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Laser irradiated enhancement of the atomic electron capture rate in search of new physics

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Electron capture processes are important for new physics searches and therefore a high capture rate is inevitably desired. We investigate a possibility of enhancement of an atomic electron capture rate by irradiating laser beam to "an atom". The possibility of the enhancement is shown as a consequence of an enhanced electron wave function at origin $\Psi(0)$ through an effectively increased mass of the electron. We find that order of magnitude enhancement can be achieved using the laser with intensity 10^{10} W/mm^2, and energy of photon, of $\mathcal{O}(10^{-3})$ eV.

1. Introduction

A capture process has a potential power to search new physics. For example, instead of neutrinoless double beta decay, the process

$$(Z, A) + e^- \to (Z-2, A) + e^+, \tag{1}$$

where (Z, A) denotes a nucleus with the atomic number Z and the mass number A, can be used for investigation of Majorana property of neutrinos.[1,2,3] The process of muonium annihilation into two γ, $\mu + e \to 2\gamma$ is also a promising one for the search of lepton flavor violation. [4] Furthermore neutrinos from a beta capture process will be very powerful source for oscillation experiments since it has a definite energy[5,6].

A capture rate, Γ, is in general given by the form

$$\Gamma = |\Psi(0)|^2 \int d\text{LIPS} |\mathcal{M}|^2, \tag{2}$$

where $\Psi(\mathbf{r})$ is a wave function of an electron and $\int d\text{LIPS}$ is the Lorentz invariant phase space , and $\mathcal{M}$ is an amplitude of the capture process with a plane wave electron. To earn a statistics, a rapid capture process is welcome. The more rapid it is, the better an experiment will be. Then how can we get more rapid process? We cannot control the amplitude $\mathcal{M}$ since it is determined by the fundamental physics. It is completely calculated, at least in principle, by the Lagrangian of the world. Thus, if we have a chance to accelerate the process, we have to control the wave function of an electron.

In this paper we will show our attempt to control the wave function so that $|\Psi(0)|$ is larger than that of the uncontrolled case. That is we will show the possibility to get a higher capture rate by enhancing the wave function since $\Gamma \propto |\Psi(0)|^2$. The wave function is determined by the electron mass , $|\Psi(0)|^2 \propto m_{eff}^3$ where m_{eff} is the effective mass of an electron. Therefore if we make the electron mass effectively heavier , we would get a higher capture rate. In a medium, wave has a different wave length from that in vacuum, that is, in a term of particle, an electron will have a different mass from that in vacuum. It leads us to an idea that we should immerse the system of atoms in a medium. As a medium what is the most persuading is a photon bath.

With this intuition we are led to irradiate the system with a laser. As we have already mentioned, an electron is in a medium of photons under an irradiation. In a term of waves, de Broglie wave length of the electron becomes *"shorter"* than that in the Coulomb electric field produced by the nucleus. Hence the Bohr radii of electrons becomes smaller due to the Bohr's quantization condition. In a term of particles, an electron propagates with being scattered by photons in the laser. Therefore, the electron must come nearer to the nucleus to go around it without falling into the nucleus. The common feature in both picture is that the electron goes around the nucleus in a closer orbit under the irradiation. This means that the binding energy of the electron becomes larger. This fact can be interpreted as the electron has heavier mass because the binding energy of the electron is proportional to its mass. Thus we will have a larger value of wave function at the nucleus and hence we can get a higher capture rate. The idea of irradiating laser was proposed by Yoshimura[3]. Indeed our idea was strongly stimulated by his proposal. However our result and the interpretation of enhancing mechanism are completely different from his idea.

2. Non-relativistic Limit of the Dirac Equation

First, we derive a non-relativistic quantum equation with effective mass, m_{eff}, from the Dirac equation with a vector potential $\boldsymbol{A}(\boldsymbol{x}, t)$ of an irradiated laser and a Coulomb potential $\phi(\boldsymbol{x})$. Throughout this paper, we use MKS unit and write down each constants explicitly.

2.1. *The Dirac equation*

A Dirac equation with a vector potential and the Coulomb potential is given by

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{x},t)=\left[c\boldsymbol{\alpha}\cdot(\hat{\boldsymbol{p}}+e\boldsymbol{A}(\boldsymbol{x},t))+\beta m_e c^2-e\phi(\boldsymbol{x})\right]\Psi(\boldsymbol{x},t), \tag{3}$$

where $-e$ and m_e are the electric charge and the mass of bound electron in a vacuum and $\boldsymbol{\alpha}$ and β are the Dirac matrices. the Coulomb potential is

$$\phi(\boldsymbol{x})=\frac{Ze}{4\pi\epsilon_0|\boldsymbol{x}|}. \tag{4}$$

The vector potential $\boldsymbol{A}(\boldsymbol{x},t)$ is given by the expectation value of photon field operator, $\hat{\boldsymbol{A}}(\boldsymbol{x},t)$, with respect to a coherent state of photons representing the laser.[7] In this paper, we consider a situation in which two identical lasers are irradiated to the atom from opposite sides. The vector potential in the case is given as follows,

$$\boldsymbol{A}(\boldsymbol{x},t)=2\sqrt{\frac{2\hbar N}{\epsilon_0\omega}}\cos(\boldsymbol{k}\cdot\boldsymbol{x})\cos(\omega t+\phi_\alpha)\boldsymbol{a}_{\boldsymbol{k}}, \tag{5}$$

where N is the average photon number density and $\boldsymbol{a}_{\boldsymbol{k}}$ is the polarization vector. $\boldsymbol{k}$, $\hbar\omega(=E_\gamma)$ and ϕ_α are the wave number vector, the energy per a photon and initial phase respectively. Note that, in Eq.(3), the external vector potential, $\boldsymbol{A}(\boldsymbol{x},t)$, appears in the form, $\hat{\boldsymbol{p}}+e\boldsymbol{A}(\boldsymbol{x},t)$, hence it modifies the momentum or wave length. As shown in Sec.2.3, this modification can be interpreted as the modification of mass.

2.2. *Effective Mass*

According to the intuition mentioned in Sec.1, we put the vector potential into the scalar mass to obtain the effective mass.

Let U be the diagonalization matrix and m_{eff} be the eigenvalue of $ce\boldsymbol{\alpha}\cdot\boldsymbol{A}+\beta m_e c^2$. After simple calculation, we obtain

$$m_{eff}(\boldsymbol{x},t)=\sqrt{m_e^2+\left(\frac{e}{c}\boldsymbol{A}(\boldsymbol{x},t)\right)^2}, \tag{6}$$

$$U(\boldsymbol{x},t)=\begin{pmatrix}\mathbf{1}\cos\Theta(\boldsymbol{x},t) & -\boldsymbol{\sigma}\cdot\boldsymbol{a}(\boldsymbol{x},t)\sin\Theta(\boldsymbol{x},t)\\ \boldsymbol{\sigma}\cdot\boldsymbol{a}(\boldsymbol{x},t)\sin\Theta(\boldsymbol{x},t) & \mathbf{1}\cos\Theta(\boldsymbol{x},t)\end{pmatrix}, \tag{7}$$

$$\boldsymbol{a}(\boldsymbol{x},t)\equiv\frac{\boldsymbol{A}(\boldsymbol{x},t)}{|\boldsymbol{A}(\boldsymbol{x},t)|}, \tag{8}$$

$$\sin\Theta(\boldsymbol{x},t)=\frac{(m_{eff}(\boldsymbol{x},t)-m_e)c^2}{\sqrt{\left(ce\boldsymbol{A}(\boldsymbol{x},t)\right)^2+(m_{eff}(\boldsymbol{x},t)-m_e)^2c^4}}, \tag{9}$$

$$\cos\Theta(\boldsymbol{x},t)=\frac{|ce\boldsymbol{A}(\boldsymbol{x},t)|}{\sqrt{\left(ce\boldsymbol{A}(\boldsymbol{x},t)\right)^2+(m_{eff}(\boldsymbol{x},t)-m_e)^2c^4}}. \tag{10}$$

Here we define m_{eff} as the effective mass of electrons in the photon medium. Note that an irradiation of laser originally modifies the momentum of bound electron, not its mass. This modification results in the change of the binding energy. From eq.(6), we see that m_{eff} becomes heavier than m_e in the presence of the external field. Since the atomic electron capture rate is proportional to $|\Psi(0)|^2 \propto m_{eff}^3$, the high intensity laser irradiation can enhance the capture rate.

Then, the Dirac equation, (3), is rewritten as follows (for simplicity, we denote $\boldsymbol{A}(\boldsymbol{x},t)$, $m_{eff}(\boldsymbol{x},t)$ and $U(\boldsymbol{x},t)$ as $\boldsymbol{A}$, m_{eff} and U.)

$$i\hbar\frac{\partial}{\partial t}\Phi = \left[cU^\dagger \boldsymbol{\alpha} U\cdot\hat{\boldsymbol{p}} + \beta m_{eff}c^2 - e\phi\right]\Phi - U^\dagger\left\{i\hbar\frac{\partial}{\partial t}U - c\alpha\cdot(\hat{\boldsymbol{p}}U)\right\}\Phi, \tag{11}$$

here we define new mass eigenstates, $\Phi \equiv U^\dagger\Psi$. Incidentally, if $|e\boldsymbol{A}|$ is much smaller than $m_e c$, m_{eff} becomes $m_e\left(1 + e^2\boldsymbol{A}^2/2m_e^2c^2\right)$. This can be interpreted as the shift of energy reference. The effective mass is also expressed using the Keldysh parameter, κ,

$$m_{eff} = m_e\sqrt{1+\left(\frac{2\alpha}{\kappa}\right)^2}, \quad \kappa = \sqrt{\frac{m_e^2\alpha E_\gamma}{8\pi(\hbar c)^2\hbar I}}, \tag{12}$$

here $\alpha = e^2/4\pi\epsilon_0\hbar c$ is the fine structure constant.

In the following, we investigate the case in which E_γ is about a $\mathcal{O}(10^{-3})$ eV or less, hence the wave length of laser is about 10^{-6} m or longer, which is 10^5 times longer than the atomic size. The period of laser, $T_{\rm Laser}$, in this region is long compared to the average time, T_e, during which electrons go around nucleus, i.e. $T_{\rm Laser} = 4.1\times10^{-15}/E_\gamma[\text{eV}]$ (s) $> 2.9\times10^{-16} = T_e$ (s). These facts indicate that dependence of U on the position and the time is very mild. Therefore we can neglect the last term in Eq.(11) and treat m_{eff} as a constant at each time. Then the Dirac equation becomes a simple form

$$i\hbar\frac{\partial}{\partial t}\Phi(\boldsymbol{x},t) = \hat{H}\Phi(\boldsymbol{x},t), \tag{13}$$

where

$$\begin{aligned}\hat{H} = {} & c\boldsymbol{\alpha}\cdot\hat{\boldsymbol{p}} + c\left\{-(\boldsymbol{\alpha}\cdot\boldsymbol{a}) + \beta\sin 2\Theta + (\boldsymbol{\alpha}\cdot\boldsymbol{a})\cos 2\Theta\right\}(\boldsymbol{a}\cdot\hat{\boldsymbol{p}}) \\ & + \beta m_{eff}c^2 - e\phi(\boldsymbol{x}).\end{aligned} \tag{14}$$

We make further approximation that under the static assumption, the vector potential, Eq.(5), is approximated as

$$\boldsymbol{A}^2(\boldsymbol{x},t) = \frac{4\hbar N}{\epsilon_0\omega}. \tag{15}$$

As we mentioned above, the effective mass, Eq.(6), is given by the vector potential at each time. For our purpose, m_{eff}^3 should be replaced by its time average because the capture rate is proportional to m_{eff}^3. But considering the time period concerning here, there is only a few % difference between the time average of m_{eff}^3 and the one which $\boldsymbol{A}^2(\boldsymbol{x},t)$ is replaced by Eq.(15). Furthermore, this replacement is often done in the laser science field and makes the form of the effective mass easy to understand. From these reasons, we simply replace $\boldsymbol{A}^2(\boldsymbol{x},t)$ in the effective mass by the Eq.(15) in the following discussion.

N is related to the laser intensity, I,

$$I \simeq c\hbar\omega N. \tag{16}$$

Thus the vector potential is expressed in terms of I as

$$\boldsymbol{A}^2(\boldsymbol{x},t) \simeq \frac{4I}{\epsilon_0\ \omega^2 c}. \tag{17}$$

We note that the vector potential is proportional to $\sqrt{I}/E_\gamma$. There are two competitive ionization processes to be suppressed for the enhancement to really work. One is a tunnel ionization and the other is a multi-photon ionization (MPI). Probabilities of tunneling ionization[8] are proportional to $\exp(-\mathcal{E}_{\rm C}/\mathcal{E}_{\rm L})$, where $\mathcal{E}_{\rm C}$ is the strength of Coulomb electric field produced by the nucleus and $\mathcal{E}_{\rm L}$ is that produced by the laser. Since $\mathcal{E}_{\rm L}$ is given by $\sqrt{I/\epsilon_0 c}$, $\mathcal{E}_{\rm L}$ becomes smaller than $\mathcal{E}_{\rm C}$ in the hydrogen atom for $I < 10^{13}$ W/mm^2. Therefore the tunneling ionization can be ignored at $I \ll 10^{13}$ W/mm^2. For instance, if I is 10^{10} W/mm^2 and $\mathcal{E}_{\rm C}$ is 5.0×10^{11} V/m, $\exp(-\mathcal{E}_{\rm C}/\mathcal{E}_{\rm L}) \sim 10^{-8\times10^4}$, hence the tunneling ionization is highly suppressed in this case. Furthermore, it is considered that there are no resonances on a photon absorption in MPI cross sections when E_γ is smaller enough than the binding energy difference of $1s$ and $2s$ state, $\Delta E_{\rm bin} = E_{2s} - E_{1s}$. In this case, the cross sections are proportional to $(N\alpha)^{E_{\rm bin}/E_\gamma} \sim (I\alpha/E_\gamma)^{E_{\rm bin}/E_\gamma}$ and go to zero as $E_\gamma \to 0$ for fixed I/E_γ. The same conclusion is derived from the discussion with the Keldysh parameter, which indicate that MPI is more suppressed than the tunnel ionization when $\kappa < 1$. For the case with $I = 10^{10}$ W/mm^2 and $E_\gamma = 10^{-3}$ eV, κ is 3.8×10^{-3} and the MPI is suppressed. Thus, the ionizing processes can be suppressed by using a low energy and intensity laser. On the contrary, m_{eff} is an increasing function of I/E_γ^2, or $1/\kappa$. Therefore one can simultaneously increase the effective mass and suppress the ionization processes as much as desired, even if I and I/E_γ are fixed at low value so that the tunnel ionization and MPI are ignored for $1s$ and $2s$ states.

2.3. *Non-relativistic Limit*

Since binding energy of atomic electrons is very small compared to their rest mass, non-relativistic equation gives a good approximation. Therefore we reduce Eq.(13) to the non-relativistic form.

To take the non relativistic limit, we separate the rest mass from the wave function.

$$\Phi(\boldsymbol{x},t) = \exp\left(-\frac{im_{eff}c^2}{\hbar}t\right)\varphi_{NR}(\boldsymbol{x},t), \quad \varphi_{NR}(\boldsymbol{x},t) = \begin{pmatrix}\varphi(\boldsymbol{x},t)\\ \eta(\boldsymbol{x},t)\end{pmatrix}. \tag{18}$$

By inserting the above relations into Eq.(13), we have

$$i\hbar\frac{\partial}{\partial t}\varphi = \Big(-e\phi + c\tilde{s}(\boldsymbol{a}\cdot\hat{\boldsymbol{p}})\Big)\varphi + \Big(c\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}} + c(\tilde{c}-1)(\boldsymbol{a}\cdot\hat{\boldsymbol{p}})(\boldsymbol{\sigma}\cdot\boldsymbol{a})\Big)\eta, \tag{19a}$$

$$i\hbar\frac{\partial}{\partial t}\eta = \Big(-e\phi - c\tilde{s}(\boldsymbol{a}\cdot\hat{\boldsymbol{p}}) - 2m_{eff}c^2\Big)\eta + \Big(c\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}} + c(\tilde{c}-1)(\boldsymbol{a}\cdot\hat{\boldsymbol{p}})(\boldsymbol{\sigma}\cdot\boldsymbol{a})\Big)\varphi, \tag{19b}$$

where $\tilde{s} = \sin 2\Theta$ and $\tilde{c} = \cos 2\Theta$. Since $|\boldsymbol{p}|/m_{eff}c \ll 1$, the second equation reduces to

$$\eta = \frac{1}{2m_{eff}c}\left[\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}} + (\tilde{c}-1)(\boldsymbol{\sigma}\cdot\boldsymbol{a})(\boldsymbol{a}\cdot\hat{\boldsymbol{p}})\right]\varphi. \tag{20}$$

Then, we obtain the non-relativistic equation by inserting the above result into the first equation,

$$i\hbar\frac{\partial}{\partial t}\varphi(\boldsymbol{x},t) = \left[\frac{1}{2m_{eff}}\left(\hat{\boldsymbol{p}}^2 - \tilde{s}^2(\boldsymbol{a}\cdot\hat{\boldsymbol{p}})^2\right) + c\tilde{s}(\boldsymbol{a}\cdot\hat{\boldsymbol{p}}) - e\phi(x)\right]\varphi(\boldsymbol{x},t). \tag{21}$$

Now one can see that the mass of the electron is replaced by m_{eff} and the vector potential is disappeared in the kinetic term. Thus, the modification of momentum is interpreted as the modification of mass.

3. Perturbative Expansion

In the previous section, we derived the non-relativistic equation with the effective mass, Eq.(21). We show here that the probability of finding the electron at the origin is mainly determined by the zeroth order wave function which is proportional to m_{eff}^3. Therefore the high intensity laser can enhance the capture rate of the atomic decay. To show this, we solve the equation perturbatively up to the second order.

First, we rewrite the Hamiltonian in Eq.(21) in a more simple form. Let us take $\boldsymbol{A} = (0,0,A)$ and complete the square on $\hat{p}_z$

$$\frac{1}{2m_{eff}}\left(\hat{p}_z^2 - \tilde{s}^2\hat{p}_z^2\right) + c\tilde{s}\hat{p}_z = \frac{\tilde{c}^2}{2m_{eff}}\left(\hat{p}_z + m_{eff}c\frac{\tilde{s}}{\tilde{c}^2}\right)^2 - \frac{1}{2}m_{eff}c^2\tilde{t}^2, \tag{22}$$

here $\tilde{t} = \tan 2\Theta = \sin 2\Theta/\cos 2\Theta$. The first term in Eq.(22) is reduced to $\hat{p}_z^2$ by replacing $\varphi(\boldsymbol{x},t)$ with $\exp(-im_{eff}c\tilde{s}z/\hbar\tilde{c}^2)\varphi(\boldsymbol{x},t)$, and the second term is dropped because it is just like vacuum energy. Furthermore, we separate time dependence. Thus we obtain the equation which we are going to solve,

$$E\varphi(\boldsymbol{x}) = (\hat{H}_0 + \hat{H}_{int})\varphi(\boldsymbol{x}), \tag{23a}$$

$$\hat{H}_0 = \frac{\hat{\boldsymbol{p}}^2}{2m_{eff}} - e\phi(x), \tag{23b}$$

$$\hat{H}_{int} = -\frac{\tilde{s}^2}{2m_{eff}}\hat{p}_z^2. \tag{23c}$$

In the following, we calculate the energy and wave functions of hydrogen-like $1s$ bound state of the atom with atomic number Z.

3.1. *The Zeroth Order*

The zeroth order energies and wave functions are the eigenvalues and the eigenfunctions of $\hat{H}_0$. These are given by

$$\varphi_{nlm}^{(0)}(\boldsymbol{x}) = R_{nl}(r)Y_{lm}(\theta,\phi), \tag{24}$$

where $n,\ l$ and m are the principal quantum number, azimuthal quantum number and magnetic quantum number. $R_{nl}(r)$ and $Y_{lm}(\theta,\phi)$ are Laguerre function and spherical harmonics function.

The energy in the zeroth order is

$$E_n^{(0)} = -\frac{\hbar^2}{2m_{eff}a_1^2}\frac{1}{n^2}, \quad a_1 = \frac{4\pi\epsilon_0\hbar^2}{Zm_{eff}e^2}. \tag{25}$$

3.2. *The First Order*

The corrections to the energy and wave function of the 1s state in the first order is

$$E_1^{(1)} = \frac{\tilde{s}^2}{3}E_1^{(0)}. \tag{26}$$

and

$$\varphi_{100}^{(1)}(\boldsymbol{x}) = \tilde{s}^2\sum_{n=2} f_{n00}^{(1)}\varphi_{n00}^{(0)} + \tilde{s}^2\sum_{n=3} f_{n20}^{(1)}\varphi_{n20}^{(0)}, \tag{27}$$

where

$$f_{n00}^{(1)} = \frac{8}{3}\frac{n^{5/2}}{(n^2-1)^2}\left(\frac{n-1}{n+1}\right)^{n-1}, \tag{28a}$$

$$f_{n20}^{(1)} = \frac{\sqrt{5}}{15}\frac{n^2}{n^2-1}\left(\frac{2}{n+1}\right)^5\sqrt{\frac{(n-3)!}{(n+2)!}} \times\sum_{r=0}^{n-3} {}_{n+2}C_{n-3-r}\frac{(r+3)!}{r!}\big(n(r+5)+1\big). \tag{28b}$$

3.3. *The Second Order*

The corrections to the energy and wave function of the 1s state in the second order is

$$E_1^{(2)} = \tilde{s}^4E_1^{(0)}\left[\frac{3}{4}{f_{200}^{(1)}}^2 + \sum_{n=3}\frac{n^2-1}{n^2}\left({f_{n00}^{(1)}}^2 + {f_{n20}^{(1)}}^2\right)\right]. \tag{29}$$

and

$$\varphi_{100}^{(2)}(\boldsymbol{x}) = \sum_{\substack{n\neq 1\\ lm}}\tilde{s}^4 f_{nlm}^{(2)}\varphi_{nlm}^{(0)} + \tilde{s}^4C^{(2)}\varphi_{100}^{(0)}, \tag{30}$$

where $C^{(2)}$ is

$$C^{(2)} = -\frac{1}{2}\sum_{n=2} f_{n00}^{(1)\,2} - \frac{1}{2}\sum_{n=3} f_{n20}^{(1)\,2}. \tag{31}$$

and $f_{nlm}^{(2)}$ is

$$\begin{aligned}
f_{n00}^{(2)} = & -\frac{1}{3}\frac{n^2}{n^2-1} f_{n00}^{(1)} + \frac{n^2}{n^2-1}\sum_{n'=2} f_{n'00}^{(1)} N_{nn'}^{00}\left(\frac{1}{3n'}F_{nn'}^{00}(2) + \frac{2}{3}F_{nn'}^{00}(1)\right) \\
& + \frac{2\sqrt{5}}{15}\frac{n^2}{n^2-1}\sum_{n'=3} f_{n'20}^{(1)} N_{nn'}^{02}\left(-\frac{1}{n'^2}F_{nn'}^{02}(2) + \left(\frac{3}{n'}+2\right)F_{nn'}^{02}(1)\,,\right. \qquad (32a) \\
& \left. + 7(n'-3)F_{nn'}^{02}(0) - 7(n'+2)G_{nn'}^{02}(0)\right),
\end{aligned}$$

$$\begin{aligned}
f_{n20}^{(2)} = & \left(C^{(1)} - \frac{1}{3}\frac{n^2}{n^2-1}\right) f_{n20}^{(1)} \\
& + \frac{2\sqrt{5}}{15}\frac{n^2}{n^2-1}\sum_{n'=2} f_{n'00}^{(1)} N_{nn'}^{20}\left(-\frac{1}{n'^2}F_{nn'}^{20}(2) - \left(\frac{3}{n'}-2\right)F_{nn'}^{20}(1)\right. \\
& \left. + 3(n'-1)F_{nn'}^{20}(0) - 3n'G_{nn'}^{20}(2)\right) \\
& + \frac{1}{7}\frac{n^2}{n^2-1}\sum_{n'=3} f_{n'20}^{(1)} N_{nn'}^{22}\left(-\frac{11}{3n'}F_{nn'}^{22}(2) + \frac{22}{3}F_{nn'}^{22}(1)\right), \qquad (32b)
\end{aligned}$$

$$\begin{aligned}
f_{n40}^{(2)} = & \frac{4\sqrt{5}}{35}\frac{n^2}{n^2-1}\sum_{n'=3} f_{n'20}^{(1)} N_{nn'}^{42}\left(-\frac{1}{n'^2}F_{nn'}^{42}(2) - \left(\frac{7}{n'}-2\right)F_{nn'}^{42}(1)\right. \\
& \left. + 7(n'-3)F_{nn'}^{42}(0) - 7(n'+2)G_{nn'}^{42}(0)\right). \qquad (32c)
\end{aligned}$$

Other $f_{nlm}^{(2)} = 0$. The coefficients, $N_{nn'}^{ll'}$, $F_{nn'}^{ll'}(p)$ and $G_{nn'}^{ll'}(p)$ are given in Appendix A.

In Figures 1, we show the coefficients of wave functions, Eqs.(28) and (32) at each principle number. As seen from the figures, the first and second order corrections are much smaller than 1 (the zeroth order) and go to zero in the limit of $n \to \infty$. It is also seen that the first order corrections are larger than those of the second order. These show that the probability at the origin is mainly determined by the zeroth order wave function.

4. Enhancement of Atomic Electron Capture Rate

We show the results of numerical calculations of the atomic electron capture rate Eq.(2), mixing angle Eq.(26) and energies Eq.(29).

From Eq.(2) and $|\Psi(0)|^2 \simeq |\varphi(0)|^2$, the ratio of capture rate is given by

$$\frac{\Gamma_{\text{laser}}}{\Gamma} = \frac{|\Psi_{\text{laser}}(0)|^2}{|\Psi(0)|^2} \propto \left(\frac{m_{eff}}{m_e}\right)^3. \tag{33}$$

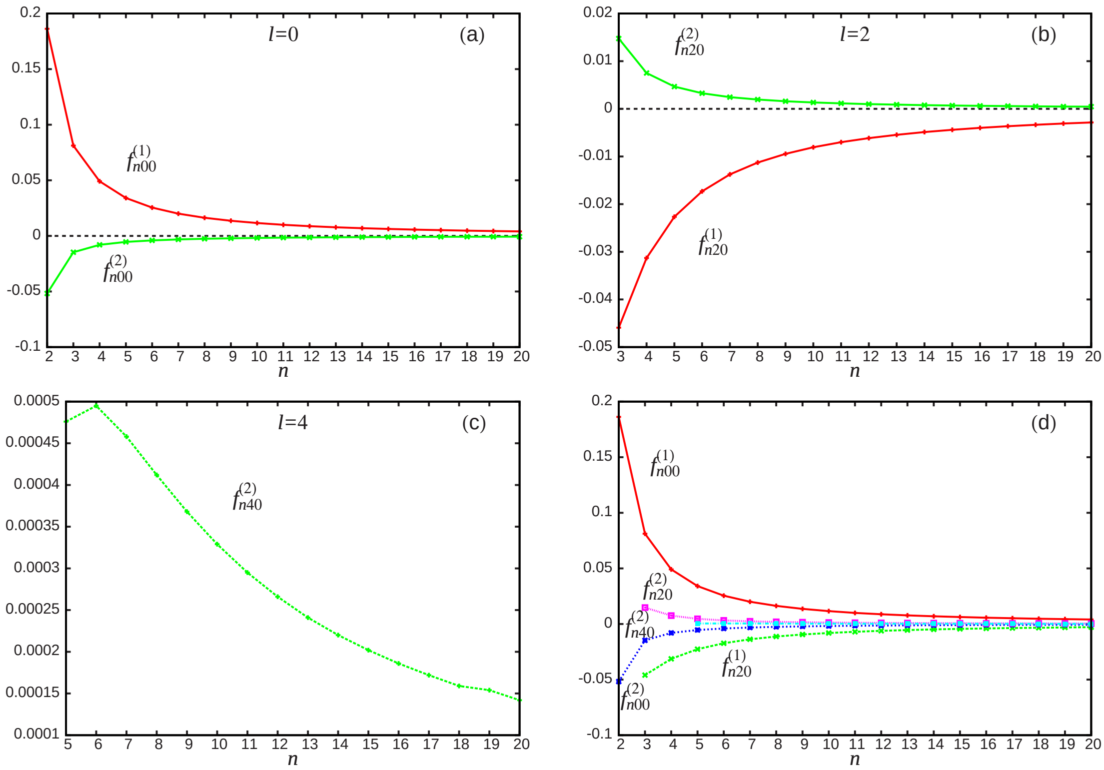

Fig. 1. Figs.(a), (b) and (c) plot the coefficients for $l = 0$, $l = 2$ and $l = 4$ to n respectively. Fig.(d) shows all coefficients in Eqs. (28) and (32).

m_{eff} is a function of the intensity given in Eq.(17).

As discussed in Sec.2, we take $E_\gamma = 10^{-3}$ eV and $I = 10^{10}$ W/mm^2 as the reference value so that MPI and tunnel ionization can be ignored. Figures 2 show the ratio of the capture rate, Eq.(33) and $\sin^2 2\Theta$ at $E_\gamma = 10^{-3}$ eV. We take $Z = 1$, $m_e = 0.5$ MeV and $4\pi\epsilon_0\hbar c/e^2 = 137$ for all figures. The intensity is varied from $I = 10^9$ to 2×10^{11} W/mm^2. It is seen that the ratio of capture rate and the mixing angle grow up as the laser intensity becomes higher. It is also seen that at $I = 10^{10}$ W/mm^2, $\sin^2 2\Theta$ is about 0.7 and the ratio is about 6 times larger than normal one.

Figure 3.(a) shows the E_γ dependence of the ratio of capture rate at the intensity 10^{10} W/mm^2. Figure 3.(b) plots the ratio of the energy to the zeroth order energy as a function of the intensity at $E_\gamma = 10^{-3}$ eV,

It is seen in Fig.3.(a) that the capture ratio and mixing angle become smaller as the photon energy becomes larger. Thus the lower energy photon is suitable for the enhancement. This can be understood in terms of the average photon number

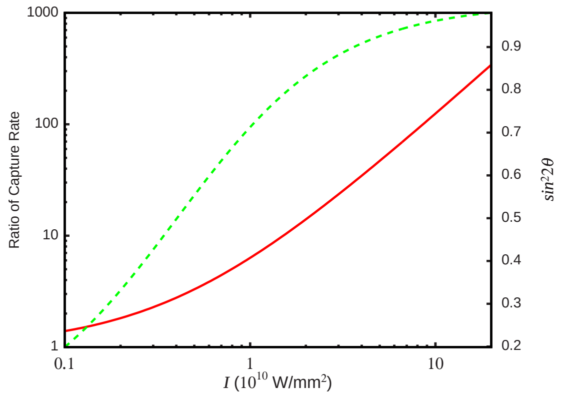

Fig. 2. Ratio of the atomic electron capture rate and $\sin^2 2\Theta$ under the laser irradiation for $E_\gamma = 10^{-3}$ eV are plotted as a function of I. The laser intensity is varied from 10^9 to 2×10^{11} W/mm^2. Red solid line represents the ratio of the capture rate and green dashed line is $\sin^2 2\Theta$. The left horizontal axis is for the ratio of the capture rate and the right axis is for $\sin^2 2\Theta$.

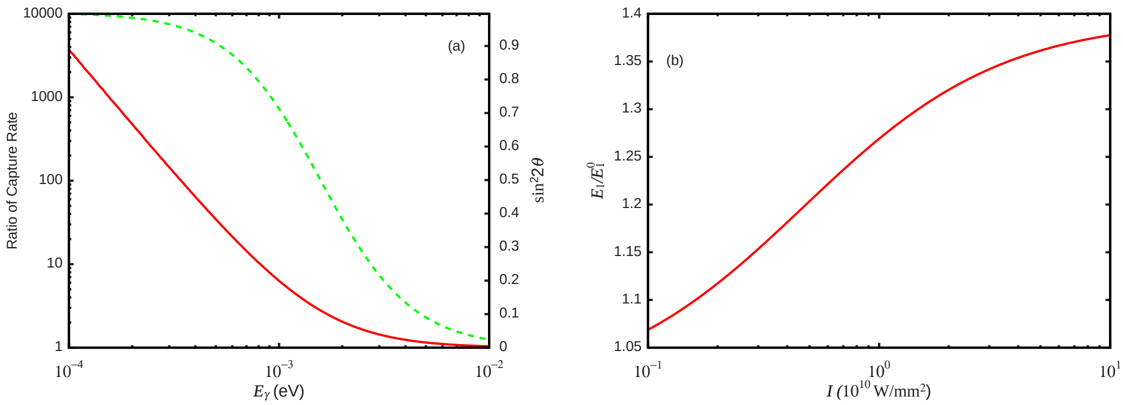

Fig. 3. Fig.(a) shows the E_γ dependence of the ratio of the capture rate at $I = 10^{10}$ W/mm^2. In Fig.(a), the red solid line represents the ratio of the capture rate and the green dashed line the mixing angle. Fig.(b) shows the intensity dependence of the energy at $E_\gamma = 10^{-3}$ eV.

within the orbital radius as follows. From Eq.(16), the average photon number within $4\pi a_1^3/3$ at $I = 10^{10}$ W/mm^2 is expressed as,

$$2N \times \frac{4\pi}{3} a_1^3 = \frac{4I}{cE_\gamma} \frac{4\pi}{3} a_1^3 \sim \frac{4.2 \times 10^{-4}}{E_\gamma[\text{eV}]}, \tag{34}$$

here the factor 2 is multiplied because two identical lasers are irradiated to the atom from opposite sizes. The photon bath becomes denser as the photon energy becomes lower. Thus one can obtain higher capture rate by low energy laser. From Fig.3.(b), we can see that the energy of bound electron increases about 27% at 10^{10} W/mm^2.

It comes mainly from the first order correction, Eq.(26). It can be interpreted as the electron is bound more closer to nucleus or muon for higher intensity laser.

5. Summary and Discussion

We have studied a possibility to enhance the atomic electron capture rate by irradiating a laser. We reduced the Dirac equation with the vector and the Coulomb potential (3) to the non-relativistic equation with effective mass m_{eff}, Eq.(21). The effective mass was , indeed, found to be heavier than that in a vacuum.

We solved this equation up to second order of the perturbation, $\hat{H}_{int}$ (23c) in Sec.3. Though at one glance the perturbative term looks large, we showed that it can be treated as perturbation. The validity of perturbative calculations is reinforced from Fig.1 in which the corrections to the wave functions are smaller than the zeroth order.

We have calculated the ratio of the capture rate and the mixing angle numerically, and investigated their intensity and photon energy dependences. We investigated the situation with a laser of 10^{-3} eV energy and 10^{10} W/mm^2 intensity as the reference value. As shown in Sec.2, the competitive ionization processes are negligible for these values. In this situation, the enhancement mechanism works for $1s$ electrons, which play a main role in capture process, and the capture rate becomes larger by ten times. This is because the effective mass is an increasing function of I/E_γ^2, while tunnel ionization and MPI are suppressed due to low energy and intensity that is discussed in Sec.2.

In Fig.2, we showed our numerical result that the enhancement of the atomic electron capture rate can be achieved by several factors even for order of 10^{10} W/mm^2. As is shown in Fig.3.(a), the low energy laser is suitable for the enhancement of capture rate. Fig.3.(b) shows that the electron becomes bound more closer to the nucleus and muon as the laser intensity becomes higher. It is naively understood by the fact that the photon number density is higher for lower energy photon as long as the laser intensity is fixed, Eq.(34). The strength of interaction between an electron and a photon does not depend on the photon energy. Therefore denser photon bath gives electron a heavier effective mass. Incidentally, the approximation employed in this paper becomes more reliable for lower energy photon which has less time and position dependence.

The most serious question in our calculation is the relaxation time. We calculated the static wave function under the photon bath. However, in a realistic situation a laser is irradiated at t=0 to the system in a vacuum. It means that we need to calculate the transition time from 1s state in a vacuum to that in a medium. If this transition time is much longer than T_{Laser}, then our approximation becomes invalid. We can expect that, with sufficiently high intensity laser, this time is sufficiently small but we need to certificate it. This calculation is very complicated and hence we leave it for future work.

Electron capture processes are important since they give a variety of methods to explore new physics, like lepton flavor violations and neutrino oscillations. To obtain high statistics, the only way to enhance the atomic electron capture rate is to raise the probability of electrons at the origin. We showed that the laser irradiation makes bound electrons heavier and hence enhances the electron capture rate. Although our result is assumed the static electromagnetic field and based on the perturbative calculation, it is expected that the enhancement will be possible in this way. If the enhancement by laser irradiation is verified by experiments, it can be directly applied for new physics searches using capture processes.

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Appendix A. Coefficients

$$N_{nn'}^{ll'} = \frac{1}{4}\left(\frac{2}{n}\right)^{l+2}\left(\frac{2}{n'}\right)^{l'+2}\sqrt{\frac{(n-l-1)!}{(n+l)!}\frac{(n'-l'-1)!}{(n'+l')!}}, \tag{A.1}$$

$$F_{nn'}^{ll'}(p) = \sum_{r=0}^{n-l-1}\sum_{r'=0}^{n'-l'-1} {}_{n+l}C_{n-l-1-r}\left(-\frac{2}{n}\right)^{r} {}_{n'+l'}C_{n'-l'-1-r'}\left(-\frac{2}{n'}\right)^{r'} \times \frac{(r+r'+l+l'+p+1)!}{r!r'!}\left(\frac{nn'}{n+n'}\right)^{r+r'+l+l'+p+1}. \tag{A.2}$$

$$G_{nn'}^{ll'}(p) = \sum_{r=0}^{n-l-1}\sum_{r'=0}^{n'-l'-2} {}_{n+l}C_{n-l-1-r}\left(-\frac{2}{n}\right)^{r} {}_{n'+l'-1}C_{n'-l'-2-r'}\left(-\frac{2}{n'}\right)^{r'} \times \frac{(r+r'+l+l'+p)!}{r!r'!}\left(\frac{nn'}{n+n'}\right)^{r+r'+l+l'+p+1}. \tag{A.3}$$

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