PAPER *Special Issue on Microwave and Millimeter Wave Technology*

Design of a Grooved Circular Cavity for Dielectric Substrate Measurements in Millimeter Wave Region

Takashi SHIMIZU[†a)], **Zhewang MA**[†], *Regular Members, and* **Yoshio KOBAYASHI**[†], *Fellow*

SUMMARY A grooved circular cavity is designed for the millimeter wave measurements of dielectric substrates. The grooves are introduced to separate the degenerate $\mathrm{TE}_{01\mathrm{p}}$ and $\mathrm{TM}_{11\mathrm{p}}$ modes in circular cavities. A rigorous mode-matching method is used to investigate the influence of grooves on both the $\mathrm{TE}_{01\mathrm{p}}$ and $\mathrm{TM}_{11\mathrm{p}}$ modes. The dimensions of the grooves are determined from the numerical results. Comparative experiments of circular cavities with and without grooves validate the design method.

key words: *dielectric substrate measurement, cut-off circular waveguide method, millimeter wave*

1. Introduction

Rapid progress of microwave and millimeter wave circuits requests cheap and low-loss dielectric materials. We have proposed the cavity resonator method [1], [2] and the cut-off circular waveguide method [3], [4] to measure the complex permittivities of low-loss dielectric substrates. In these methods, it is needed to separate the degenerate $\mathrm{TM}_{11\mathrm{p}}$ mode from the $\mathrm{TE}_{01\mathrm{p}}$ mode, because the dimensions and relative conductivity σ_r of the circular cavity can be measured from the $\mathrm{TE}_{01\mathrm{p}}$ modes. Here $\sigma_r = \sigma/\sigma_0$ is the effective relative conductivity including influence of oxidation and roughness of the copper surface, σ is the conductivity, $\sigma_0 = 58 \times 10^6\,\mathrm{S/m}$ is the conductivity of the standard copper. Moreover, the degenerate $\mathrm{TM}_{11\mathrm{p}}$ mode affects these measurements, especially relative conductivity measurement.

The study of the degenerate TE and TM modes has been presented in [5], [6]. However, these studies were not performed by the rigorous analysis.

In this paper, a grooved circular cavity is designed for the millimeter wave measurements of dielectric substrates. The grooves at the both ends in a circular cavity are introduced to separate the degenerate $\mathrm{TE}_{01\mathrm{p}}$ and $\mathrm{TM}_{11\mathrm{p}}$ modes. The TE_{01} mode is cut off in a radial waveguide constituted by the grooves; hence the resonant frequency of the $\mathrm{TE}_{01\mathrm{p}}$ mode is affected little by the grooves. On the other hand, the $\mathrm{TM}_{11\mathrm{p}}$ mode propagates forth and back in the grooves; hence the resonant frequency of the $\mathrm{TM}_{11\mathrm{p}}$ mode is affected significantly. A rigorous mode-matching method is used to investigate the influence of grooves on both the $\mathrm{TE}_{01\mathrm{p}}$ and $\mathrm{TM}_{11\mathrm{p}}$ modes. The dimensions of the grooves are determined from the results calculated numerically. The measured results validate the design method.

2. Analysis

A cross sectional view of a circular cavity with diameter D and height H is shown in Fig. 1. The circular cavity is cut into two parts in the middle of the height for clamping a dielectric plate sample. At both upper and lower ends of the cavity, grooves with depth d_g and width w are cut at both ends of the cylinder for separating the degenerate $\mathrm{TE}_{01\mathrm{p}}$ and $\mathrm{TM}_{11\mathrm{p}}$ modes. The cavity with these grooves can be viewed as coaxially cascaded circular waveguides with different diameters [7], [8]. Then electromagnetic (EM) fields in each of these circular waveguides are expressed by series of incident and reflected normal modes of the respective circular waveguide. At the step-junction between two neighboring circular waveguides with different diameters, the EM boundary conditions are applied. As a result, the generalized scattering matrix of the incident and reflected normal modes including higher order modes is obtained at the step-junction. By combining the generalized scattering matrices at each of the step-junctions, and using the EM boundary conditions at the top and bottom of the cavity, we get finally the eigenvalue ma-

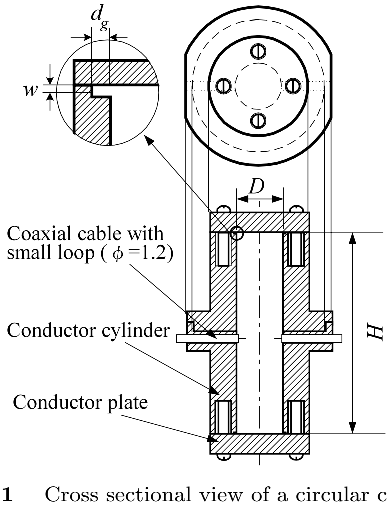


Fig. 1 Cross sectional view of a circular cavity.

Manuscript received November 29, 2002.
Manuscript revised March 7, 2003.

[†] The authors are with the Faculty of Engineering, Saitama University, Saitama-shi, 338-8570 Japan.

a) E-mail: shimizu@reso.ees.saitama-u.ac.jp trix equation for calculating the resonant frequencies and field distributions of different resonant modes.

In addition, the unloaded Q_u of the $\mathrm{TE_{01p}}$ modes can be calculated from a simple equation for a circular cavity without grooves, because the grooves constructed the radial waveguide do not almost affect to the fields distributions of the $\mathrm{TE_{01p}}$ modes.

3. Design of the Cavity

3.1 Determination of the Diameter and Height of the Circular Cavity without Grooves

For the circular cavity as shown in Fig. 1, but without grooves, the resonant frequencies f_0 are calculated by

$$\left(\frac{f_0 D}{c}\right)^2 = \frac{1}{4}\left(\frac{pD}{H}\right)^2 + \left(\frac{j^{(\prime)}_{nm}}{\pi}\right)^2 \tag{1}$$

where c is the velocity of light, and j_{nm} and j'_{nm} are the m-th root of the n-th Bessel function of the first kind and its differential, respectively. Also, j'_{nm} is for the TE_{nmp} modes and j_{nm} for the TM_{nmp} modes.

At first, the ratio D/H were chosen as 0.294 from Fig. 2, so that unwanted modes do not appear near the degenerate $\mathrm{TE_{01p}}$ and $\mathrm{TM_{11p}}$ modes. Then the value of D is chosen as 7.0 mm so that the resonant frequency of the $\mathrm{TE_{011}}$ mode becomes approximately 50 GHz. Thus, the value of H is determined to be 23.8 mm.

3.2 Control of the Degenerate $\mathrm{TM_{11p}}$ Modes

We need to suppress the degenerate $\mathrm{TM_{11p}}$ mode and to separate from the $\mathrm{TE_{01p}}$ mode, because the dimension and relative conductivity of a circular cavity are measured by using the $\mathrm{TE_{01p}}$ mode. This is realized by the position of the excitation, the plane of small loop of coaxial cable and the grooves machined at both ends of the cylinder as shown in Fig. 1.

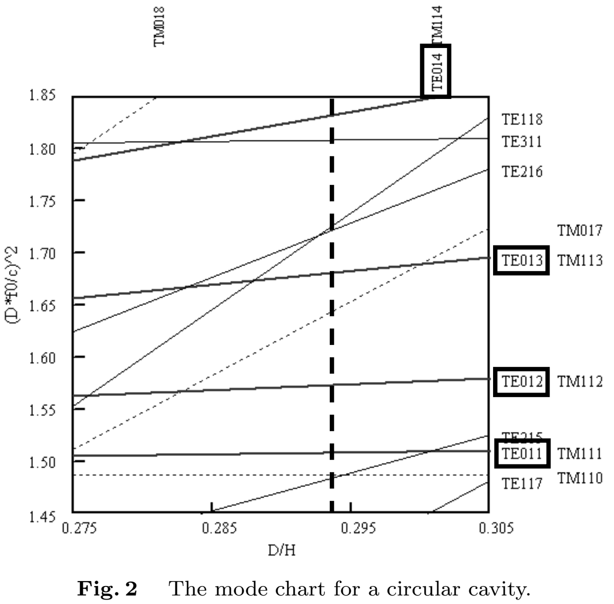


Fig. 2 The mode chart for a circular cavity.

3.2.1 The Position of Excitation

The position of the excitation is investigated to suppress the degenerate $\mathrm{TM_{11p}}$ modes in consideration the electromagnetic field distributions of the circular empty cavity. The electromagnetic fields of the $\mathrm{TE_{01p}}$ and $\mathrm{TM_{11p}}$ modes are shown in Fig. 3.

At first, we consider the case of exciting the H_r-components at $z = \pm H/2$ and $r = D/4$ by a pair of coaxial cables with small loops at their top ends. In this case, the both modes are excited and the degenerate $\mathrm{TM_{11p}}$ modes cannot be suppressed, because the both modes have the H_r-components at the conductor plates of the both cylinder ends.

Secondly, we consider the case of exciting the H_z-components at $z = 0$ and $r = \pm D/2$. In this case, the degenerate $\mathrm{TM_{11p}}$ modes are not excited because this mode doesn't have the H_z-component at the middle of the cylinder wall. However, the $\mathrm{TE_{01p}}$ modes are not also excited by the same reason where mode number p is even. We determine the position of the excitation at $z = -0.5$ mm to excite all $\mathrm{TE_{01p}}$ modes.

3.2.2 Determination of Groove Size

The degenerate $\mathrm{TM_{11p}}$ modes are suppressed if the each plane of the small loop of the coaxial cables is vertical to z-axis perfectly. However, it is difficult to realize this condition, practically. The grooves are introduced to separate the degenerate $\mathrm{TE_{01p}}$ and $\mathrm{TM_{11p}}$ modes in circular cavities. The field distributions of the $\mathrm{TE_{011}}$ and $\mathrm{TM_{111}}$ modes of the grooved circular cavity are shown in Fig. 4. To compare the filed distributions of these

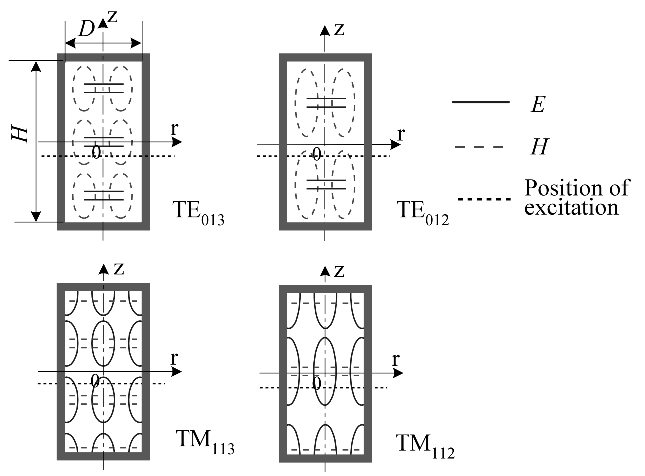


Fig. 3 The field distributions of the $\mathrm{TE_{01p}}$ and $\mathrm{TM_{11p}}$ modes of the circular cavity.

modes at $z = \pm H/2$, the TE$_{011}$ mode does not have the electric fields. However, the TM$_{111}$ mode has the E_z-component. Therefore, the degenerate TM$_{11\mathrm{p}}$ modes are separated from the TE$_{01\mathrm{p}}$ modes by the grooves machined at both ends.

The resonant frequencies of the TE and TM modes are calculated by a computer program developed based on the mode-matching method described in Sect. 2.

At first, we check the convergence of the solution with number of expansion modes N. When D=7.0 mm, H= 23.8 mm, d_g=0.2 mm and w=0.2 mm, the calculated resonant frequencies for the TE$_{011}$ and TM$_{111}$ modes are shown in Fig. 5. It is seen that the solution converges to the sixth effective figure when $N = 25$.

Secondly, the f_0 values for the TE$_{01\mathrm{p}}$ and TM$_{11\mathrm{p}}$ modes were calculated as functions of d_g and w. The calculated results are shown in Fig. 6, where the values of f_0 changed by grooves are normalized by the values of f_n without grooves, and d_g and w are normalized by D and H, respectively. As the d_g and w values are increased, the f_0 value of the TM$_{11\mathrm{p}}$ mode is decreased significantly. However, the change of f_0 value for the TE$_{01\mathrm{p}}$ mode is negligibly small. Thus, we can separate the TM$_{11\mathrm{p}}$ mode from the TE$_{01\mathrm{p}}$ mode by changing the size of the groove.

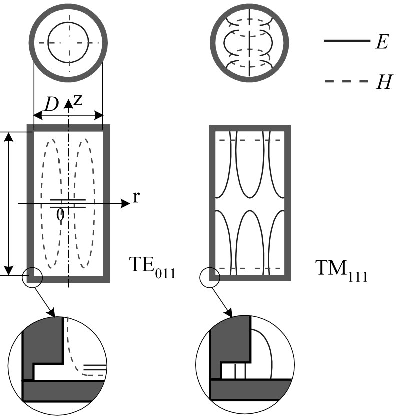


Fig. 4 The field distributions of the TE$_{011}$ and TM$_{111}$ modes of circular cavity with grooves.

Moreover, the calculated results of the TM$_{11\mathrm{p}}$ modes by the perturbation method [9] are also shown in Figs. 6(a) and (c) by small open circles. In this case, it is found the results by perturbation method is useful when d_g/D is smaller than 0.05.

The ratio of d_g/D and w/H were determined to be 0.085 and 6.3×10^{-3}, respectively, so that the TM$_{11\mathrm{p}}$ mode decreases about 0.5%, compared with the case without grooves. As a result, d_g and w are determined to be 0.6 mm and 0.15 mm, respectively.

4. Measurement

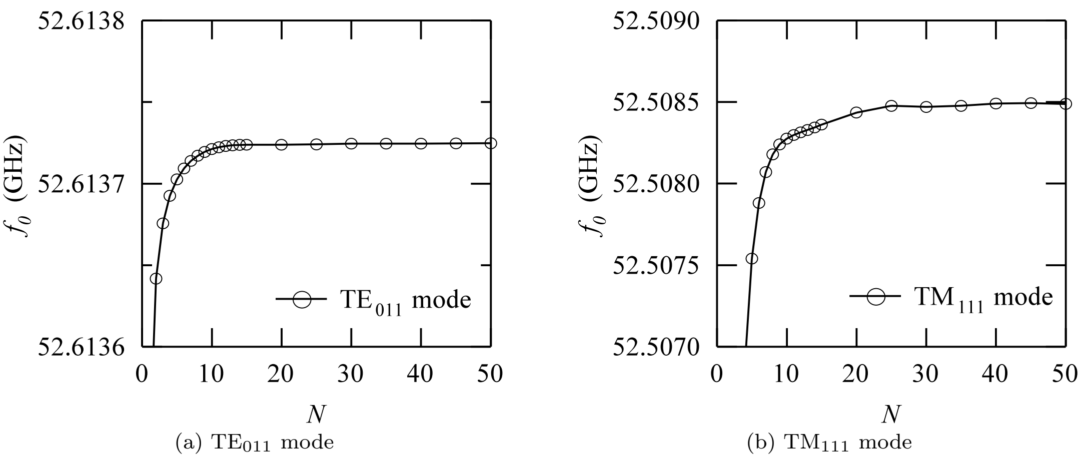
A circular cavity with grooves is manufactured by using oxygen free copper. The dimensions determined above are D=7.0 mm, H=23.8 mm, d_g=0.6 mm and w=0.15 mm. Actual value of D and H are calculated from the measured resonant frequencies of the TE$_{01\mathrm{p}}$ and TE$_{01\mathrm{q}}$ modes by using following equations,

$$D = \frac{cj'_{01}}{\pi}\sqrt{\frac{q^2 - p^2}{(qf_{0p})^2 - (pf_{0q})^2}},$$

$$H = \frac{c}{2}\sqrt{\frac{q^2 - p^2}{f_{0q}^2 - f_{0p}^2}}$$

where c is velocity of light and j'_{01}=3.83171. The measured results are D=6.990 mm and H=23.770 mm, which are the average values calculated from 6 sets of the TE$_{01\mathrm{p}}$ and TE$_{01\mathrm{q}}$ modes (p, $q = 1 \cdots 4$).

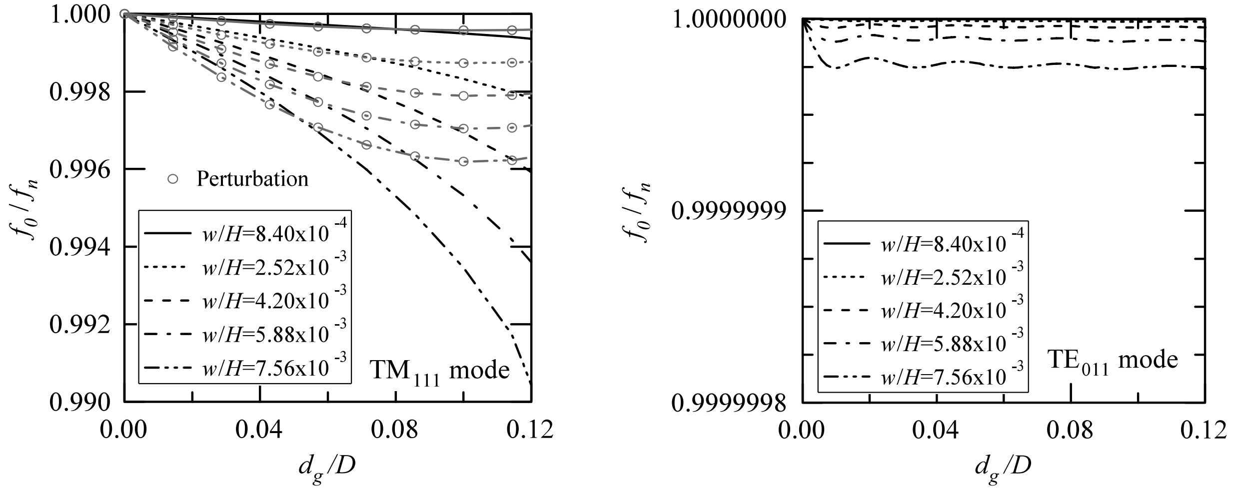
The experiments are conducted with two cavities. The first cavity is the one with grooves described above. Then the planer copper plates at the both ends shown in Fig. 1 are replaced with two convex copper plates shown in Fig. 7. In this case, a cavity without grooves



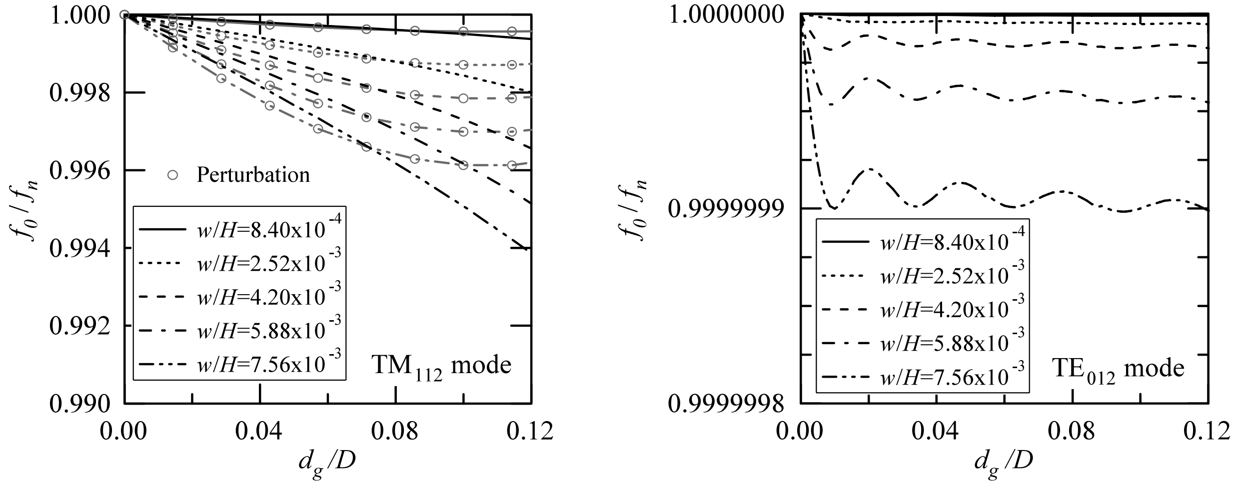
(a) TE$_{011}$ mode

(b) TM$_{111}$ mode

Fig. 5 The convergence of the solution of the TE$_{011}$ and TM$_{111}$ modes with number of expansion modes N. (D = 7.0 mm, H = 23.8 mm, d_g = 0.2 mm, w = 0.2 mm)



(a) TM_{111} mode (b) TE_{011} mode



(c) TM_{112} mode (d) TE_{012} mode

Fig. 6 Variation of the resonant frequencies of the TE and TM modes with the groove depth d_g and width w.

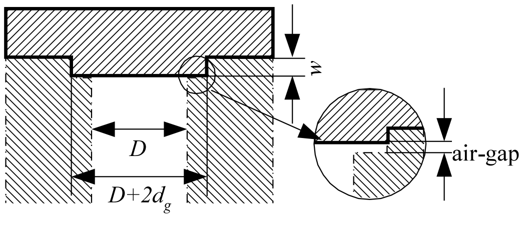


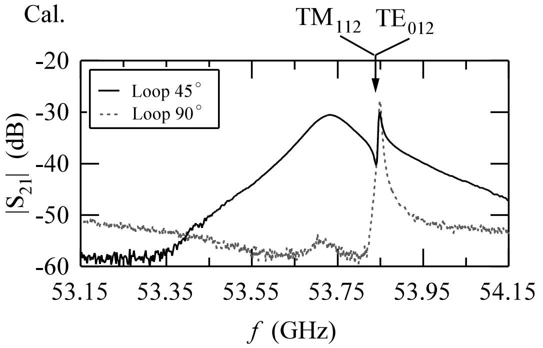
Fig. 7 Cross sectional view of a convex copper plate.

having D=6.990 mm, H=23.470 mm is formed.

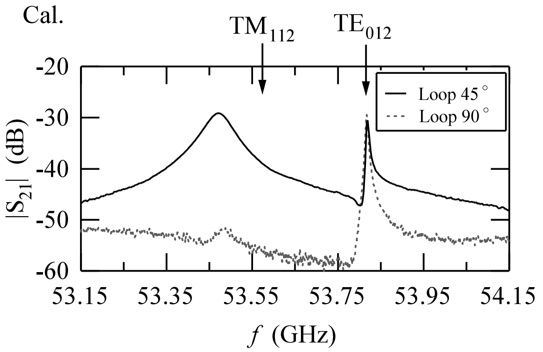
These cavities are excited and detected by a pair of coaxial cables with small loops at their top ends. The plane of the small loops is rotated by 45 degrees to z-axis to excite both the $\mathrm{TE}_{01\mathrm{p}}$ and $\mathrm{TM}_{11\mathrm{p}}$ modes. The measured results are shown in Figs. 8(a) and (b) for the cavities with and without grooves, respectively. The resonant frequencies calculated from the computer program are indicated on the top of Fig. 8. Moreover, the comparison between calculated and measured f_0 values for the TM_{112} mode is shown in Table 1.

In Fig. 8(a), it is seen that the degenerate TM_{112} mode is separated from the TE_{012} mode due to the air-gap effect between the convex copper plate and the cylinder. On the other hand, it is found from Fig. 8(b) that the f_0 of the TE_{012} mode is shifted a little due to the reduction of H from 23.770 mm to 23.470 mm. The resonant frequency of the TM_{112} mode is decreased significantly, and is very close to the designed value.

In actual measurement for dimensions and relative conductivity, the plane of the small loops is rotated by 90 degree to z-axis to excite only the $\mathrm{TE}_{01\mathrm{p}}$ modes. The frequency responses are shown in Figs. 8(a) and (b) by dash lines, respectively. The comparison of measured results of cavity without and with grooves for the TE_{012} mode is shown in Table 2. The value of σ_r was calculated accurately from the value of Q_u measured for the $\mathrm{TE}_{01\mathrm{p}}$ mode by following equation,



(a) Cavity without grooves



(b) Cavity with grooves of $d_g = 0.6$ mm, $w = 0.15$ mm

Fig. 8 Frequency response measured for the TE_{012} and TM_{112} modes of the circular cavity and the resonant frequencies calculated by mode-matching method.

Table 1 Comparison between calculated and measured f_0 values for the TM_{112} mode.

Cavity Type & Dimension (mm)	Calculated f_0 (GHz)	Measured f_0 (GHz)	Error (%)
No groove D=6.990, H=23.470	53.847	53.733	0.2
Groove d_g=0.6, w=0.15 D=6.990, H=23.770	53.567	53.471	0.2
Shift of f_0 (GHz)	0.280	0.262	6.4
$\Delta f_0 / f_0$ (%)	0.52	0.49	6.1

Table 2 Comparison between cavity without grooves and with grooves for the TM_{012} mode.

Cavity Type	Calculated f_0 (GHz)	Measured f_0 (GHz)	Measured Q_u	Measured σ_r (%)
No groove D=6.990, H=23.470	53.847	53.842 ±0.001	10940 ±950	73.3 ±9.0
Groove d_g=0.6, w=0.15 D=6.990, H=23.770	53.809	53.816 ±0.001	11450 ±290	84.7 ±2.9

$$\sigma_r = \frac{4\pi f_{0p} Q_{up}^2 \left\{ j_{01}'^2 + 2\left(p\pi\right)^2 \left(\frac{D}{2H}\right)^3 \right\}^2}{\sigma_0 \mu_0 c^2 \left\{ j_{01}'^2 + \left(\frac{p\pi D}{2H}\right)^2 \right\}^3}$$

It is found that the cavity with grooves has high Q_u value and small measurement error, compared with one without grooves. As a result, the value of σ_r of this cavity is determined 84.7%.

Table 3 Measured result of some low loss dielectric planes by using cavity with grooves ($\sigma_r = 84.7\%$).

Sample	thickness (mm)	f_0 (GHz)	Q_u	ε_r	$\tan\delta$ ($\mathrm{x}10^{-4}$)
PTFE	1.073 ±0.004	46.571 ±0.001	5020 ±70	2.016 ±0.005	1.65 ±0.05
Crythnex	0.823 ±0.046	46.645 ±0.003	4240 ±60	2.333 ±0.011	2.61 ±0.07
Modified polyolefin	2.050 ±0.001	38.010 ±0.001	3950 ±100	2.310 ±0.002	1.29 ±0.08
Modified polystyrene	1.178 ±0.001	42.833 ±0.002	1620 ±70	2.472 ±0.003	7.22 ±0.37
Polyimide	0.515 ±0.001	46.833 ±0.002	340 ±10	3.083 ±0.006	62.0 ±1.0
MgO	0.505 ±0.001	31.785 ±0.001	9030 ±70	9.801 ±0.017	0.19 ±0.02
Sapphire	0.524 ±0.001	32.032 ±0.001	8310 ±40	9.354 ±0.015	0.28 ±0.05
GaAs	0.108 ±0.001	45.050 ±0.001	4970 ±10	12.79 ±0.11	2.77 ±0.08
$\mathrm{LaAlO_3}$	0.521 ±0.001	21.054 ±0.002	6770 ±50	24.04 ±0.04	0.38 ±0.03
BMT	0.508 ±0.001	21.127 ±0.001	5640 ±10	24.34 ±0.04	0.72 ±0.01

Some low loss dielectric plates were measured by using this cavity. The values of relative permittivity ε_r and loss tangent $\tan\delta$ can be calculated from the measured values of the f_0 and Q_u of the TE_{011} mode [3], [4]. The measured results are shown in Table 3. When the value of $\sigma_r = 73.3\%$ is used to calculate $\tan\delta$, the value of $\tan\delta$ of PTFE plate is 1.51×10^{-4} and the one of sapphire plate is 0.20×10^{-4}. These errors of the correct value are about 8.5% and 28.6%. It is found that $\tan\delta$ can be evaluated accurately when the cavity with grooves is used.

5. Conclusion

It is verified numerically and experimentally that the grooved circular cavity is useful to separate degenerate TM_{11p} mode from the TE_{01p} mode. The computer program by the mode matching method is powerful in designing such a grooved circular cavity.

Acknowledgements

The authors wish to thank Mr. M. Kato for his assistance in the measurement. This work is supported in part by the Grant-in-Aid for Scientific Research (KAKENHI14550318) from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

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Takashi Shimizu was born in Tokyo, Japan on January 6, 1977. He received the B.E. and M.E. degrees in electrical and electronic engineering from Saitama University, Saitama, Japan, in 1999 and 2001, respectively. Now, he is a doctor course student at the same university. His current main interests include measurements of low-loss dielectric materials in microwave and millimeter-wave region. He is a student member of the Institute of Electrical and Electronics Engineers, Inc.

Zhewang Ma was born in Anhui, China, on July 7, 1964. He received the B.Eng. and M.Eng. degrees from the University of Science and Technology of China (USTC), Hefei, China, in 1986 and 1989, respectively. In 1995, he was granted the Dr. Eng. degree from the University of Electro-Communications, Tokyo, Japan. He was a Research Assistant in 1996, in the Department of Electronic Engineering, the University of Electro-Communications, and became an Associate Professor there in 1997. Since 1998, he has been an Associate Professor in the Department of Electrical and Electronic Systems, Saitama University, Japan. From 1985 to 1989, he was engaged in research works on dielectric waveguides, resonators and leaky-wave antennas. From 1990 to 1997, he did studies on computational electromagnetics, analytical and numerical modeling of various microwave and millimeter wave transmission lines and circuits. His current research works are mainly on the design of microwave and millimeter wave filters, measurements of dielectric materials and high temperature superconductors. He received Japanese Government (Ministry of Education, Culture, Sports, Science and Technology) Graduate Scholarship from 1991 to 1993. He was granted the URSI Young Scientist Award in 1993. From 1994 to 1996, he was a Research Fellow of the Japan Society for the Promotion of Science (JSPS). Dr. Ma is a member of IEEE. He has served on the Editorial Board of IEEE Transactions on Microwave Theory and Techniques, Review Board of IEEE Microwave and Wireless Components Letters. He was a member of the Steering Committee for 2002 Asia Pacific Microwave Conference (APMC2002) held in Kyoto, Japan.

Yoshio Kobayashi was born in Japan on July 4, 1939. He received the B.E., M.E., and D.Eng. degrees in electrical engineering from Tokyo Metropolitan University, Tokyo, Japan, in 1963, 1965, and 1982, respectively. Since 1965, he has been with Saitama University, Saitama, Japan. He is now a professor at the same university. His current research interests are in dielectric resonators and filters, measurements of low-loss dielectric and high-temperature superconductive (HTS) materials, and HTS filters, in microwave and millimeter wave region. He served as the Chair of the Technical Group on Microwaves, IEICE, from 1993 to 1994, as the Chair of the Technical Group of Microwave Simulators, IEICE, from 1995 to 1997, as the Chair of Technical Committee on Millimeter-wave Communications and Sensing, IEE Japan, from 1993 to 1995, as the Chair of Steering Committee, 1998 Asia Pacific Microwave Conference (APMC'98) held in Yokohama, as the Chair of the National Committee of APMC, IEICE from 1999 to 2000, and as the Chair of the IEEE MTT-S Tokyo Chapter from 1995 to 1996. He also serves as a member of the National Committee of IEC TC49 since 1991, the Chair of the National Committee of IEC TC49 WG10 since 1999 and a member of the National Committee of IEC TC90 WG8 since 1997. Prof. Kobayashi received the Inoue Harushige Award on "Dielectric filters for mobile communication base stations" in 1995. He is a Fellow of IEEE and a member of IEE Japan.