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On the Error of the Unloaded Q-Factor of a Transmission-Type Resonator Measured by the Insertion Loss Method and the Return Loss Method

Zhewang MA[†a)], ***Regular Member and*** **Yoshio KOBAYASHI**[†], ***Fellow***

SUMMARY Two type measurement methods of the unloaded Q-factor of a microwave resonator, the insertion loss method and the return loss method, are reexamined theoretically and compared experimentally. An error formula is derived to estimate the errors between the unloaded Q-factors measured by the two different methods. Measured results of a stripline resonator verified well the derived formula, and proved that the return loss method is more accurate and reliable than the traditional insertion loss method.

key words: *microwave resonator, Q measurement*

1. Introduction

In the measurements of dielectric materials and superconductors, accurate determination of the unloaded Q-factor of a microwave resonator is of paramount importance [1]–[5]. A transmission type circuit configuration and a network analyzer are usually used in the measurement. After we get the loaded Q-factor Q_L from the measured resonant frequency and the 3-dB bandwidth of the transmitted signal, we can then determine the unloaded Q-factor Q_u from Q_L by choosing two different methods. The first method requires the measurement of insertion loss only at the resonant frequency, and is known as the insertion loss method. The second method demands the measurement of the return loss at both the input and output of the resonator, and is referred later as the return loss method.

Although the insertion loss method is widely accepted and used in the Q-measurements, it is valid only when the couplings at the input and output of the resonator are equal to each other. However, in many practical measurements, this condition is very difficult to be satisfied, and the unequal couplings may make the measured results unreliable [3]. Instead of the insertion loss method, the return loss method is considered to be more accurate and is recommended by some researchers [1], [4]. However, neither theoretical error analysis was made to convince the readers, nor comparison of the Q_u values measured by the two different methods was provided to support the recommendations. A transmission-mode Q factor technique (TMQF) is described in [5], which used a complicated circuit model of the test fixture in order to take into account of noise, crosstalk between the input and output, the coupling loss and reactance, as well as unequal couplings at the input and output. Measurements of S_{11}, S_{22}, and S_{21} parameters at multiple frequencies are needed, and processing of these S-parameters, including fitting to the Q circles and phase correction, is also required. As a result, while this method can provide precise measurements of Q factors, it is time-consuming and not easy to conduct.

In this paper, we make an error analysis of the unloaded Q-factors of a transmission-type resonator measured by the insertion loss method and the return loss method. In Sect. 2, after brief derivation of the insertion loss method and the return loss method for measuring Q_u, an error-formula is provided to estimate the errors of Q_u measured by using the two methods. The simulated results show that the weaker the coupling at the input and output of the resonator, the bigger the errors between the two methods. In Sect. 3, the unloaded Q_u of a stripline circular patch resonator is measured by both the insertion loss method and the return loss method with unequal couplings at the input and output. It is found that the errors of the measured Q_u by the two different methods agree well with the theoretical predictions by the error-formula derived in Sect. 2. Examinations of the values of the measured Q_u reveal also that the return loss measurement method provides more reliable results than the insertion loss method.

2. Analysis

Figure 1 shows the equivalent circuit of a transmission-type resonator. The series RLC resonator has a resonant frequency f_0 and an unloaded Q_u. It is coupled with external circuits through ideal transformers with transform ratios 1:n_1 and 1:n_2, respectively.

In order to obtain the insertion loss and return loss characteristics of the resonator, we derive the $ABCD$ matrix of the circuit by multiplying the $ABCD$ matrices of the cascaded left-hand transformer, the series RLC resonator, and the right-hand transformer in

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[†]The authors are with Department of Electrical and Electronic Systems, Saitama University, Saitama-shi, 338-8570 Japan.
a) E-mail: ma@ees.saitama-u.ac.jp

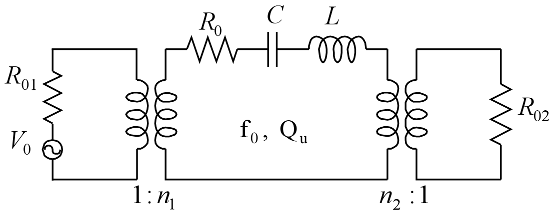

Fig. 1 Equivalent circuit of a transmission-type resonator.

Fig. 1 in sequence [6], [7]. We get the following expression:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \dfrac{n_2}{n_1} & \dfrac{R_0}{n_1 n_2}\left[1 + jQ_u\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)\right] \\ 0 & \dfrac{n_1}{n_2} \end{bmatrix} \tag{1}$$

where

$$\omega_o = \frac{1}{\sqrt{LC}}, \quad Q_u = \frac{\omega_o L}{R_0} \tag{2}$$

By substituting the above A, B, C, and D into the conversion equations between $ABCD$ matrix and scattering matrix [8], we get the reflection and transmission coefficients of the resonator. At the resonant angular frequency ω_0, we have

$$S_{11}(\omega_0) = \frac{1 - \beta_1 + \beta_2}{1 + \beta_1 + \beta_2} \tag{3a}$$

$$S_{21}(\omega_0) = \frac{2\sqrt{\beta_1\beta_2}}{1 + \beta_1 + \beta_2} \tag{3b}$$

$$S_{22}(\omega_0) = \frac{1 + \beta_1 - \beta_2}{1 + \beta_1 + \beta_2} \tag{3c}$$

where the external coupling coefficients β_1 and β_2 at the input and output are defined as

$$\beta_1 = \frac{n_1^2 R_{01}}{R_0}, \quad \beta_2 = \frac{n_2^2 R_{02}}{R_0} \tag{4}$$

Also we have

$$|S_{21}(\omega)|^2 = \frac{|S_{21}(\omega_0)|^2}{1 + Q_L^2\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^2} \tag{5}$$

In (5), if we choose two frequencies $\omega_1 = 2\pi f_1$ ($\omega_1 < \omega_0$) and $\omega_2 = 2\pi f_2$ ($\omega_2 > \omega_0$) at which

$$\frac{|S_{21}(\omega_0)|^2}{|S_{21}(\omega_{1\mathrm{or}2})|^2} = 1 + A^2\,(A > 0) \tag{6}$$

then

$$Q_L\left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1}\right) = -A \tag{7a}$$

$$Q_L\left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right) = A \tag{7b}$$

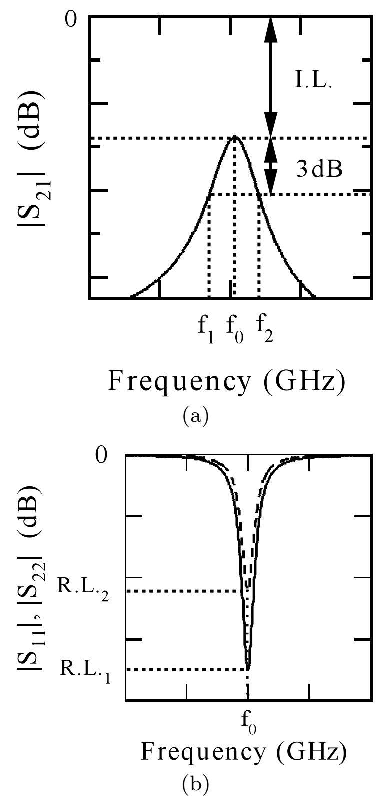

Fig. 2 (a) Transmission coefficient of a resonator, with the resonant frequency f_0 and the 3-dB frequencies f_1 and f_2. (b) Reflection coefficients with the return losses $R.L._1$ and $R.L._2$ at the resonant frequency f_0.

By adding and subtracting (7a) and (7b), we get

$$\omega_0 = \sqrt{\omega_1 \omega_2} \tag{8}$$

$$Q_L = \frac{A\omega_0}{\omega_2 - \omega_1} \tag{9}$$

Equation (9) indicates that the loaded Q_L of a resonator can be obtained by measuring the transmission coefficient S_{21}(related to A by (6)) at two arbitrary frequencies, ω_1 and ω_2, at which $|S_{21}(\omega_1)| = |S_{21}(\omega_2)|$. A special case is to measure the half-power (the 3-dB) frequencies f_1 and f_2, at which $A = 1$, i.e., $|S_{21}(\omega_{1\mathrm{or}2})|^2/|S_{21}(\omega_0)|^2 = 1/2$ (referring to (6)). Equation (9) is then simplified to the following well-known formula

$$Q_L = \frac{\omega_0}{\Delta\omega_{3\,\mathrm{dB}}} = \frac{f_0}{\Delta f_{3\,\mathrm{dB}}} \tag{10}$$

where $\Delta\omega_{3\,\mathrm{dB}} = 2\pi\Delta f_{3\,\mathrm{dB}} = 2\pi(f_2 - f_1)$. Figure 2(a) illustrates a typical transmission coefficient curve of a resonator, with the resonant frequency f_0 and the 3-dB frequencies f_1 and f_2 indicated.

From (2) and (4), it is straightforward to get

$$Q_L = \frac{\omega_0 L}{R_0 + n_1^2 R_{01} + n_2^2 R_{02}} = \frac{Q_u}{1 + \beta_1 + \beta_2} \tag{11}$$

By combining (3a), (3c), and (11), we get

$$Q_u = Q_L(1+\beta_1+\beta_2) = \frac{2Q_L}{S_{11}(\omega_0)+S_{22}(\omega_0)} = \frac{2Q_L}{10^{(-R.L._1/20)}+10^{(-R.L._2/20)}} \quad (12)$$

where $R.L._1$ and $R.L._2$ are the return loss in dB at the input and output, respectively, measured at the resonant frequency f_0. The above equation shows that the unloaded Q_u of a resonator can be obtained by measuring the return losses at both the input and output of the resonator. The formula (12) is thereby named as the return loss formula, and the measurement method the return loss method. Figure 2(b) shows typical reflection coefficient curves of a resonator, with the return losses $R.L._1$ and $R.L._2$ at the resonant frequency f_0 illustrated.

When the couplings of the resonator with external circuits at the input and output are equal to each other, $\beta_1 = \beta_2$, i.e., $S_{11}(\omega_0) = S_{22}(\omega_0)$, then from (3) we have $1-S_{21}(\omega_0) = [S_{11}(\omega_0)+S_{22}(\omega_0)]/2$. Formula (12) then becomes

$$Q'_u = \frac{Q_L}{1-S_{21}(\omega_0)} = \frac{Q_L}{1-10^{(-I.L./20)}} \quad (13)$$

where $I.L.$ is the insertion loss of the resonator at the resonant frequency f_0. In (13), we use Q'_u to make a difference with Q_u in formula (12). Equation (13) is the well-known insertion loss formula for measuring the unloaded Q-factor. It reveals that we need only to measure the insertion loss $I.L.(\omega_0)$ of the resonator to determine Q_u from the measured Q_L.

From the above derivation, we see that the insertion loss formula (13) is valid only when the couplings at the input and output are equal to each other. This assumption is, however, very difficult to be satisfied in many measurements, even careful adjustment of the couplings is made. With unequal couplings at the input and output, the measured Q'_u by the insertion loss formula (13) will have a different value with Q_u measured by the return loss formula (12). The error between Q'_u and Q_u is derived and expressed by the following formula:

$$\frac{Q'_u}{Q_u} - 1 = \frac{S_{11}(\omega_0)+S_{22}(\omega_0)}{2[1-S_{21}(\omega_0)]} - 1 = \frac{\Delta r + 2S_{22}(\omega_0)}{2[1-S_{21}(\omega_0)]} - 1 \quad (14)$$

where

$$\Delta r = S_{11}(\omega_0) - S_{22}(\omega_0) \quad (15)$$

From (3), we have

$$S_{21}^2(\omega_0) = [1-S_{11}(\omega_0)][1-S_{22}(\omega_0)] = 1 - \Delta r - 2S_{22}(\omega_0) + \Delta r S_{22}(\omega_0) + S_{22}^2(\omega_0) \quad (16)$$

Then $S_{22}(\omega_0)$ is solved from (16) as

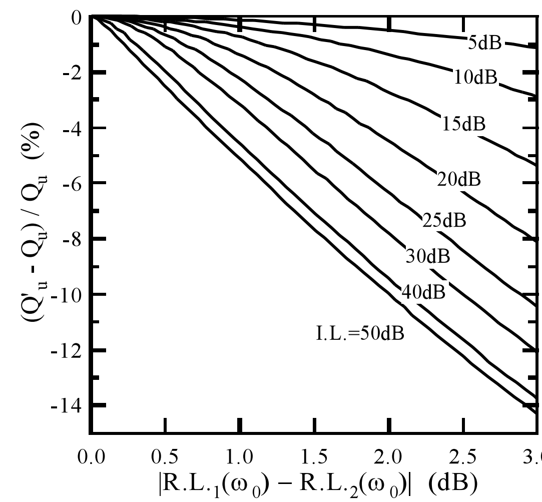

Fig. 3 The calculated error between Q'_u and Q_u versus $|R.L._1(\omega_0) - R.L._2(\omega_0)|$. The insertion loss $I.L.(\omega_0)$ is varied from 5 dB to 50 dB.

$$S_{22}(\omega_0) = 1 - \Delta r/2 - \sqrt{S_{21}^2(\omega_0) + (\Delta r/2)^2} \quad (17)$$

Substitute (17) into (14), we have

$$\frac{Q'_u - Q_u}{Q_u} = \frac{S_{21}(\omega_0) - \sqrt{S_{21}^2(\omega_0) + [S_{11}(\omega_0) - S_{22}(\omega_0)]^2/4}}{1 - S_{21}(\omega_0)} \quad (18)$$

It becomes evident from (18) that the error between Q'_u and Q_u is a function of $S_{21}(\omega_0)$ and the difference $S_{11}(\omega_0) - S_{22}(\omega_0)$. In the case of undercouplings, i.e., when the value of $S_{21}(\omega_0)$ is very small, $Q'_u - Q_u$ becomes approximately proportional to $|S_{11}(\omega_0) - S_{22}(\omega_0)|$. If $S_{11}(\omega_0) = S_{22}(\omega_0)$, then $Q'_u = Q_u$.

Numerical results calculated by (18) are drawn in Fig. 3. Instead of $|S_{11}(\omega_0) - S_{22}(\omega_0)|$, the return loss difference $|R.L._1(\omega_0) - R.L._2(\omega_0)|$ in dB is chosen as the x-axis for convenience, because the return loss in dB can be read directly on an network analyzer. In Fig. 3, the insertion loss $I.L.(\omega_0)$ is varied from 5 dB to 50 dB. It is seen that the error between Q'_u and Q_u increases monotonously with the increase of $|R.L._1(\omega_0) - R.L._2(\omega_0)|$. Also it becomes evident that the larger the insertion loss, the bigger the error between Q'_u and Q_u. This result is important because in many Q measurement experiments, we try to use very weak input and output couplings in order to make the influence of external circuits on the resonator ignorable. In these cases, if we like to use the insertion loss formula, we should make careful adjustment of the input and output couplings to reduce the error introduced by the unequal couplings.

3. Measurements

To verify the error formula (18) and the numerical results in Fig. 3, we measured the unloaded Q-factor of a stripline circular patch resonator. The resonator, as shown in Fig. 4, is built by sandwiching a circular conductor patch between two dielectric (TEFLON) substrates. The TEFLON substrates are further sandwiched between two conductor plates. The circular conductor patch has a diameter $D = 23.5$ mm and a thickness $t = 50\,\mu$m. The TEFLON substrate has a dielectric constant ε_r=2.03 and a thickness $d = 1.006$ mm. Coaxial lines coaxially connected to the circular patch are used as the input and output feeds. TM_{010} mode is excited in the resonator for measurements. The couplings at the input and output are controlled by adjusting the distances, L and L', between the coaxial lines and the dielectric substrates as shown in Fig. 4. A photograph of the resonator is given in Fig. 5.

The resonator is connected to a network analyzer HP8510B through coaxial cables. The measured resonant frequency is about 10.55 GHz. While keeping the insertion loss $I.L.(\omega_0)$ approximately 20, 30, and 40 dB, respectively, we measured three groups of data by varying the return loss at the input and output of the resonator, respectively. From the measured Q_L, the insertion loss $I.L.(\omega_0)$, and the return losses $R.L._1(\omega_0)$ and $R.L._2(\omega_0)$, we get Q_u from (12), Q'_u from (13), and then $Q'_u - Q_u$. The measured errors $(Q'_u - Q_u)/Q_u$ are depicted in Fig. 6 by squares, triangles, and circles, which correspond to $I.L.(\omega_0) \approx 20$, 30, and 40 dB, respectively. The theoretical predictions of $(Q'_u - Q_u)/Q_u$ by (20) are drawn in Fig. 6 by solid lines, and they agree well with the measured data.

In Table 1, one group of the measured data are provided. The insertion loss is kept approximately 20 dB. The return losses at the input and output of the stripline resonator are varied for each measurement. Examinations of the values of Q'_u and Q_u in Table 1 reveal that with unequal couplings at the input and output of the resonator, the value of Q'_u varies in a much larger range $(1128 - 1020 = 108)$ than that of Q_u

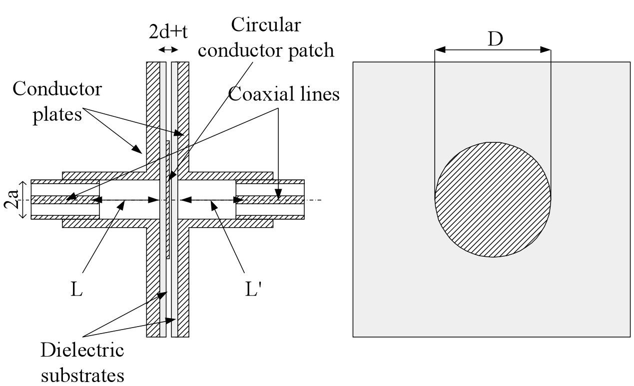

Fig. 4 Configuration of a stripline circular patch resonator operating at TM_{010} mode.

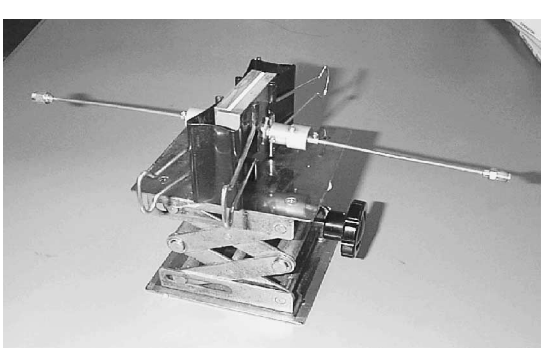

Fig. 5 A photograph of the stripline resonator illustrated in Fig. 4.

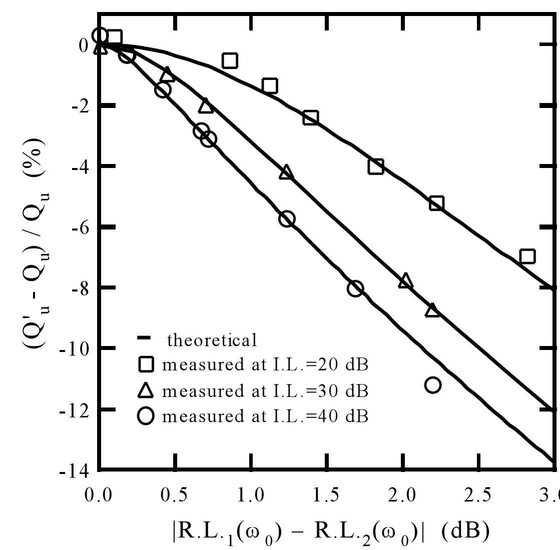

Fig. 6 Comparison of the measured and predicted errors between Q'_u and Q_u by the insertion loss and return loss method, respectively.

Table 1 Measured Q_L, Q'_u, and Q_u, with varied return loss at the input and output of the stripline resonator. The resonant frequency is about 10.55 GHz, and the insertion loss is kept approximately 20 dB.

I.L. (dB)	f_0 (GHz)	R.L.$_1$ (dB)	R.L.$_2$ (dB)	Q_L	Q'_u	Q_u
20.21	10.55	0.52	1.38	998.6	1107	1113
20.30	10.55	0.31	2.53	940.1	1041	1098
20.28	10.55	0.43	1.82	971.7	1076	1103
20.36	10.55	0.37	2.19	959.0	1061	1105
20.12	10.55	0.48	1.6	986.7	1095	1110
20.22	10.55	0.82	0.92	1017	1128	1125
20.16	10.54	0.23	3.05	920.1	1020	1097
Ave.					1075	1107
Error					±14	±4

$(1125 - 1097 = 28)$. The mean value of Q'_u and Q_u are 1075 and 1107, respectively, and their errors are ± 14 and ± 4, respectively. It is evident that with unequal couplings at the input and output of the resonator, the measured Q_u by the return loss formula has more stable values and smaller errors than the measured Q'_u by the traditionally used insertion loss formula.

4. Conclusions

The insertion loss method and the return loss method for measuring the unloaded Q-factor of a microwave resonator are reexamined theoretically and compared experimentally. An error formula is derived to estimate the errors between the unloaded Q-factors measured by the two different methods, and the simulated results make it evident that the weaker the coupling at the input and output of the resonator, the bigger the errors between the two methods. Measured results of a stripline resonator verified well the derived formula, and proved that the return loss method can provide more accurate and reliable unloaded Q-factors than the traditional insertion loss method.

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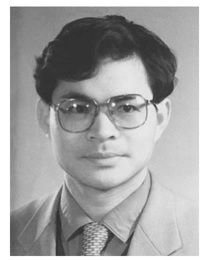

Zhewang Ma was born in Anhui, China, on July 7, 1964. He received the B.Eng. and M.Eng. degrees from the University of Science and Technology of China (USTC), Hefei, China, in 1986 and 1989, respectively. In 1995, he was granted the Dr. Eng. degree from the University of Electro-Communications, Tokyo, Japan. He was a Research Assistant in 1996, in the Department of Electronic Engineering, the University of Electro-Communications, and became an Associate Professor there in 1997. Since 1998, he has been an Associate Professor in the Department of Electrical and Electronic Systems, Saitama University, Japan. From 1985 to 1989, he was engaged in research works on dielectric waveguides, resonators and leaky-wave antennas. From 1990 to 1997, he did studies on computational electromagnetics, analytical and numerical modeling of various microwave and millimeter wave transmission lines and circuits. His current research works are mainly on the design of microwave and millimeter wave filters, measurements of dielectric materials and high temperature superconductors. He received Japanese Government (Ministry of Education, Culture, Sports, Science and Technology) Graduate Scholarship from 1991 to 1993. He was granted the URSI Young Scientist Award in 1993. From 1994 to 1996, he was a Research Fellow of the Japan Society for the Promotion of Science (JSPS). Dr. Ma is a member of IEEE. He has served on the Editorial Board of *IEEE Transactions on Microwave Theory and Techniques*, Review Board of *IEEE Microwave and Wireless Components Letters*, and Review Board of *IEICE Transactions on Electronics*, Japan. He was a member of the Steering Committee for 2002 Asia Pacific Microwave Conference (APMC2002) held in Kyoto, Japan.

Yoshio Kobayashi was born in Japan on July 4, 1939. He received the B.E., M.E., and D.Eng. degrees in electrical engineering from Tokyo Metropolitan University, Tokyo, Japan, in 1963, 1965, and 1982, respectively. Since 1965, he has been with Saitama University, Saitama, Japan. He is now a professor at the same university. His current research interests are in dielectric resonators and filters, measurements of low-loss dielectric and high-temperature superconductive (HTS) materials, and HTS filters, in microwave and millimeter wave region. He served as the Chair of the Technical Group on Microwaves, IEICE, from 1993 to 1994, as the Chair of the Technical Group of Microwave Simulators, IEICE, from 1995 to 1997, as the Chair of Technical Committee on Millimeter-wave Communications and Sensing, IEE Japan, from 1993 to 1995, as the Chair of Steering Committee, 1998 Asia Pacific Microwave Conference (APMC'98) held in Yokohama, as the Chair of the National Committee of APMC, IEICE from 1999 to 2000, and as the Chair of the IEEE MTT-S Tokyo Chapter from 1995 to 1996. He also serves as a member of the National Committee of IEC TC49 since 1991, the Chair of the National Committee of IEC TC49 WG10 since 1999 and a member of the National Committee of IEC TC90 WG8 since 1997. Prof. Kobayashi received the Inoue Harushige Award on "Dielectric filters for mobile communication base stations" in 1995. He is a Fellow of IEEE and a member of IEE Japan.