

Theory for the Design of a Filter Having One Cross Coupling Path to Realize Transmission Zeros

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SUMMARY A general circuit model of a filter having one cross coupling path is analyzed, and a new theory is developed for the design of a filter with transmission zeros in its stopband. By using the derived formulas, the reactance element values in the cross coupling path are determined readily. The transmission zeros can thus be assigned at desired frequencies. Various design examples are provided, together with simulated results, which validate the proposed theory.

key words: filter, transmission zero, cross coupling

1. Introduction

In order to realize a high performance filter with low insertion-loss in its passband and sharp attenuation in its stopband, it is usually tried to introduce transmission zeros in the stopband of the filter. A representative is the elliptic function filter which requires cross couplings between pairs of nonadjacent resonators of the filter [1]. The problem associated with the elliptic function filter is that it is usually rather difficult to realize physically the demanded multiple cross couplings. As an alternative, a technique easier to implement is to introduce only one cross coupling path between a pair of nonadjacent resonators [2], [3]. As a consequence, while the elliptic function filter exhibits equal ripple in its stopband, a filter designed with one cross coupling path has generally one finite transmission zero (or one pair of transmission zeros) only in its stopband, and exhibits drastic attenuation at frequencies around the pole(s). Since specifications of many filters require sharp attenuation at some frequencies only in its stopband, this technique is getting recent interest [2]–[5]. Levy [2] developed an approximate synthesis method based on a lowpass prototype filter. This method is simple and useful in many cases. But it suffers from inaccuracy, and can even fail for very highly selective filters [3], [4]. Hong [3] provided some tables and approximate formulas that were obtained by curve fitting. However, they are of limited usage, particularly they are only feasible for filters with even numbers of resonators.

In this paper, we develop a new theory for the design of a filter having one cross coupling path to pro-

duce transmission zeros in its stopband. We provide answers, including new formulas, to the following questions encountered in the design of such a filter

- (1) Where can the cross coupling path be introduced.
- (2) Which type of coupling should be used.
- (3) How to determine the reactance element values of the cross coupling path in order to get transmission zeros at the assigned frequencies.

The theory in this paper is derived by using a novel *ABCD* matrix technique, and is verified by numerical design examples. The results show that both symmetrical and asymmetrical transmission zeros can be realized by the proposed method. The design of our filter begins with the use of the well-known formulas for Butterworth or Chebyshev bandpass filters, and complicated synthesis process and/or formulas are avoided.

2. Theory

We assume that we begin with a Butterworth or Chebyshev bandpass filter (BPF). A generalized Butterworth or Chebyshev BPF circuit, using impedance-inverters and lossless series-type resonators, is shown in the *dotted-line* box in Fig. 1 [6]. The values of the impedance-inverter $K_{i,i+1}$ ($i = 0, 2, \dots, n$) and resonator reactance $X_i(\omega)$ ($i = 1, 2, \dots, n$) are calculated from the specifications of the filter, using formulas provided in [6]. In the case a Chebyshev filter, the formulas are as follows

$$\begin{aligned}
 K_{01} &= \sqrt{\frac{R_0 \chi_1 w}{g_0 g_1 \omega'_1}}, & K_{i,i+1} &= \frac{w}{\omega'_1} \sqrt{\frac{\chi_i \chi_{i+1}}{g_i g_{i+1}}} \\
 K_{n,n+1} &= \sqrt{\frac{R_0 \chi_n w}{g_n g_{n+1} \omega'_1}}, & \omega'_1 &= 1 \text{ (rad/sec)} \\
 \chi_i &= \frac{\omega_0}{2} \frac{dX_i(\omega)}{d\omega}, & w &= \frac{\omega_2 - \omega_1}{\omega_0}, & \omega_0 &= \sqrt{\omega_1 \omega_2}
 \end{aligned} \tag{1}$$

where g_i is the lumped element value of the prototype lowpass filter, R_0 is the source/load impedance, ω_1 and ω_2 are the lower and upper edge-frequencies of the equal-ripple passband, respectively, ω_0 is the center frequency of the filter, w is the fractional bandwidth, χ_i is the reactance slope-parameter of the resonator.

The Butterworth or Chebyshev filter has transmission zeros at dc and infinite frequencies only. In order

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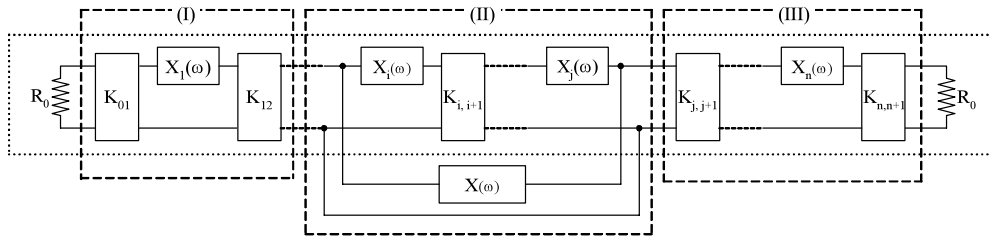


Fig. 1 A generalized bandpass filter circuit using impedance-inverters and series-type resonators. A cross coupling path with reactance $X(\omega)$ is introduced between the i -th and j -th resonator.

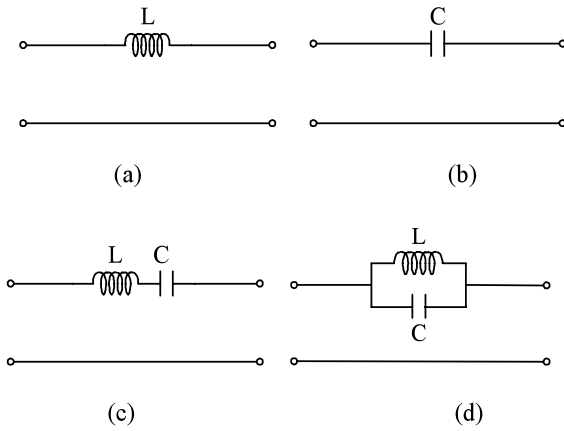


Fig. 2 Four types of cross couplings. (a) An inductor L , (b) a capacitor C , (c) a series LC resonator, and (d) a shunt LC resonator.

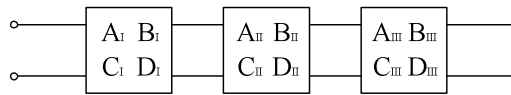


Fig. 3 $ABCD$ matrix representation of the filter shown in Fig. 1.

to get finite transmission zeros in the stopband, a cross coupling path with reactance $X(\omega)$ is introduced between the i -th and j -th resonator, as shown in Fig. 1. The cross coupling reactance $X(\omega)$ can be an inductor L , a capacitor C , a series or shunt LC resonator, as shown in Fig. 2, to represent electric, magnetic, and mixed electromagnetic coupling, respectively.

In order to get the frequency response of the filter with an additional cross coupling path, we divide the circuit into three parts, as indicated by the *dashed-line* boxes in Fig. 1. We use three $ABCD$ matrices, $A_I B_I C_I D_I$, $A_{II} B_{II} C_{II} D_{II}$, and $A_{III} B_{III} C_{III} D_{III}$, to represent these three parts, as shown by Fig. 3. The first and the third part in Fig. 1 consist of cascaded impedance-inverters and series-type resonators. Therefore, $A_I B_I C_I D_I$ and $A_{III} B_{III} C_{III} D_{III}$ are the products of the $ABCD$ matrix of the impedance-inverter

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & jK_{i,i+1} \\ \frac{j}{K_{i,i+1}} & 0 \end{bmatrix} \quad (2)$$

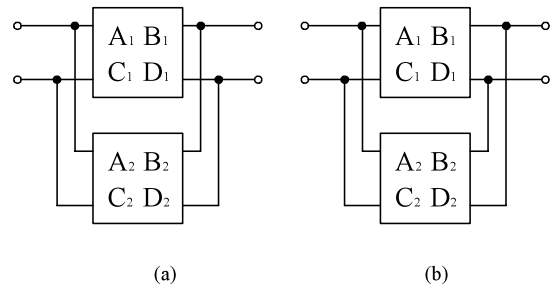


Fig. 4 Two shunt $ABCD$ matrices with, (a) inphase connection, and (b) outphase connection.

and that of the series-type resonator

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & jX_i(\omega) \\ 0 & 1 \end{bmatrix} \quad (3)$$

The second part in Fig. 1 has two paths. The upper path consists of cascaded impedance-inverters and series-type resonators, and its overall $ABCD$ matrix, $A_I B_I C_I D_I$, is the products of the $ABCD$ matrices expressed by (2) and (3). The lower path is the cross coupling path with a reactance $X(\omega)$, whose $ABCD$ matrix, $A_2 B_2 C_2 D_2$, is expressed by (3). Therefore, the second part in Fig. 1 can be represented by two shunt $ABCD$ matrices, as shown in Fig. 4. Figures 4(a) and (b) represent inphase and outphase connections, respectively. Note that in Fig. 1, only the inphase connection is drawn for brevity. The overall $ABCD$ matrix of the second part, $A_{II} B_{II} C_{II} D_{II}$, is related to the two shunt $ABCD$ matrices by the following expressions:

$$A_{II} = \frac{A_1 B_2 + A_2 B_1}{B_2 \pm B_1} \quad (4a)$$

$$B_{II} = \frac{B_1 B_2}{B_2 \pm B_1} \quad (4b)$$

$$C_{II} = (C_1 \pm C_2) + \frac{(A_2 \mp A_1)(D_1 \mp D_2)}{B_2 \pm B_1} \quad (4c)$$

$$D_{II} = \frac{B_1 D_2 + B_2 D_1}{B_2 \pm B_1} \quad (4d)$$

where in the sign \pm or \mp , the upper one corresponds to the inphase connection, and the lower one outphase connection.

Finally, the overall $ABCD$ matrix of the filter is

readily obtained by multiplying the $ABCD$ matrices of these three parts. The reflection and transmission response of the filter are calculated then by using the following equations:

$$S_{11}(\omega) = \frac{A_T R_0 + B_T - C_T R_0^2 - D_T R_0}{A_T R_0 + B_T + C_T R_0^2 + D_T R_0} \quad (5a)$$

$$S_{21}(\omega) = \frac{2R_0}{A_T R_0 + B_T + C_T R_0^2 + D_T R_0} \quad (5b)$$

where A_T , B_T , C_T , and D_T are the elements of the overall $ABCD$ matrix of the filter. We observed that all the elements A_T , B_T , C_T , and D_T in (5a) and (5b) have a common denominator, $(B_2 + B_1)$ in the case of inphase connection, or $(B_2 - B_1)$ in the case of outphase connection. Here B_1 and B_2 are the elements of the shunt $ABCD$ matrices shown in Fig. 4. Therefore, at an assigned frequency ω_p , if we let

$$B_2(\omega_p) = -B_1(\omega_p) \quad (6)$$

in the case of inphase connection, or

$$B_2(\omega_p) = B_1(\omega_p) \quad (7)$$

in the case of outphase connection, we will have $S_{21}(\omega_p) = 0$, i.e., we will get a transmission zero at the assigned frequency. For a Butterworth or Chebyshev bandpass filter having one cross coupling path, (6) or (7) provides the condition (at least sufficient condition) to introduce one transmission zero at an assigned frequency.

By referring to Fig. 1 and Fig. 4, we see that B_1 in (6) and (7) can be easily calculated by cascading the $ABCD$ matrices of the impedance-inverters and series-type resonators. On the other hand, if the cross coupling element is an inductor L , (6) and (7) become

$$B_2(\omega_p) = j\omega_p L = \mp B_1(\omega_p) \quad (8)$$

and if the cross coupling element is a capacitor C , (6) and (7) become

$$B_2(\omega_p) = 1/j(\omega_p C) = \mp B_1(\omega_p) \quad (9)$$

From (8) or (9), we can calculate the value of L or C with which the transmission zero will appear at the assigned frequency ω_p .

If we want to get two transmission zeros simultaneously at frequencies ω_{p1} and ω_{p2} , we can use a cross coupling path with a series or shunt LC resonator, as shown in Figs. 2(c) and (d), instead of a single L or C . We have then

$$\begin{aligned} B_2(\omega_{pi}) &= j \left[\omega_{pi} L - 1/(\omega_{pi} C) \right] \\ &= \mp B_1(\omega_{pi}), \quad i = 1, 2 \end{aligned} \quad (10)$$

in the case of a series LC resonator, or

$$\begin{aligned} B_2(\omega_{pi}) &= j\omega_{pi} L / (1 - \omega_{pi}^2 LC) \\ &= \mp B_1(\omega_{pi}), \quad i = 1, 2 \end{aligned} \quad (11)$$

in the case of a shunt LC resonator. Both (10) and (11) are simple linear equations of two variables L and C , from which L and C are solved readily.

3. Design Examples

Based on the theory proposed above and the derived formulas, a computer program is developed to implement the design of a filter having one cross coupling path. As an example, a Chebyshev filter having five resonators is considered. The passband attenuation ripple $L_A = 0.01$ dB, the lower and upper passband edge-frequencies $f_1 = 9.9$ GHz and $f_2 = 10.1$ GHz, respectively. The center frequency of the filter is about 10 GHz, and the equal-ripple fractional bandwidth about 2%. The ideal Chebyshev characteristics of this filter are shown in Figs. 6, 8, 9, 11–13, and 15–17 by solid lines. The cross coupling path can be connected to the filter, as shown in Fig. 1, at a number of different places. Because of the limited space, we can not provide here all of the results obtained with possible connections of the cross coupling path. We show below some typical cases with discussions.

First, the cross coupling path is connected to the filter between the input and output feed lines, as shown in Fig. 5. When we want to introduce a transmission zero at $f_p = 9.6$ GHz in the lower stopband, our computation shows that we can use a capacitor $C = 2.1053355 \times 10^{-15}$ F in the case of inphase coupling path, or an inductor $L = 1.3054986 \times 10^{-7}$ H in the case of outphase coupling path. If we use an inductor in the case of inphase coupling path, or a capacitor in the case of outphase coupling path, we get negative values of L or C , which is physically meaningless. The frequency response of the filter with the cross coupling element is drawn in Fig. 6(a) by dashed lines. It is seen from Fig. 6(a) that by introducing a cross coupling path in the filter, we can get a transmission zero exactly at the assigned frequency. On the other hand, the attenuation in the upper stopband is reduced to some extent as the penalty. The passband response of the filter with a cross coupling path exhibits minor difference with that of the ideal Chebyshev filter (solid lines) without a cross coupling path. However, if the transmission zero is chosen very close to the passband, it is observed that the return loss of the filter in the passband will increase significantly.

The simulated frequency response of a filter with an assigned transmission zero at 10.4 GHz in the upper stopband is drawn in Fig. 6(b) by dashed lines. Similar discussions as above can be made.

The above filter circuits with our calculated element values, including the cross coupling elements, are also simulated by using the commercial simulator ADS [7]. In the filter circuit for ADS simulation, the K-inverters are replaced by T-type inductors, and the series resonators are replaced by half-wavelength uniform

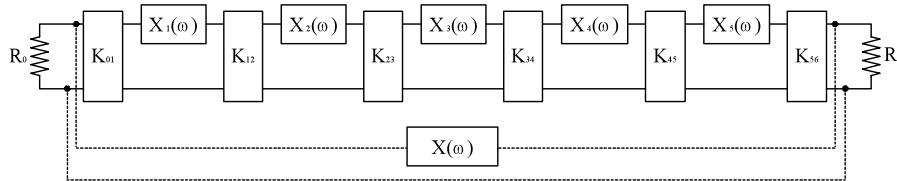
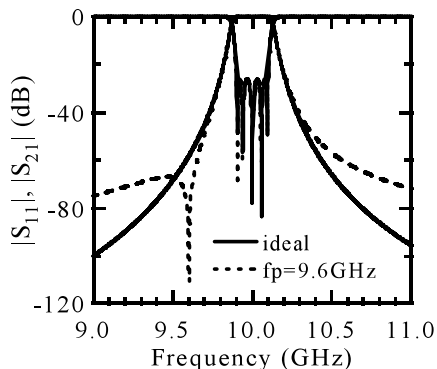
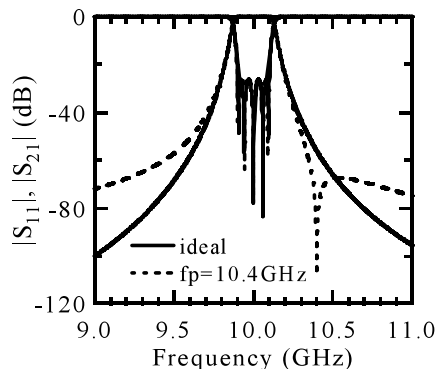


Fig. 5 The cross coupling path is introduced between the input and output feed lines.

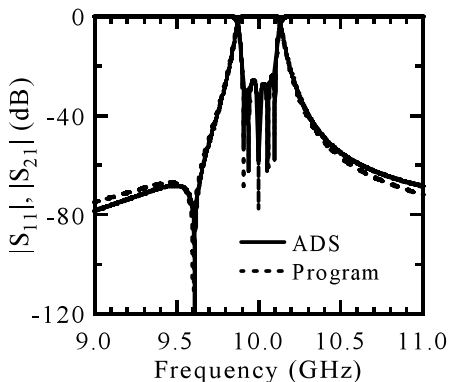


(a) Inphase *C*-coupling ($C = 2.1053355 \times 10^{-15}$ F).

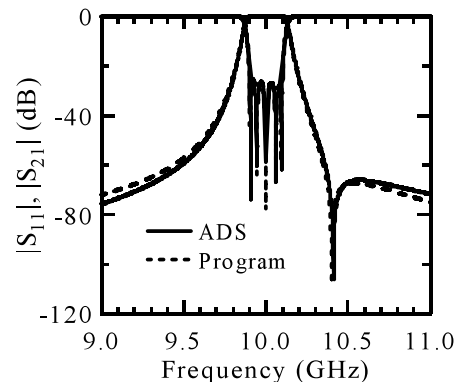


(b) Inphase *L*-coupling ($L = 1.2143900 \times 10^{-7}$ H).

Fig. 6 Frequency responses of filters with and without an attenuation pole in, (a) the lower stopband, and (b) the upper stopband.



(a) Inphase *C*-coupling ($C = 2.1053355 \times 10^{-15}$ F).



(b) Inphase *L*-coupling ($L = 1.2143900 \times 10^{-7}$ H).

Fig. 7 Comparison of the frequency responses of the filters calculated by our computer program and the commercial simulator ADS.

transmission lines. The obtained results are drawn in Figs. 7(a) and (b) by solid lines for comparison with the dashed lines calculated by our computer program. The agreement is excellent, and this validates our formulas derived in Sect. 2, as well as our computer program.

If we want to get two transmission zeros simultaneously, we can use a coupling path having both *L* and *C* elements. As an example, let $f_{p1}=9.4$ GHz, $f_{p2}=9.6$ GHz, both in the lower stopband, or $f_{p1}=10.4$ GHz, $f_{p2}=10.6$ GHz, both in the upper stopband. By using an inphase coupling path with a series *LC* resonator, or an inphase coupling path with a shunt *LC* resonator, we get the simulated results shown in Figs. 8(a) and (b), respectively. In Fig. 8(a), in addition to the assigned two transmission zeros, an un-

expected but favorable transmission zero appeared at about 11.0 GHz in the upper stopband. There is another unfavorable attenuation zero at about 9.68 GHz. This attenuation zero is caused by the resonance of the series *LC* resonator in the cross coupling path, because at this resonant frequency, the cross coupling path (a series *LC* resonator) is short-circuited.

We can also make one transmission zero appear in the lower stopband, and the other in the upper stopband of the filter. As an example, let $f_{p1}=9.6$ GHz and $f_{p2}=10.4$ GHz. When we use an inphase coupling path with a series *LC* resonator, we get the result drawn in Fig. 9(a) by dashed lines. Because of the resonance of the series *LC* resonator which makes the coupling path short-circuited, an unfavorable transmission zero

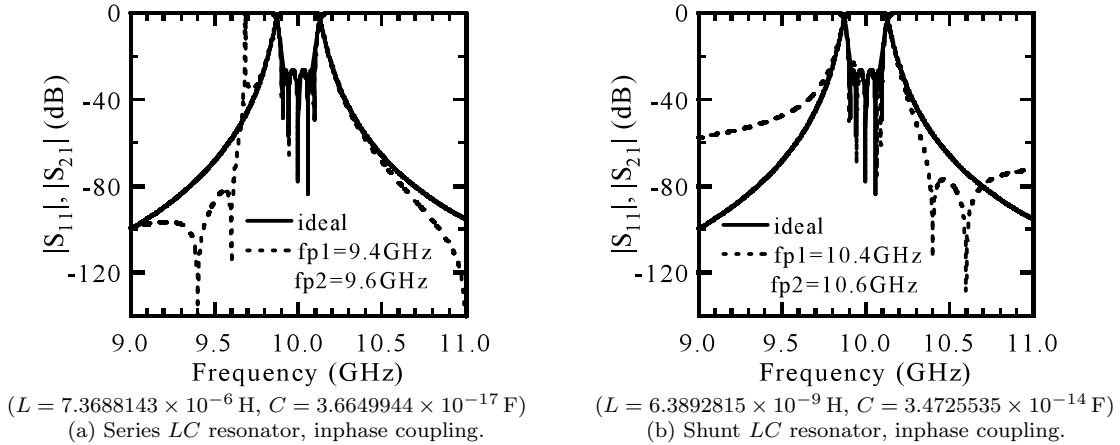


Fig. 8 Frequency responses of filters with and without two attenuation poles in, (a) the lower stopband, and (b) the upper stopband.

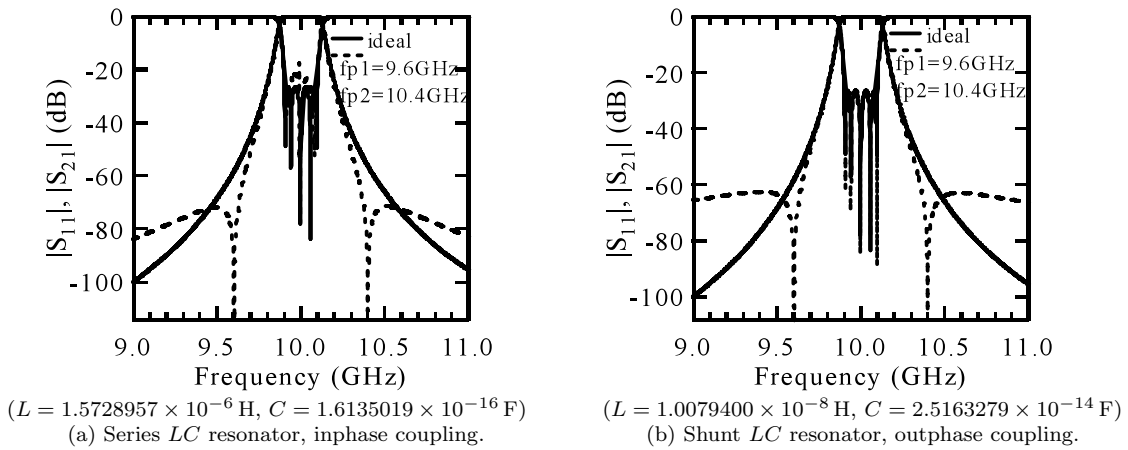


Fig. 9 Frequency responses of filters with and without one attenuation pole in the lower stopband, and the other in the upper stopband.

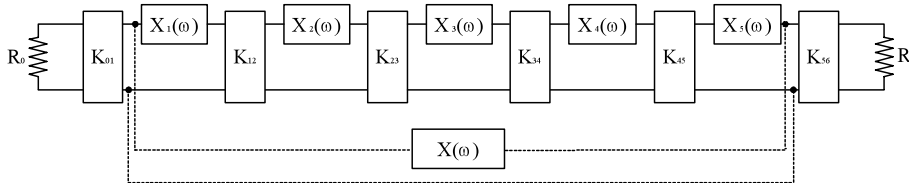
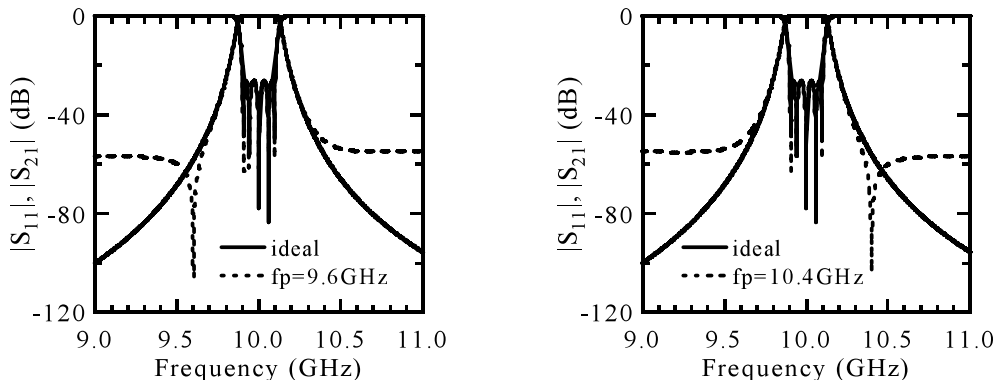


Fig. 10 The cross coupling path is introduced between the 1st and 5th resonator.

appeared also in the passband, in addition to the two assigned transmission zeros in the stopband. When we use an outphase coupling path with a shunt LC resonator, we get the filter response plotted in Fig. 9(b) by dashed lines.

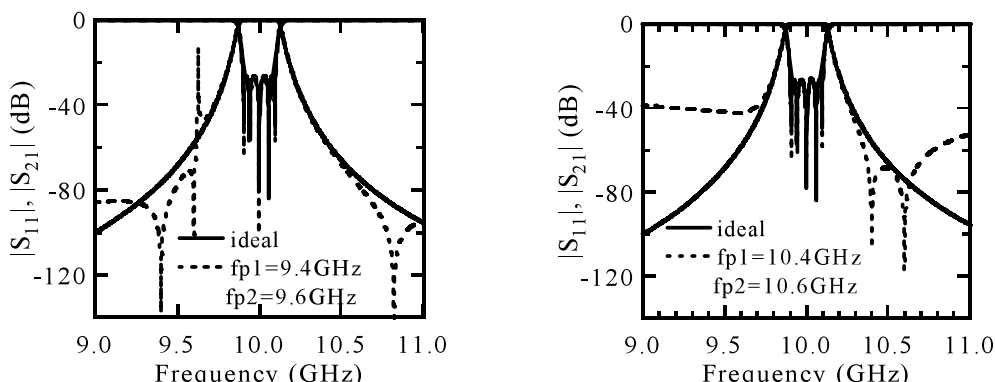
When the cross coupling path is introduced between the first and the fifth resonator, as shown in Fig. 10, the corresponding results for filters with assigned transmission zeros are shown in Figs. 11–13. Similar discussions as above can be made. Compared with results in Figs. 6, 8, and 9, the attenuation in the stopband in Figs. 11–13 is reduced by about 20 dB.

When the cross coupling path is introduced between the second and the fourth resonator, as shown in Fig. 14, the corresponding frequency responses of the filters are shown in Figs. 15–17. The results are quite close to those in Figs. 6, 8, and 9. However, the values of the reactance elements, L and C , in the cross coupling path are different with those in Figs. 6, 8, and 9, by 1 or 2 orders. Therefore, in the design of the cross coupling path, we can choose realizable values of reactance elements by changing the place where the cross coupling path is introduced.



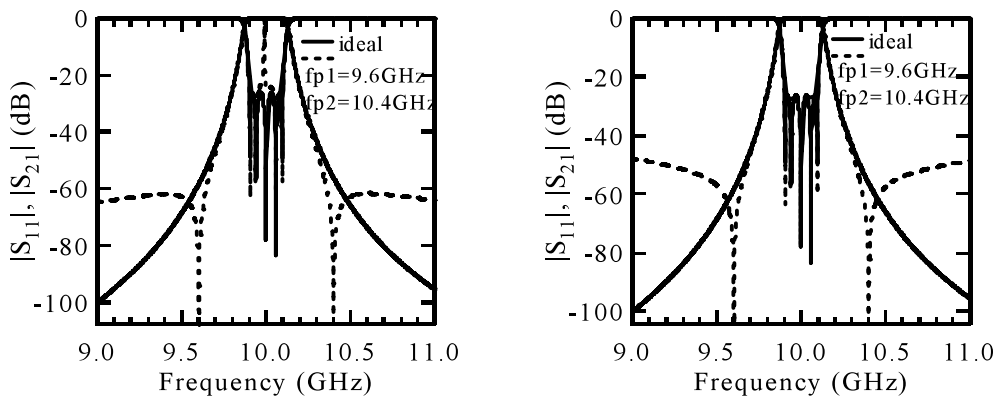
(a) Inphase C -coupling ($C = 6.2947109 \times 10^{-15}$ F). (b) Inphase L -coupling ($L = 4.0851745 \times 10^{-8}$ H).

Fig. 11 Frequency responses of filters with and without an attenuation pole in, (a) the lower stopband, and (b) the upper stopband.



($L = 7.6303004 \times 10^{-6}$ H, $C = 3.5816077 \times 10^{-17}$ F) (a) Series LC resonator, inphase coupling. ($L = 1.7306676 \times 10^{-9}$ H, $C = 1.2958661 \times 10^{-13}$ F) (b) Shunt LC resonator, inphase coupling.

Fig. 12 Frequency responses of filters with and without two attenuation poles in, (a) the lower stopband, and (b) the upper stopband.



($L = 5.2766183 \times 10^{-7}$ H, $C = 4.8107639 \times 10^{-16}$ F) (a) Series LC resonator, inphase coupling. ($L = 3.3804686 \times 10^{-9}$ H, $C = 7.5010968 \times 10^{-14}$ F) (b) Shunt LC resonator, outphase coupling.

Fig. 13 Frequency responses of filters with and without one attenuation pole in the lower stopband, and the other in the upper stopband.

4. Conclusions

We have developed a theory for the design of a filter having one cross coupling path in order to realize trans-

mission zeros in its stopband. The examples given show that when we want to get transmission zeros at assigned frequencies, we can use an inductor or a capacitor, a series or a shunt LC resonator. We can choose an inphase or outphase cross coupling path. We can introduce

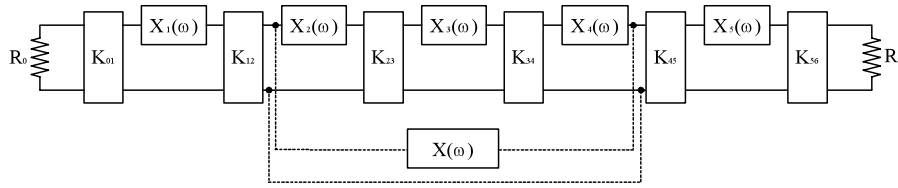


Fig. 14 The cross coupling path is introduced between the 2nd and 4th resonator.

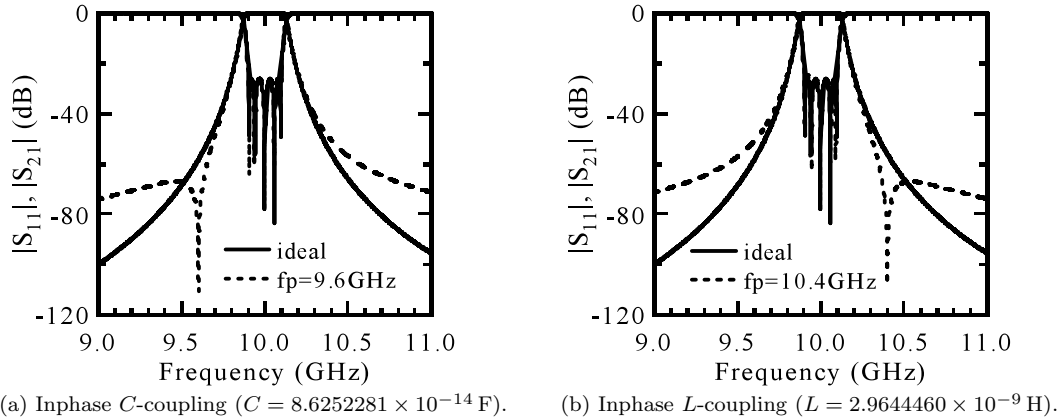


Fig. 15 Frequency responses of filters with and without an attenuation pole in, (a) the lower stopband, and (b) the upper stopband.

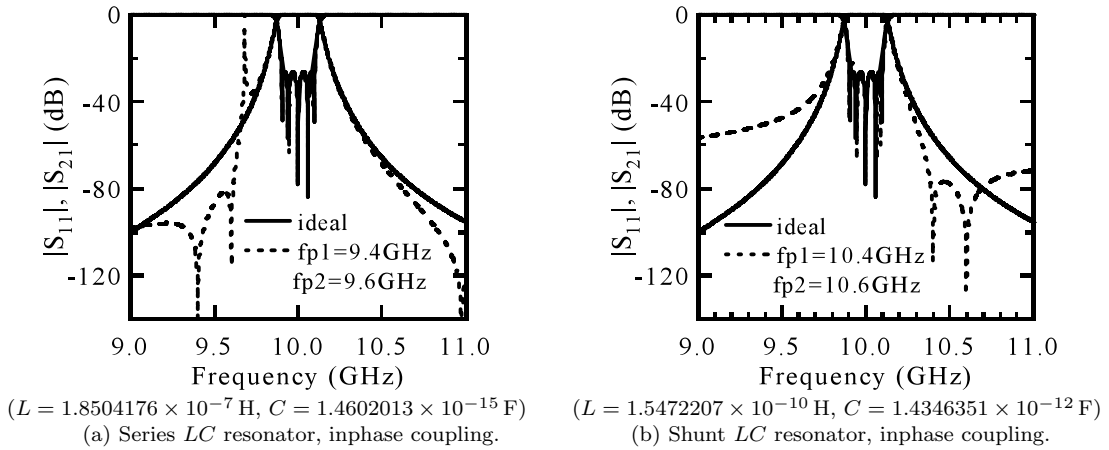


Fig. 16 Frequency responses of filters with and without two attenuation poles in, (a) the lower stopband, and (b) the upper stopband.

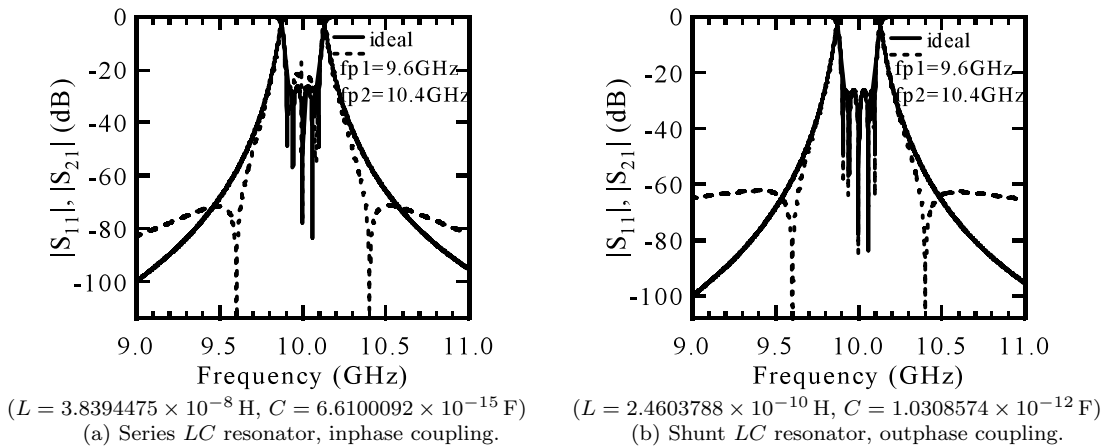


Fig. 17 Frequency responses of filters with and without one attenuation pole in the lower stopband, and the other in the upper stopband.

the coupling path to the filter at a number of different places. The developed theory and derived formulas provide solutions to all these possible choices, from which we can get the best frequency response and/or the easiest filter structure to realize.

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