LETTER

Amplitude Banded RLS Approach to Time Variant Channel Equalization

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SUMMARY This paper proposes a non-linear adaptive algorithm, the amplitude banded RLS (ABRLS) algorithm, as an adaptation procedure for time variant channel equalizers. In the ABRLS algorithm, a coefficient matrix is updated based on the amplitude level of the received sequence. To enhance the capability of tracking for the ABRLS algorithm, a parallel adaptation scheme is utilized which involves the structures of decision feedback equalizer (DFE). Computer simulations demonstrate that the novel ABRLS based equalizer provides a significant improvement relative to the conventional RLS DFE on a rapidly time variant communication channel.
key words: *time variant channel, decision feedback equalizer, RLS algorithm, amplitude banded technique*

1. Introduction

Two commonly used filter structures for communications channel equalization are a linear transversal equalizer (LTE) [1] and a decision feedback equalizer (DFE) [2]. The computational complexity of the DFE is similar with that of the LTE. The DFE, however, can alleviate the noise enhancement involved in the LTE, and often provides better performance. This is due to the decision circuit in the structure of the DFE, which provides a noise-free output if the decision is correct.

In a time variant environment, rapid tracking is required for the equalizers. Although the least mean square (LMS) algorithm [6] is a simple and widely used adaptation procedure for the equalizer coefficients, its adaptation speed may not be satisfied on a rapidly time variant channel. The alternative to the LMS algorithm is the recursive least squares (RLS) [7]. The RLS algorithm may achieve more rapid adaptation than the LMS algorithm in the case where the input sequence to the equalizers makes an ill-conditioned correlation matrix. In [3]–[5], the use of the DFE involving the coefficient adaptation with the RLS algorithm has been proposed on HF channels containing multipath characteristics with rapid time variation.

In this paper, a technique for the adaptation of equalizer coefficients, the amplitude banded technique [8], is used to cope with time variant multipath channels. The amplitude banded technique takes advantage of amplitude information of the input sequence, and switches the coefficients to be updated for each iteration. This provides an effect of restricting the possible range of movement of the adapting coefficients, and as a result alleviates the influence of time variation of the channel. Based on the principle of the amplitude banded technique, we derive a novel adaptation algorithm, the amplitude banded RLS (ABRLS) algorithm, in which a non-linear adaptation process on a coefficient matrix is implemented.

We also present a filter structure to make the ABRLS algorithm work effectively, which has a parallel adaptation scheme. By computer simulations, it is shown that the ABRLS algorithm provides superior performance in the structure of the DFE by being aided by the standard RLS algorithm.

2. ABRLS Algorithm and Parallel Adaptation

It is assumed that the channel model is given by

$$x_k = \sum_{i=0}^{L} h_i(k) u_{k-i} + n_k \tag{1}$$

where $h_o(k), h_1(k), ..., h_L(k)$ are the channel coefficients, u_k is the transmitted sequence, and n_k is a white Gaussian noise uncorrelated with u_k. The channel output x_k becomes the input for an equalizer.

Now let us consider that a tap coefficient vector $\mathbf{c}(k)$ is updated by means of the RLS algorithm [7]. The input vector is $\mathbf{x}(k)$. When $\mathbf{c}(k)$ and $\mathbf{x}(k)$ are given by $\mathbf{c}(k) = (c_0(k), c_1(k), ..., c_{M_f+M_b}(k))^T$ and $\mathbf{x}(k) = (x_k, x_{k-1}, ..., x_{k-M_f}, \hat{u}_{k-d-1}, \hat{u}_{k-d-2}, ..., \hat{u}_{k-d-M_b})^T$, respectively, the RLS algorithm becomes the adaptation procedure for an $M_f + M_b + 1$ length DFE, where $\hat{u}_{k-d}$ is an estimate of the transmitted sequence delayed by d and M_f and M_b correspond to the order of the feedforward and feedback filters, respectively.

For the proposed algorithm, a Q by M_f+M_b+1 coefficient matrix $\mathbf{A}(k)$ is prepared, elements of which are given by $a_{ij}(k), i = 1, 2, ..., Q, j = 1, 2, ..., M_f + M_b + 1$. The $\mathbf{A}(k)$ is initialized at $k = 0$ where all the elements are set to zeros. For the adaptation, the elements of $\mathbf{A}(k)$ are updated based on the operation of switching the elements to be updated. Among the Q by

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$M_f + M_b + 1$ elements of $\mathbf{A}(k)$, only $M_f + M_b + 1$ elements, $a_{q(j)j}(k), j = 1, 2, ..., M_f + M_b + 1$, are selected for each iteration and a coefficient vector is set as $\mathbf{a}(k) = (a_{q(1)1}(k),\ a_{q(2)2}(k), ..., a_{q(M_f+M_b+1)M_f+M_b+1}(k))^T$ where $q(j)$ is an integer. The $q(j)$ is determined based on the amplitude level of each element $x_j(k)$ of the input vector $\mathbf{x}(k)$ for $j = 1, 2, ..., M_f + M_b + 1$ as follows:

if $|x_j(k)| \geq X_{max}(1 - 1/Q)$ then $q(j) = 1$
if $X_{max}(1 - (i-1)/Q) > |x_j(k)| \geq X_{max}(1 - i/Q)$
then $q(j) = i \quad (i = 2, 3, ..., Q)$

where X_{max} denotes the maximum amplitude of the received sequence. The X_{max} is measured at the receiver side. The Q corresponds to a division number to classify the level of the amplitude of the received sequence. This value affects the performance of the equalizer. Basically, as Q is increased, the performance is improved (the setting of the values from 2 to 10 for Q may be adequate in practice). The output of the equalizer is obtained by convolution between $\mathbf{a}(k)$ and $\mathbf{x}(k)$. Thus the coefficient vector $\mathbf{a}(k)$ is updated by the RLS algorithm, and then the elements of the updated coefficient vector $\mathbf{a}(k+1)$ are inserted into the coefficient matrix $\mathbf{A}(k+1)$. For the next iteration, a coefficient vector is again built up based on the elements of the input vector, and updated by the RLS algorithm.

The adaptation described above is made in a nonlinear operation based on the *amplitude banding* of the received sequence. This is the so-called amplitude banded adaptation [8]. The above RLS-based algorithm provides the ABRLS algorithm for an M_f+M_b+1 length DFE. The ABRLS algorithm would update all the elements of the coefficient matrix $\mathbf{A}(k)$, because the input sequence is statistically distributed on a time variant channel.

For each iteration, the ABRLS algorithm has $M_f + M_b + 1$ coefficients to be updated. This is the same situation as the RLS algorithm. Therefore, the computational complexity of the ABRLS is comparable with the RLS.

The ABRLS algorithm itself has the potential to track time variant channels more rapidly than the RLS algorithm, as shown in Sect. 3. The coefficient selection for each iteration in the ABRLS algorithm is based on the amplitude information of the received sequence. Because we can determine the pattern of the channel impulse response from the amplitude of the channel output, it is obvious that the amplitude of the received sequence is directly associated with the channel coefficients. Whenever the received sequence x_k is allocated to one range among Q ranges based on its amplitude level, the coefficient corresponding to the one range is always selected and updated in the ABRLS algorithm. In this case, the time variation influenced from the channel for the coefficient corresponding to the one range may be decreased approximately by a factor of Q [9]. This results in the adaptation for which the influence of time variation of the channel is alleviated. Although all the elements of the coefficient matrix in the ABRLS algorithm are not updated for each iteration, the coefficients to be selected and updated for each iteration are strongly related with the previously updated coefficients for each banded range. In such a case, even if the number of updates for the coefficient corresponding to each banded range is decreased, the convergence speed of the adaptive algorithm does not deteriorate; rather we can preserve it adequately [10]. From this reason, combined with the property being less influenced by the time variation of the channel, the ABRLS algorithm more rapidly tracks the channel.

However, the superior tracking performance of the ABRLS algorithm might not be always guaranteed for all the adaptation process, due to the non-linearity which the amplitude banded technique essentially has. Amplitude ambiguities sometimes cause an unstable phenomenon which appears as "spikes" in the mean square error convergence [9]. This is possibly because the amplitude of the channel output can not be perfectly and uniquely associated with the channel coefficients. For example, it is possible that a different channel coefficients pair makes the same amplitude channel outputs. Therefore, a parallel adaptation scheme is here utilized to achieve at least the performance of the conventional RLS DFE. Figure 1 depicts the whole configuration of the ABRLS-RLS DFE to be proposed in this paper. In the structure of the ABRLS-RLS DFE, two DFEs, DFE(I) and DFE(II), are used to produce the equalizer output. The two DFEs are individually updated based on the error sequences e_{ak} and e_k, respectively. The comparator provides $f_k = e_{ak}$ if $(e_{ak})^2 \leq (e_k)^2$ and $f_k = e_k$ otherwise. Based on the comparator output, the ABRLS-RLS DFE outputs y_{ak} when $f_k = e_{ak}$, and y_k when $f_k = e_k$.

The ABRLS-RLS DFE requires a parallel adaptation of the ABRLS algorithm with the RLS algorithm. This results in that the whole computational complexity for implementing the ABRLS-RLS DFE is approximately twice that for implementing the RLS DFE.

3. Simulation Results

Two channel models are used in our simulations. One is that the transfer function of which is given by

$$\textit{Channel 1:}\ H_1(z) = 1 + \sin\left(\frac{2\pi}{T}k\right)z^{-1} \tag{2}$$

where T is the period to control the rate of time variation of the channel. The other is given by

$$\textit{Channel 2:}\ H_2(z) = h_0(k) + h_1(k)z^{-1} + h_2(k)z^{-2} \tag{3}$$

where the time variant coefficients, $h_0(k)$, $h_1(k)$ and $h_2(k)$ are generated by passing Gaussian white noise

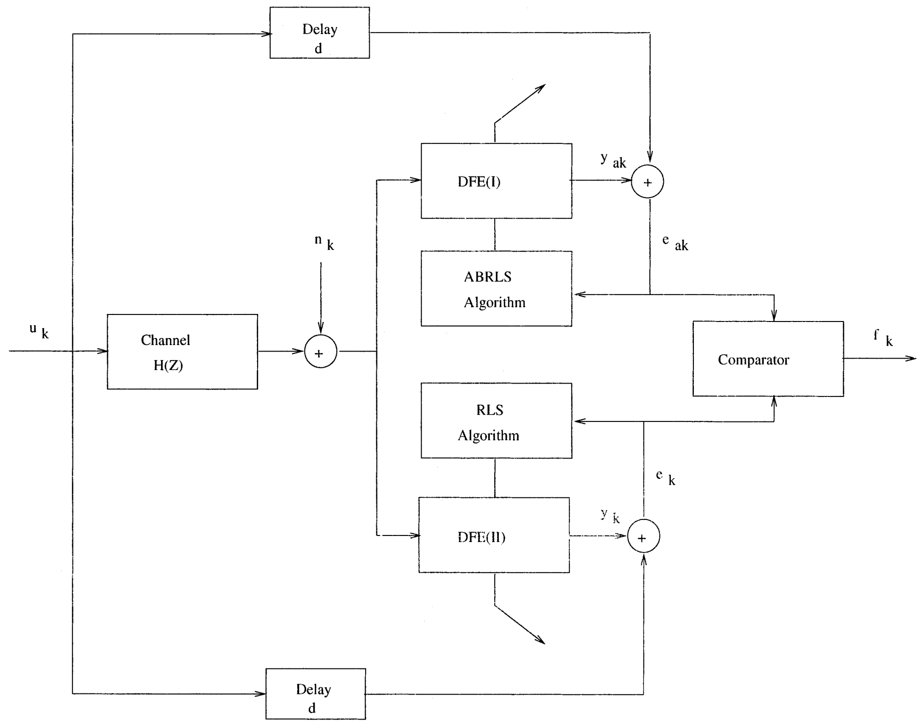

Fig. 1 Configuration of the ABRLS-RLS DFE.

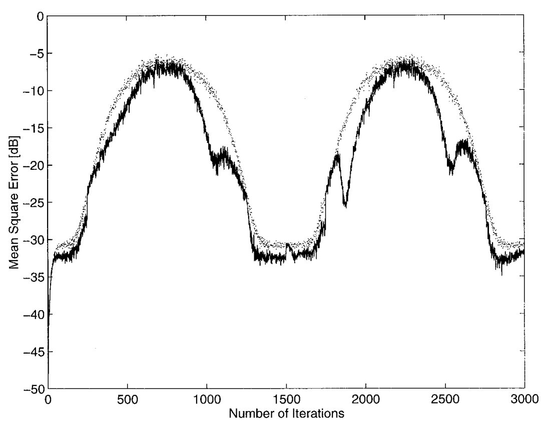

Fig. 2 Convergence of the RLS LTE (dotted line) and the ABRLS-RLS LTE (solid line) on channel 1.

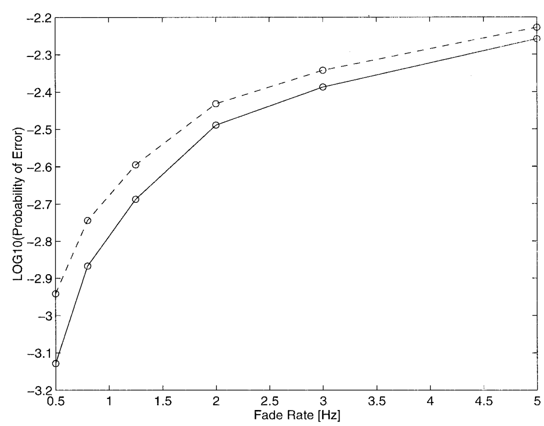

Fig. 3 BER performance of the RLS DFE (dashed line) and the ABRLS-RLS DFE (solid line) against channel fade rates on channel 2.

at 2400 sample/s through second order Butterworth filters with 3 dB bandwidths on the order of the fade rate. The input sequence of both channels is a uncorrelated, pseudo-random sequence with values of +1 or −1. Channel 2 is an HF channel model $H_3(z)$ used in [4].

Figure 2 illustrates the convergence of the RLS LTE and ABRLS-RLS LTE for $M_f = 3$, $d = 0$, and the forgetting factor $\lambda = 0.92$ on channel 1 with the value of $T = 3000$. The RLS LTE and ABRLS-RLS LTE are DFEs without feedback, having the setting of $M_b = 0$. For the ABRLS-RLS LTE, two LTEs are used in parallel, the form of which is the same as that shown in Fig. 1. The curves in Fig. 2 are the average of 100 individual trials. The additive noise is −100 dB. The division number Q for the ABRLS algorithm is 5. It should be noted that among the factors causing channel distortion, time variation predominates over the additive noise and the delay in channel 1. Thus, by using channel 1, we can investigate the capability of the equalizers only against the time variation. Figure 2 shows that the tracking done by the ABRLS algorithm aided by the RLS algorithm is superior to that done by only the RLS algorithm. This would also validate the superior tracking capability which the ABRLS algorithm itself has.

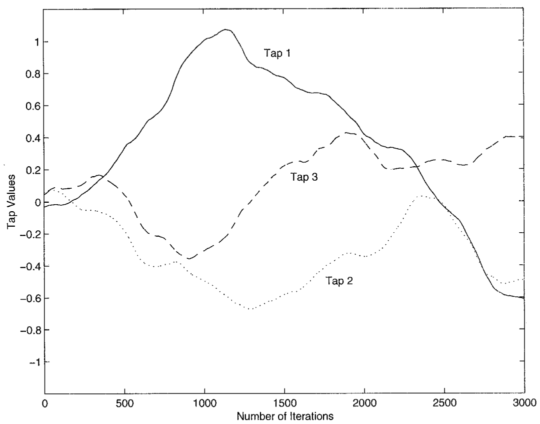

Fig. 4 An example of coefficient trajectory for channel 2. Taps 1-3 correspond to $h_0(k)$, $h_1(k)$ and $h_2(k)$, respectively.

Figure 3 illustrates the bit error rate (BER) performance against channel fade rates on channel 2 with a signal-to-noise ratio of 20 dB where the RLS DFE and the ABRLS-RLS DFE are compared (Fig. 4 is an example of coefficients generated for channel 2 with a fade rate of 2 Hz). In Fig. 3, the equalizers have $M_f = 4$ and $M_b = 2$. The forgetting factor has been optimized as $\lambda = 0.94$ so that both equalizers provide the best performance. For the ABRLS algorithm, $Q = 5$. From Fig. 3, we observe that the ABRLS-RLS DFE provides better performance than the RLS DFE in a wide range of fade rates.

4. Conclusion

This paper has proposed a novel DFE, the ABRLS-RLS DFE, which implements a non-linear adaptation based on the amplitude level of the received sequence. Computer simulations have demonstrated that the ABRLS-RLS DFE provides better performance than the conventional RLS DFE in a rapidly time variant environment. The ABRLS-RLS DFE, however, involves a parallel adaptation scheme, resulting in an increment of computational complexity. Future work will aim at a non-uniform division of the amplitude of the received sequence in the ABRLS-RLS DFE.

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