However, we actually do not obtain the specific description of the gradient version of the RIV algorithm from literature [9]. Thus, we specifically derive it here. The GIV algorithm is derived from the RIV algorithm by approximating the correlation matrix $P(n)$ by a scaled unit matrix μI where I denotes the unit matrix. The GIV algorithm is given by

$$\hat{\theta}(n) = \hat{\theta}(n-1) + \mu z(n)\varepsilon(n) \quad (12)$$

$$\varepsilon(n) = y(n) - \varphi^T(n)\theta(n-1) \quad (13)$$

where μ corresponds to the step-size parameter.

This algorithm is similar to the LMS equation error (LMSEE) algorithm [11]:

$$\hat{\theta}(n) = \hat{\theta}(n-1) + \mu\varphi(n)\varepsilon(n) \quad (14)$$

$$\varepsilon(n) = y(n) - \varphi^T(n)\theta(n-1). \quad (15)$$

The LMSEE is an efficient algorithm for adaptive IIR filtering. The parameter estimate by the LMSEE algorithm, however, is biased if the additive noise $v(n)$ is present. This is explained by the fact that the relationship

$$E[\varepsilon(n)\varphi(n)] \neq 0 \quad (16)$$

is satisfied at the convergence point of the LMSEE algorithm. Inserting (13) into (16) obviously leads to a biased estimate of θ . On the other hand, the GIV algorithm gives consistent estimates if the instrumental variable vector is appropriately chosen, because at the convergence point of the GIV algorithm the following equation

$$E[\varepsilon(n)z(n)] = 0 \quad (17)$$

is valid. In this case, an unbiased estimate is obtained.

The GIV algorithm preserves good properties of the RIV algorithm. However, implementation of the GIV algorithm is significantly simpler than that of the RIV algorithm, because the computational complexity of the GIV algorithm is proportional to $O(na + nb + 1)$ while that of the RIV algorithm is $O((na + nb + 1)^2)$.

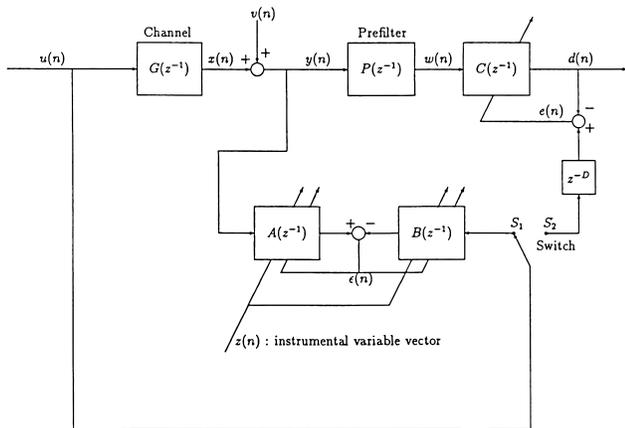


Fig. 1 Scheme of the proposed equalization system. After the channel estimation in which the switch is on S_1 , the switch is connected with S_2 and the FIR filter $C(z^{-1})$ is adapted.

3. Channel Equalization

In this section, we consider a channel equalization scenario. The basic scheme of the proposed channel equalization system is illustrated in Fig. 1.

3.1 Transmission Model

Assume that the channel $G(z^{-1})$ has an infinite impulse response. The channel output, $y(n)$, is given by

$$y(n) = \sum_{i=0}^{\infty} g_i u(n-i) + v(n) \quad (18)$$

where $g_i, i = 0, 1, \dots, \infty$ is the channel impulse response, $u(n)$ is a discrete message sequence of ± 1 (pseudo-random sequence) with zero mean and variance σ_d^2 , and $v(n)$ is additive white Gaussian noise with zero mean and variance σ_v^2 . $u(n)$ is assumed uncorrelated with $v(n)$.

The transmission model is true when the channel filter has poles close to the unit circle of the z -plane. A very long impulse response as visualized in (18) has examples in magnetic recording and telephone channels [12], [13].

3.2 Channel Estimation by GIV

For the training mode, the message sequence is available at the receiver side. In this case, we can identify the unknown channel filter from the input and output data.

We assume the channel model described by (2) to the input-output data given by (18). The channel filter is estimated as

$$G(z^{-1}) = \frac{\hat{B}(z^{-1})}{\hat{A}(z^{-1})} \quad (19)$$

by using the GIV algorithm, (12) and (13).

In general, adaptive IIR filtering algorithms based on the output errors suffer from their slow convergence speed. However, the GIV algorithm is based on the equation errors and its convergence speed is faster than those based on the output errors, because the GIV algorithm has no local convergence and needs no filter stability monitoring. Such convergence property of the GIV algorithm is extremely preferable for the purpose of channel estimation.

3.3 Design of ARMA Prefilter

The transfer function obtained by the GIV algorithm, $\hat{B}(z^{-1})/\hat{A}(z^{-1})$, provides a good estimate of the true channel filter if its structure is known *a priori*. Even if its structure is unknown, the GIV algorithm will provide a good approximation of the channel filter, because the GIV algorithm is essentially based on the RIV algorithm that guarantees a good matching of the whole system weighting sequence [9].

In order to design the prefilter $P(z^{-1})$, we use an inverse system of $\hat{B}(z^{-1})/\hat{A}(z^{-1})$. The prefilter can be designed as

$$P(z^{-1}) = \frac{\hat{A}(z^{-1})}{\hat{B}(z^{-1})}. \quad (20)$$

However, to guarantee the stability of $P(z^{-1})$, we factorize the denominator polynomial $B(z^{-1})$ as

$$\hat{B}(z^{-1}) = (1 - \beta_1 z^{-1})(1 - \beta_2 z^{-1}) \cdots (1 - \beta_{nb} z^{-1}). \quad (21)$$

Then we replace the zeros out of the unit circle, β_i , by their reciprocals, and construct again the polynomial $B_s(z^{-1})$. Therefore, the stable filter can be obtained as

$$P(z^{-1}) = \frac{\hat{A}(z^{-1})}{\hat{B}_s(z^{-1})}. \quad (22)$$

In previous work [10], we proposed a structure of adaptive IIR equalizer in which a prefilter is cascaded with an adaptive FIR equalizer. The remarkable feature was that during the adaptation of the cascaded adaptive FIR filter, the prefilter coefficients are fixed. In [10], we used an AR filter as the prefilter which was designed based on the estimation results obtained by the channel estimator with FIR structure. The channel estimator was adapted by the LMS algorithm. In this paper, we propose to use the ARMA prefilter (22) instead of the AR prefilter in the cascaded structure.

Advantages of using the ARMA filter ((22)) over the AR filter (which corresponds to the setting of $\hat{A}(z^{-1}) = 1$ in (22)) are to decrease the number of coefficients for the prefilter in order to achieve the same performance and to decrease the computational complexity required in factorizing the polynomial $\hat{B}(z^{-1})$

(which corresponds to (21)). For the transmission system such as (18), if we use an adaptive FIR filter for the purpose of channel estimation, a very long filter is required and the factorization of the polynomial becomes terrible computation. In such a case, the channel estimation by the GIV algorithm becomes effective and efficient. Also, we can flexibly control the structure for the model of the channel filter so that the order of $\hat{B}(z^{-1})$ is small and that of $\hat{A}(z^{-1})$ is large, comparatively. This property of the GIV algorithm cannot be shared with the AR prefiltering approach. Furthermore, although the LMS algorithm for the channel estimation in [10] cannot give consistent estimates for colored noise, the GIV algorithm can do that. Such robustness to additive noise will lead to more accurate equalization performance.

3.4 Adaptive Equalization with ARMA Prefilter

The prefiltered signal

$$w(n) = \frac{\hat{A}(z^{-1})}{\hat{B}_s(z^{-1})}y(n) \quad (23)$$

is used for the cascaded adaptive FIR filter with $M + 1$ adjustable tap coefficients as the input signal. The adaptive FIR filter output, $d(n)$, is given by

$$d(n) = \sum_{i=0}^M c_i(n)w(n-i) \quad (24)$$

where $c_i(n)$, $i = 0, 1, \dots, M$ are the adjustable tap coefficients at time n . The difference between the adaptive FIR filter output $d(n)$ and the desired signal $u(n-D)$,

$$e(n) = u(n-D) - d(n), \quad (25)$$

is the error sequence of the adaptive FIR filter where $u(n-D)$ is the delayed message signal obtained for the training mode. The tap coefficients of the adaptive FIR filter are updated to minimize the mean squares of $e(n)$ using the LMS algorithm:

$$\hat{C}(n) = \hat{C}(n-1) + \gamma\zeta(n)e(n) \quad (26)$$

$$e(n) = u(n-D) - \hat{C}^T(n)\zeta(n) \quad (27)$$

where $\hat{C}(n)$ is the estimate at time n of the Wiener solution $C = [c_0 c_1 \dots c_M]^T$ and $\zeta(n) = [w(n)w(n-1) \dots w(n-M)]^T$. γ is the step-size parameter to control the convergence of the LMS algorithm.

4. Simulations

The behavior of the proposed adaptive IIR equalizer has been examined with a series of computer simulation experiments. In this section we present some of the experimental results.

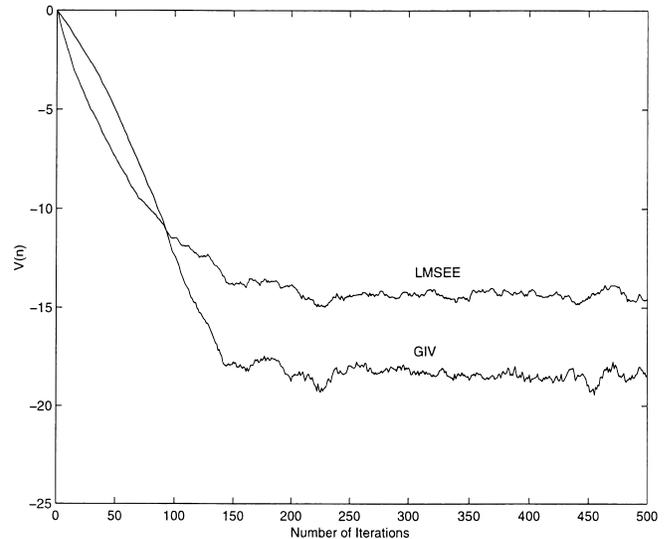


Fig. 2 Convergence of the GIV and LMSEE algorithms.

In the first example, the performance of the channel estimation by the GIV algorithm has been examined. The message signal $u(n)$ was generated as ± 1 pseudo-random sequence with zero mean and unit variance. The channel output $y(n)$ was generated by filtering the pseudo-random sequence by a first-order all-pole filter:

$$G(z^{-1}) = \frac{1}{1 - 0.9z^{-1}} \quad (28)$$

whose output was disturbed by unit variance white noise. Under the setting of model orders of $na = 1$ and $nb = 0$, the step-size $\mu = 0.02$ was used for the update and $z_1(n)$ was chosen as the instrumental variable vector. For initialization of the parameter vector, all-zero set was used. For comparison, the LMSEE algorithm was also implemented under the same model order setting, with the step-size $\mu = 0.014$ and with all-zero initialization. Figure 2 plots the normalized norm squared parameter error:

$$V(n) = 10 \log_{10} \left(\frac{\|\theta(n) - \theta_*\|^2}{\|\theta_*\|^2} \right) \quad (29)$$

where θ_* denotes the true parameter vector. Figure 2 shows that while both algorithms achieve the convergence for the same iterations, which are about 150, the GIV algorithm gives more reduced level of $V(n)$. This means that the GIV algorithm gives more accurate estimate of the parameter vector than the LMSEE algorithm. This result is suggested by the fact that the estimate by the GIV algorithm is unbiased, while that by the LMSEE algorithm is biased. Figure 2 also indicates that the GIV algorithm is a fast converging algorithm for adaptive IIR filtering.

Secondly, we give an example of the channel equalization. The output signal $y(n)$ was generated by filtering the pseudo-random sequence by a first-order pole /

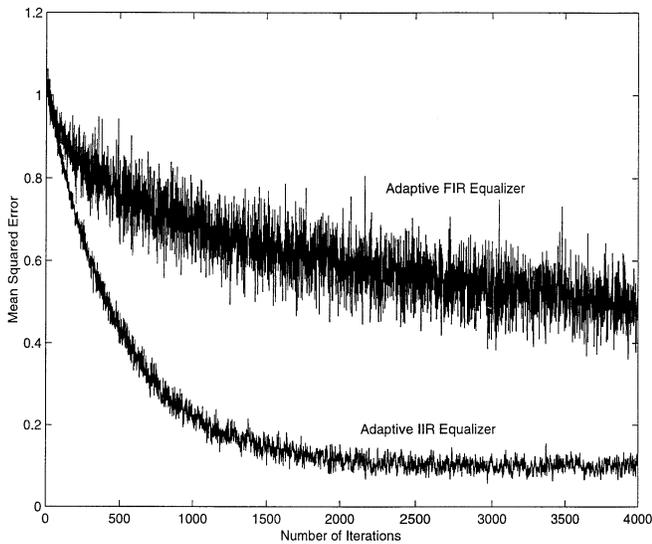


Fig. 3 Comparison of the convergence rate for the proposed adaptive IIR equalizer and the adaptive FIR equalizer.

first-order zero filter:

$$G(z^{-1}) = \frac{0.27 - 0.26z^{-1}}{1 + 0.85z^{-1}} \quad (30)$$

whose output was disturbed by white noise with variance of 0.001. This channel impulse response itself had unit variance and hence the signal-to-noise ratio of the channel output was 30 dB. For the proposed equalizer, the GIV algorithm gave the channel estimates from a sample set of 250 input-output data with the step-size $\mu = 0.03$. The model orders were set to $na = 1$ and $nb = 1$. For the instrumental variable vector, $z_1(n)$ was again chosen. For initialization of the parameter vector, all-zero set was used. The cascaded adaptive FIR filter was set to the order of $M = 17$ and was updated with the step-size of $\gamma = 0.001$. For comparison, the commonly used adaptive FIR equalizer was also implemented with the above step-size 0.001 and with the order 20. The delay D was set to 10 in both equalization algorithms. To compare the convergence rate of both equalizers, Fig. 3 plots the MSE performance. The MSE for the proposed adaptive IIR equalizer was evaluated from the prefiltered data and the delayed message sequence. The MSE for the adaptive FIR equalizer was evaluated from the received data and the delayed message sequence. From Fig. 3, we see that the proposed adaptive IIR equalizer gives more reduced MSE than the adaptive FIR equalizer. Also, it is observed that even if the number of iterations wasted on the channel estimation (250 in this simulation case) is considered, the convergence of the proposed adaptive IIR equalizer is faster than that of the adaptive FIR equalizer.

5. Conclusion

In this paper, an adaptive IIR equalizer has been pro-

posed. The equalizer has the ARMA prefilter designed based on the channel estimation by the GIV algorithm. The coefficients of the ARMA prefilter are fixed during the adaptation of the cascaded adaptive FIR filter. The ARMA prefilter has an effect to reduce the correlations of the received signals from the channel, leading to acceleration of the convergence of the cascaded adaptive FIR filter. Even if the channel distortion is severe, the convergence speed of the GIV algorithm is fast and the proposed adaptive IIR equalizer provides better performance than the commonly used adaptive FIR equalizer.

References

- [1] R.W. Chang, "A new equalizer structure for fast start-up digital communication," *The Bell System Technical Journal*, vol.50, no.6, pp.1969–2014, July 1971.
- [2] E.H. Satorious and S.T. Alexander, "Channel equalization using adaptive lattice algorithms," *IEEE Trans. Commun.*, vol.27, no.6, pp.899–905, 1979.
- [3] E.H. Satorious and J.D. Pack, "Application of least squares lattice algorithms to adaptive equalization," *IEEE Trans. Commun.*, vol.29, no.2, pp.136–142, 1981.
- [4] D.N. Godard, "Channel equalization using a Kalman filter for fast data transmission," *IBM J. Rev. Develop.*, vol.18, pp.267–273, 1974.
- [5] K.A. Hamied, M. Rahman, and M.S. El-Hennawey, "A new channel estimator for fast start-up equalization," *IEEE Trans. Commun.*, vol.39, no.2, pp.177–181, Feb. 1991.
- [6] B. Widrow and D. Stearns, *Adaptive Signal Processing*, Prentice Hall, Englewood Cliffs, New Jersey, 1985.
- [7] B. Mulgrew and C.F.N. Cowan, "An adaptive Kalman equalizer: Structure and performance," *IEEE Trans. Acoust., Speech & Signal Process.*, vol.36, no.12, pp.1727–1735, Dec. 1987.
- [8] L. Ljung and T. Soderstrom, *Theory and Practice of Recursive Identification*, MIT Press, Cambridge, MA, 1983.
- [9] T. Soderstrom and P. Stoica, "On some system identification techniques for adaptive filtering," *IEEE Trans. Circuits & Syst.*, vol.35, no.4, pp.457–461, 1988.
- [10] K. Itoh, T. Shimamura, H. Yashima, and J. Suzuki, "IIR adaptive equalizer using all-pole prefilter," *IEICE Trans.*, vol.J76-A, no.9, pp.1279–1285, Sept. 1993.
- [11] J.M. Mendel, *Discrete Technique of Parameter Estimation: The Equation Error Formulation*, Marcel Dekker, 1973.
- [12] A. Duel-Hallen and C. Heegard, "Delayed decision feedback sequence estimation," *IEEE Trans. Commun.*, vol.37, no.5, pp.428–436, May 1989.
- [13] T. Ernst and A. Kaelin, "Using noise-feedback in approximating ML sequence estimation for channels with infinite intersymbol interference," *Proc. IEEE ISCAS*, pp.393–396, 1994.
- [14] W.Y. Chen, "Adaptive pilot filtering for LMS algorithm," *Proc. IEEE ICASSP*, pp.1509–1512, 1991.
- [15] S.H. Leung and B.L. Chan, "Fast converging LMS equalizer," *Electron. Lett.*, vol.29, no.10, pp.829–830, 1993.
- [16] T. Katayama, *Introduction to System Identification*, Asakura-Syoten Co., Japan, 1993.