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Analysis of Dielectric Resonators Using the FDTD Method Combined with the Pade Interpolation Technique

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SUMMARY The finite-difference time-domain (FDTD) method incorporating Berenger's PML absorbing boundary condition is developed to model three-dimensional dielectric resonators. The fast Fourier transform (FFT) coupled with the Pade interpolation technique is employed to obtain frequency domain results with satisfactory resolution and accuracy, and to reduce the computation time significantly compared with that needed when the conventional FFT algorithm is used. Computed resonant frequencies of two types of cylindrical dielectric resonators are compared with theoretical and measured results. A good agreement is observed.

key words: *dielectric resonator, FDTD method, Pade approximation, interpolation method*

1. Introduction

Accurate analysis of dielectric resonators is of great importance because dielectric resonators are widely used in various microwave circuits, like filters and oscillators. The mode-matching method has been extensively used for the characterization of dielectric resonators because it is accurate and computationally efficient [1], [2]. However, for arbitrarily shaped three-dimensional dielectric resonators, this method can be hardly applied. In recent years, the finite-difference time-domain (FDTD) method has been increasingly used to analyze various types of microwave circuits, including the dielectric resonators [3]–[7]. To obtain the resonant frequencies of dielectric resonators by the FDTD method, we need to transform the temporal response of the FDTD simulation to frequency domain results by using, generally speaking, the fast Fourier transform (FFT). The resolution of the frequency response by the FFT is reciprocal to the product of the number of FDTD iterations and the time step size. Therefore, in order to achieve a reasonably good frequency resolution, it is necessary to conduct the FDTD simulation for a sufficiently long time.

To reduce the computation time of resonant structures by the FDTD method, a number of signal processing techniques have been introduced. These include the Prony's method [4] and the matrix pencil (MP) method [8], [9]. Roughly speaking, these methods are based on fitting the early time-domain response to a model. This allows the extrapolation of the future temporal response from the past result of FDTD computation. As an alternative, the Pade interpolation method was employed in [10] for analyzing rectangular conductor cavities.

In this paper, we develop a three dimensional FDTD algorithm for accurate characterization of dielectric resonators. For open structured dielectric resonators, Berenger's perfectly matched layer (PML) absorbing boundary condition is employed to reduce the computation space. The fast Fourier transform coupled with the Pade interpolation technique [10], [11] is employed to obtain frequency domain results with significantly improved resolution and much reduced computation time compared with those by the conventional FFT algorithm. Computed resonant frequencies of two types of cylindrical dielectric resonators are compared with theoretical and measured results, and a good agreement is found.

2. Theory

To start the FDTD simulation of a dielectric resonator, we excite one electric field component at one cell inside the dielectric resonator by using a Gaussian pulse with a desired spectrum width. Next we update alternatively the electric and magnetic field components at all of the cells in the computation space by using Yee's algorithm. The iteration process is continued for a certain number of time steps. The electric field component recorded each time step at one selected observing cell inside the resonator is used as the desired time domain response. This temporal response is then transformed to frequency response by using the FFT. The frequencies corresponding to the local maximums of the frequency response are the resonant frequencies of the dielectric resonator. In order to get the resonant frequencies accurately, a high resolution of the frequency response is necessary. This in turn requires a large number of iterations because the resolution of the frequency response Δf is determined by $\Delta f = 1 / (N_{max} \times \Delta t)$, here N_{max} is the maximum number of iterations, Δt the time step.

In order to improve the resolution of the frequency response obtained by the FFT, we employ the Pade interpolation technique [10], [11]. Assume that the complex FFT output obtained from the time domain response is $P(\omega)$. Then according to Pade approximation, $P(\omega)$ can be expressed by a rational function as

$$P(\omega) = \frac{Q_N(\omega)}{D_M(\omega)} \tag{1}$$

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where $Q_N(\omega)$ and $D_M(\omega)$ are polynomials of angular frequency ω expressed by

$$Q_N(\omega) = \sum_{i=0}^{N} \alpha_i \, \omega^i \tag{2}$$

$$D_M(\omega) = \sum_{i=0}^{M} \beta_i \, \omega^i \tag{3}$$

To solve the unknown complex coefficients α_i and β_i in the above expressions, we substitute the FFT output $P(\omega_j)$ into (1) and have

$$P(\omega_j)\, D_M(\omega_j) = Q_N(\omega_j), \quad j = 0, 1, \cdots, N_s \tag{4}$$

here N_s is the number of sampled data from the FFT output.

In the Pade approximation, β_0 is usually set to equal unity, then (4) is rewritten as

$$P(\omega_j)\sum_{i=1}^{M} \beta_i \, \omega_j^i - \sum_{i=0}^{N} \alpha_i \, \omega_j^i = -P(\omega_j), \quad j = 0, 1, \cdots, N_s \tag{5}$$

It is seen from (5) that the $M+N+1$ unknown coefficients of the Pade approximation can be solved from a system of linear inhomogeneous complex equations. In this paper, M is chosen equal to N, therefore, $N_s=2N+1$ data samples are required for solving the unknown coefficients.

The matrix coefficients of the linear equations in (5) contain some power of the frequency, ω_j^i. For large values of M's and N's, the value of ω_j^i will become too large to be kept within the allowed ranges of computer variables. To avoid this problem, we rewrite (2), (3) and (5) as follows:

$$Q_N(\omega) = \sum_{i=0}^{N} \alpha_i \cdot \omega_0^i \cdot \left(\frac{\omega}{\omega_0}\right)^i = \sum_{i=0}^{N} \alpha_i' \cdot \left(\frac{\omega}{\omega_0}\right)^i \tag{6}$$

$$D_M(\omega) = \sum_{i=0}^{N} \beta_i \cdot \omega_0^i \cdot \left(\frac{\omega}{\omega_0}\right)^i = \sum_{i=0}^{N} \beta_i' \cdot \left(\frac{\omega}{\omega_0}\right)^i \tag{7}$$

and

$$P(\omega_j)\sum_{i=1}^{M} \beta_i' \cdot \left(\frac{\omega_j}{\omega_0}\right)^i - \sum_{i=0}^{N} \alpha_i' \cdot \left(\frac{\omega_j}{\omega_0}\right)^i = -P(\omega_j), \quad j = 0, 1, \cdots, N_s \tag{8}$$

where $\alpha_i' = \alpha_i \times \omega_0^i$, $\beta_i' = \beta_i \times \omega_0^i$, and ω_0 is the central angular frequency between the maximum and minimum angular frequencies of the samples used. After the unknown coefficients α_i' and β_i' are solved from (8), we can then calculate the frequency response with desired resolution by using (6) and (7), and substituting them into (1).

3. Numerical Results

Figure 1 shows a dielectric rod resonator sandwiched between two parallel perfect conductor plates. The structural parameters are R=5.25 mm, h=4.6 mm, and ε_r=38. The PML absorbing boundaries are placed on the four sides surrounding the cylindrical surface of the dielectric resonator to define the computation domain. Rectangular Yee cells are used to discretize the computation space with grid increments dx=0.25 mm, dy=0.23 mm, and dz=0.25 mm. The number of cells are $120 \times 20 \times 120$.

Figure 2 shows the temporal response of an electric field component inside the dielectric resonator. Figure 3 provides the corresponding frequency response calculated by the conventional FFT for 2^{12}=4096 time steps, and by the FFT-Pade interpolation technique with 13 input samples. For comparison, the two curves are normalized independently. It is seen that while the resonant frequencies, especially resonant fre-

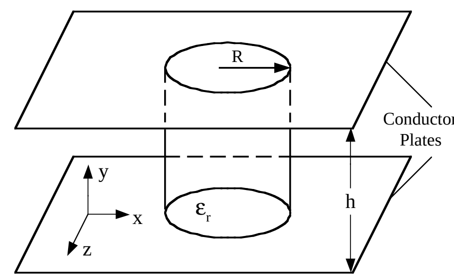

Fig. 1 A dielectric rod resonator sandwiched between two parallel conductor plates. R=5.25 mm, h=4.6 mm, and ε_r=38.

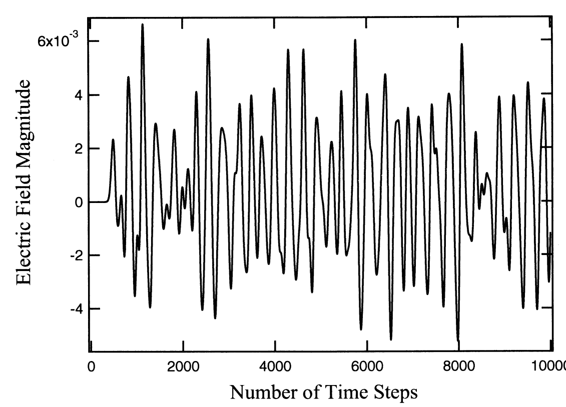

Fig. 2 Temporal response of an electric field component inside the parallel plate dielectric resonator.

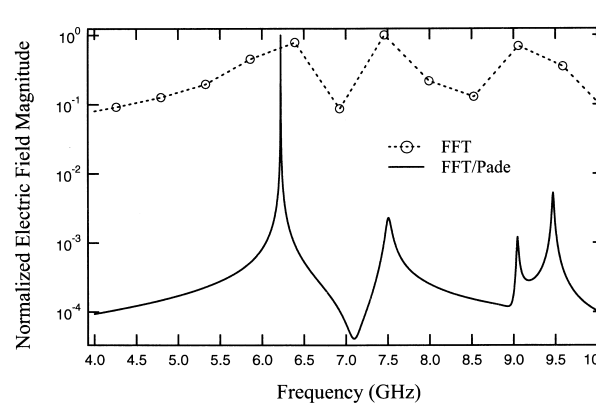

Fig. 3 Frequency response calculated by the conventional FFT for 2^{12}=4096 time steps, and by the FFT-Pade interpolation method with 13 input samples.

Table 1 Comparison of the resonant frequencies of the parallel plate dielectric resonator calculated by different methods.

Mode	Resonant Frequency (GHz)				
	Theoretical	FFT-Pade (2^{12} time steps)	Error (%)	FFT (2^{15} time steps)	Error (%)
HE_{111}	6.214	6.223	0.14	6.195	-0.31
HE_{211}	7.514	7.506	-0.11	7.527	0.17
HE_{311}	9.003	9.046	0.48	9.058	0.61
HE_{131}	9.499	9.471	-0.29	9.525	0.27

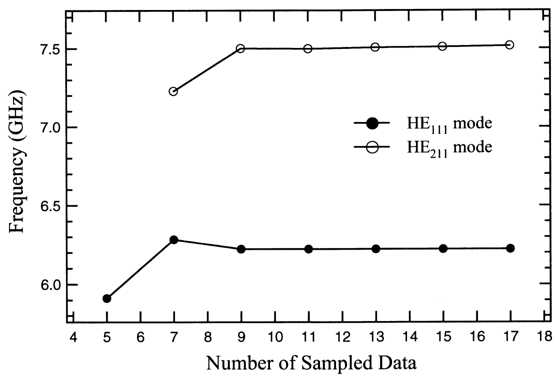

Fig. 4 Variation of the resonant frequencies of HE_{111} and HE_{211} modes with the number of data samples used in the Pade approximation for 2^{12}=4096 time iterations.

quencies close to each other, can be hardly resolved from the FFT output, they can be read accurately from the peak points in the curve calculated by the FFT-Pade method. The obtained resonant frequencies are listed in Table 1, where results by the FFT for 2^{15}=32768 iterations, as well as the theoretical results of [5], are also given. A comparison of the results reveals that the resonant frequencies by the FFT-Pade method for 2^{12}=4096 iterations have the same order of accuracy as that of the results computed by the FFT for 2^{15}=32768 iterations.

In the above computation, the transition part of the time-domain response is used in the FFT. If the transition part is not included in the FFT, our computation results show that the resonant frequency of HE_{111} mode by the FFT-Pade method has little variation. However, the resonant frequencies of higher HE modes can not be obtained accurately even by using the FFT-Pade method.

Figure 4 depicts the variation of resonant frequencies of HE_{111} and HE_{211} modes with the number of data samples used in the Pade approximation for 2^{12}=4096 iterations. When the number of data samples is equal to 9 or larger integers, the resonant frequencies of both the HE_{111} and HE_{211} mode show nearly no variation.

Figure 5 shows variation of the resonant frequency of the HE_{111} mode with the number of FDTD iterations. The resonant frequency is calculated by both the regular FFT and the FFT-Pade approximation approach with 13 data samples. It is evident that with a small number of FDTD iterations, e.g., 2^{11}=2046 time steps, the FFT-Pade approach can yield

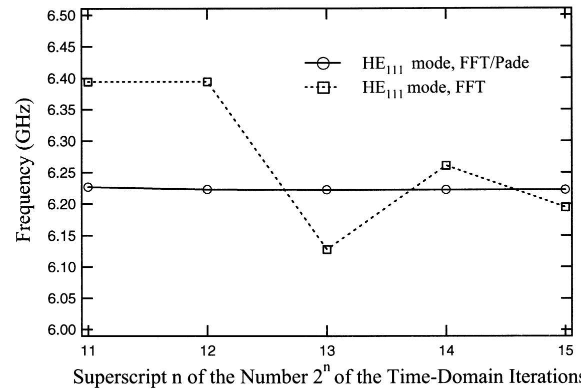

Fig. 5 Variation of the resonant frequency of the HE_{111} mode with the number of FDTD iterations. The resonant frequency is calculated by both the conventional FFT algorithm and the FFT-Pade approximation approach with 13 data samples.

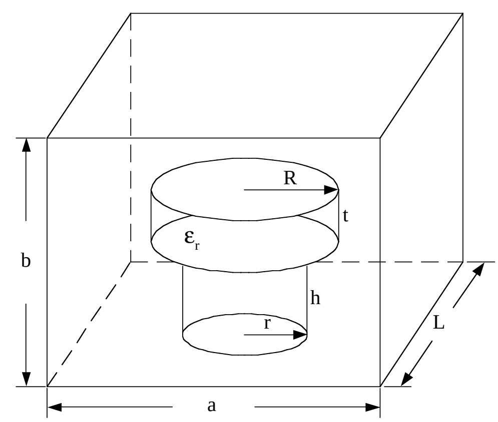

Fig. 6 Cylindrical dielectric resonator placed inside a conductor cavity. a=1″, b=1″, r=0, h=0.275″, L=0.92″, and ε_r=38.

results even more accurate than that by the conventional FFT with a much larger number of iterations, e.g., 2^{15}=32768 iterations.

The second structure examined in this paper is a cylindrical dielectric resonator placed inside a perfect conductor cavity, as shown in Fig. 6. The structural parameters are a=1″, b=1″, r=0, h=0.275″, L=0.92″, and ε_r=38. Two dielectric resonators with different geometrical dimensions are calculated by the FFT-Pade method with 13 data samples for 2^{13}=8192 iterations. The frist resonator has a diameter $2R$=0.689″ and a thickness t=0.23″. The grid increments of FDTD cells are dx=dz=$2R$/70=0.009843″ and dy=t/23=0.01″. The second resonator has a diameter $2R$=0.767″ and a thickness t=0.253″. The grid increments are dx=dz=$2R$/78=0.009705″ and dy=t/23=0.011″. The resonant frequencies of the HE_{111} modes of the resonators are given in Table 2, and are compared with those calculated by the mode-matching method, as well as the measured data [2]. A very good agreement is also observed. The minor discrepancies are mainly caused by two reasons. First, as the orthogonal uniform Yee cells are used to discretize the computation space, the geometrical dimensions

Table 2 Resonant frequencies of the HE_{111} modes of the shielded dielectric resonators with different dimensions. The resonant frequencies are obtained by three different methods.

Dimensions		Resonant Frequency (GHz)		
2R (inch)	t (inch)	Mode-Matching Method [2]	Present FDTD	Measured [2]
0.689	0.23	4.1605	4.136	4.153
0.767	0.253	3.721	3.737	3.777

of the simulated object are not exactly the same as those assigned above. Second, the curvilinear surface of the cylindrical dielectric resonator is approximated by staircases.

4. Conclusions

A three-dimensional FDTD method combined with the FFT and the Pade interpolation technique was developed for accurate characterization of dielectric resonators. Compared with the conventional FFT, the coupled FFT-Pade interpolation method can yield frequency response with significantly improved resolution. The number of the FDTD iterations and thereby the computation time are considerably reduced without sacrificing the accuracy of the obtained resonant frequencies. The method was verified by numerical examples. The computed resonant frequencies of two types of cylindrical dielectric resonators agreed well with previously published theoretical and measured results.

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