Monoenergetic Neutrino Beam for Long-Baseline Experiments

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In an electron capture process by a nucleus, emitted neutrinos are monoenergetic. By making use of this, we study how to get a completely monoenergetic neutrino beam in a long-baseline experiment.

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Introduction.—Numerous observations on neutrinos from the Sun [1], the atmosphere [2], reactors [3], and the accelerator [4] suggest that neutrinos are massive and hence there is mixing in the lepton sector.

Within the three generation framework, two of the mixing angles and the two mass squared differences are well determined [5]. To determine these parameters much more precisely and to observe effects from the other two mixing parameters, θ_{13} and the CP phase δ, there were several ideas proposed for the next generation neutrino oscillation experiments [6–9].

For a precision measurement, it is obviously better to have an experiment using neutrinos with controllable and precisely known energy. To achieve this, we consider making use of a nucleus that absorbs an electron and emits a neutrino:

$$(Z, A) + e^- \to (Z-1, A) + \nu_e, \tag{1}$$

where Z is the electric charge of the mother nucleus and A is its mass number. In this case neutrinos have a line spectrum and their energy is precisely known. Therefore, by accelerating the mother nuclei appropriately with the Lorenz boost factor γ_m, we can control the neutrino energy and make use of monoenergetic neutrinos in an oscillation experiment.

The experimental setup is very simple. We need an accumulating ring as usual [8] to circulate the nuclei. This ring is equipped with an electron injected at the entrance of the decay section which has length X and an apparatus for separation of nuclei and electrons at the end of the decay section. The energy of injected electrons must be tuned precisely so that their boost factor γ_e is the same as that of the nuclei γ_m, $\gamma_e = \gamma_m$ [10]. The separation section at the end must be constructed such that it can separate the nuclei and electrons properly in order to circulate the nuclei until they capture an electron. It may be implemented by photon injection and a strong magnet.

The range of neutrino energies, E_ν, in the laboratory frame is given by

$$0 < E_\nu < 2\gamma_m Q, \tag{2}$$

where Q is the energy difference between the mother and the daughter nuclei and $Q \ll M$. The range of E_ν that can be made use of is the important point of this Letter.

It is most probable that the beam is focused in the direction of a detector. In the center of the beam direction $E_\nu = 2\gamma_m Q$. Therefore the appropriate boost factor γ_m for the experiment is derived from the baseline length L and the relevant mass square difference δm^2:

$$\left.\frac{\delta m^2 L}{4E_\nu}\right|_{E_\nu = 2\gamma_m Q} = P, \tag{3}$$

where P is the desired oscillation phase at the maximum neutrino energy, which is determined by the physics goal. For example, if one wants to observe the oscillation at the first maximum, then $P = \pi/2$. From Eq. (3),

$$\gamma_m = \frac{\delta m^2 L}{8P}\frac{1}{Q}. \tag{4}$$

Since in the rest frame of the mother nuclei, the distance between the decay section and the detector is $L' \equiv L/\gamma_m$, larger γ_m means higher neutrino flux at the detector. It scales proportionally to γ_m^2. We observe from Eq. (4) that a lower Q value is better. However, a lower Q means, in general, a larger half-life τ. The mother nuclei should capture an electron frequently enough; otherwise, we cannot get a neutrino beam of a sufficient strength. This means

$$\tau\gamma_m < T \Rightarrow \frac{\delta m^2 L}{8PT} < Q/\tau, \tag{5}$$

where T is an appropriate time interval within which we require that all the mother nuclei should experience the process (1). Therefore, since in this kind of experiments data are taken for several years, T is of the order of a month or at most a year. This requires that γ_m should be smaller, which conflicts with the requirement of getting a higher-flux neutrino beam mentioned below Eq. (4). To satisfy both the requirements, we have to find a nucleus that has a smaller Q value and a shorter half-life τ. In the following $\gamma_m \gg 1$ and nucleus mass $M \gg Q$ are used in our derivations.

Let us now examine the theoretical aspects of this idea in more detail.

Case (i) Purely monoenergetic neutrinos.—As one of the first candidates we study here $^{110}_{50}$Sn. Theoretically this gives the best example for our scenario. Its half-life τ_{Sn} is 4.11 h. Its J^P is 0^+. It decays into the excited state of ${}^{110}_{49}\mathrm{In}$, with 1^+ whose energy level is 343 keV. Since the mass difference is 638 keV [11], the energy difference between neutral ${}^{110}_{50}\mathrm{Sn}$ and ${}^{110}_{49}\mathrm{In}$, Δ_{Sn}, is 295 keV; that is, the energy of the emitted neutrino is 295 keV minus the binding energy. For example, since the K shell binding energy, E^K_{In} of ${}^{110}_{49}\mathrm{In}$ is 28 keV [12,13], the emitted neutrino energy in the rest frame of Sn, $Q_{\mathrm{Sn}} = \Delta_{\mathrm{Sn}} - E^K_{\mathrm{In}}$ is 267 keV [14].

Then the appropriate acceleration of ${}^{110}_{50}\mathrm{Sn}$ is

$$\gamma_{\mathrm{Sn}} = 378\left(\frac{\delta m^2}{2.5\times 10^{-3}\ \mathrm{eV}^2}\right)\left(\frac{L}{100\ \mathrm{km}}\right)\left(\frac{\pi/2}{P}\right). \tag{6}$$

In the rest frame of ${}^{110}_{50}\mathrm{Sn}$, the distance L'_{Sn} is given by

$$L'_{\mathrm{Sn}} = 264\ \mathrm{m}\left(\frac{2.5\times 10^{-3}\ \mathrm{eV}^2}{\delta m^2}\right)\left(\frac{P}{\pi/2}\right). \tag{7}$$

Therefore if the "fiducial" detector radius is larger than $264(\frac{P}{\pi/2})$ m, half of the neutrinos will hit the detector. Because of the reason mentioned below the theoretically most interesting oscillation phase is $P = \pi/3$ and hence $264(\frac{P}{\pi/2}) = 176$ m. This size of a detector is not unrealistic. Incidentally, since $\gamma_{\mathrm{Sn}} = 567$, $\gamma_{\mathrm{Sn}}\tau_{\mathrm{Sn}} = 96$ days, satisfying Eq. (5). This efficiency should be compared with the case of a neutrino factory or a beta beam. In a neutrino factory [7] the distance L'_μ corresponding to L'_{Sn} is $O(10)$ km, and hence even if the area of the detector perpendicular to the neutrino beam is of $(\mathrm{O}(100)\ \mathrm{m})^2$, only 0.01% of the neutrinos are used. Similarly in a beta beam experiment L'_β is $O(1)$ km and only 1% of neutrinos are used. Therefore, even if we have an amount of ${}^{110}_{50}\mathrm{Sn}$ which is 2 orders of magnitude smaller than the number of nuclei in a beta beam experiment, say ${}^{62}\mathrm{He}$, we will have the same physics reach. That is, the "quality factor" [8] is much better. The quality factor is given by the inverse of L'. Furthermore, since the neutrino energy is much more clearly determined in this experiment, we have a better precision experiment.

There is another interesting feature for sufficiently high γ_m. As we have seen, almost all neutrinos go through the detector. Therefore we have a wide range of neutrino energies, and by measuring the interaction point the neutrino energy can be "measured" precisely. The energy of a neutrino, which is detected at a distance R from the center of the beam, is easily calculated (in the large γ_m limit):

$$E_\nu(R) = \frac{2\gamma_m Q}{1 + R^2/L'^2}. \tag{8}$$

The neutrino energy range is determined by Eq. (8),

$$\frac{2\gamma_m Q}{1 + D^2/L'^2} < E_\nu < 2\gamma_m Q, \tag{9}$$

where D is the fiducial detector diameter. For example, if $D = L'$, then half of the emitted neutrinos hit the detector and their energy range is $\gamma_m Q \le E_\nu \le 2\gamma_m Q$. The range of the oscillation phase varies from $\pi/3$ to $2\pi/3$, from which we can explore the oscillation shape around the oscillation maximum very precisely.

For the position resolution $\delta R(\delta R^2 = 2R\delta R)$, the energy resolution is given by

$$|\delta E_\nu| = \frac{2\gamma_m Q\delta R^2/L'^2}{(1 + R^2/L'^2)^2} \Rightarrow \left|\frac{\delta E_\nu}{E_\nu}\right| = \frac{\delta R^2/L'^2}{(1 + R^2/L'^2)}. \tag{10}$$

In the rest frame of the mother nucleus, monoenergetic neutrinos are emitted isotropically. In a solid angle $d\Omega$ in the rest frame, the number of neutrinos is distributed uniformly. The solid angle $d\Omega = 2\pi\sin\theta d\theta$ corresponds to

$$2\pi\sin\theta d\theta = \frac{4\pi}{(1 + R^2/L'^2)^2}\frac{dR^2}{L'^2} \tag{11}$$

and in terms of the neutrino energy

$$d\Omega = 2\pi\sin\theta d\theta = \frac{2\pi}{\gamma_m Q}dE_\nu. \tag{12}$$

Thus we have a neutrino beam uniformly distributed in its energy. As a detector can measure the energy and the interaction point, by combining these two measurements, we can determine the neutrino energy very precisely. This specific feature in a beta-capture beam arises from the fact that neutrinos are monoenergetic in the rest frame of the mother nucleus.

In Table I, we list candidate nuclei for this case (i).

Case (ii) Monoenergetic neutrino and continuous energy neutrino.—Next we consider the nucleus ${}^{48}_{24}\mathrm{Cr}$. It decays into an excited state of ${}^{48}_{23}\mathrm{V}$ whose energy level is 420 keV. The mass difference is 1659 keV and Δ_{Cr} is 1239 keV. The half-life is 21.56 h [11]. The K shell binding energy, E^K_{V}, of the daughter nucleus ${}^{48}_{23}\mathrm{V}$ is 5.465 keV [12]. Since $Q_{\mathrm{Cr}} = \Delta_{\mathrm{Cr}} - E^K_{\mathrm{V}}$ is larger than $2m_e$, twice of the electron mass, it can not only capture an electron but also emit a positron:

$${}^{48}_{24}\mathrm{Cr} + e^- \to {}^{48}_{23}\mathrm{V} + \nu_e \quad \text{and} \quad {}^{48}_{24}\mathrm{Cr} \to {}^{48}_{23}\mathrm{V} + e^+ + \nu_e. \tag{13}$$

Assuming that there are $2K$ shell electrons in the mother nucleus ${}^{48}_{24}\mathrm{Cr}$, the rate for the capture process, Γ_c, is proportional to [15]

$$\Gamma_c \propto 2\pi\{(Q_{\mathrm{Cr}})/m_e\}^2(\alpha Z)^3 = 0.196. \tag{14}$$

Here m_e is the electron mass. The rate for positron emission, Γ_{e^+}, is proportional to [15]

$$\Gamma_{e^+} \propto \int_1^{w_0} x\sqrt{x^2 - 1}(w_0 - x)^2 F(x, Z)dx = 0.004, \tag{15}$$

$$F(x, Z) = 2(1 + \gamma)\{2pr\}^{2\gamma - 2}\exp(-\pi\nu)\frac{|\Gamma(\gamma - i\nu)|^2}{[\Gamma(2\gamma + 1)]^2}. \tag{16}$$

TABLE I. Candidate nuclei for case (i). γ_m is determined by $P = \pi/3$ for a detector at $L = 100$ km and $\delta m^2 = 2.5 \times 10^{-3}$ eV^2 using Eq. (4). The energy unit is keV. $N^*[E]$ means the excited state of the nucleus N with energy E [keV]. Also the energy difference between mother and daughter nuclei is given in units keV. The unit for the lifetime τ (rest frame) and $\tau\gamma_m$ (laboratory frame) is given by m (minute), h (hour), d (day). "Detector size" indicates the radius within which half of the emitted neutrinos is contained at the detector distance; see Eq. (9).

Mother, E^K [12]	Daughter, E^K [12]	Δ [11]	τ [11]	γ_m	$\tau\gamma_m$	Detector size
${}^{110}_{50}$Sn, 29	${}^{110}_{49}$In* [343], 28	295	4.11 h	567	97 d	176 m
${}^{111}_{49}$In, 28	${}^{111}_{48}$Cd* [417], 27	449	2.80 d	359	1005 d	278 m

Here $F(x, Z)$ is the Fermi function [$\gamma \equiv (1 - \alpha Z)^{1/2}$, $\nu \equiv \alpha Z x/p$, $p = \sqrt{x^2 - 1}$, α the fine structure constant $= 1/137$, and r the radius for a nucleus in units of m_e^{-1}] and $w_0 = (\Delta_{\mathrm{Cr}} - m_e)/m_e$ is the maximum positron energy in units of the electron mass [16]. Thus the electron capture process is dominant (98.0%), and hence a neutrino beam with well-controlled energy is available.

In Table II we list other examples of nuclei, which have even lower Q and shorter τ [11]. ${}^{18}_{9}$F dominantly decays by positron emission while for ${}^{48}_{24}$Cr and ${}^{113}_{50}$Sn* the electron capture process dominates.

Since Q_{Cr} is higher than in the previous case, the appropriate γ_{Cr} is lower and hence the quality factor is worse than in the previous case. Therefore, we need to store much more ${}^{48}_{24}$Cr nuclei than ${}^{110}_{50}$Sn:

$$\gamma_{\mathrm{Cr}} = 82\left(\frac{\delta m^2}{2.50 \times 10^{-3}\ \mathrm{eV}^2}\right)\left(\frac{L}{100\ \mathrm{km}}\right)\left(\frac{\pi/2}{P}\right), \qquad (17)$$

which means that the neutrinos at the detector are completely monoenergetic as can be seen from Eq. (9). There is essentially no position dependence of neutrino energy at the detector.

Therefore, we cannot explore the energy dependence of the oscillation without changing the beam energy as previously discussed. However, this problem may be solved by the use of continuous neutrino associated with positron emission. We can control the boost factor γ_m very well, and hence the highest neutrino energy at a detector is completely determined by it. This allows a very accurate calibration of neutrino energy. Furthermore, the energy of the line spectrum and that of the continuous one are clearly separated, and simultaneous observation of two distinct energy regions gives useful information on the unitarity triangle [17]. Thus having a line and a continuous spectrum simultaneously, we may get a better oscillation parameter reach.

We have studied how the neutrino energy in oscillation experiments can be controlled better than with other ideas that are currently discussed. By electron capture, a nucleus emits a monoenergetic neutrino. Therefore by accelerating the mother nuclei, we can get a well-controlled neutrino beam. To achieve 100% electron capture rate, we need to use a nucleus with a low Q value, lower than $2m_e$. In general, such a nucleus has a long half-life. Furthermore, since we accelerate it with a very large boost factor γ_m, it becomes almost stable. Though this conflicts with the fact that the nucleus must decay within a sufficiently short time interval [see Eq. (5)], there are several candidates listed in Table II. With these nuclei, we can control the neutrino energy. Since γ_m is very large, a neutrino beam is so well concentrated in the forward direction that almost all neutrinos can be used for oscillation experiments. This significantly reduces the required number of mother nuclei. As a result of such a high γ_m, in principle, we do not need to measure the neutrino energy at the detector since by measuring the detected position we can calculate its energy, and hence we have another way of observing the energy dependence of the oscillation.

Theoretically there are only advantages, but these nuclei are so heavy that it is very energy consuming to accelerate them to the ideal γ_m. Also it may be hard to get enough nuclei even if the required number of nuclei is significantly small. As a compromise, we have also studied nuclei with a higher Q value. These nuclei not only capture an electron but also emit a positron. From the latter process neutrinos with a continuous spectrum are emitted. Furthermore, as Q is higher, γ_m must be smaller. These facts spoil some of the good features mentioned above. However, since we have

TABLE II. Candidate nuclei for case (ii). γ_m is determined by $P = \pi/2$ instead of $\pi/3$. For ${}^{113}_{50}$Sn, the lifetime is adjusted by its branching ratio to EC, 8.9%. The last column, the branching ratio for the electron capture and the positron emission, is calculated by using Eqs. (14) and (15).

Mother, E^K [12]	Daughter, E^K [12]	Δ [11]	τ [11]	γ_m	$\tau\gamma_m$	EC:e^+ emission
${}^{18}_{9}$F, 0.7	${}^{18}_{8}$O, 0.5	1656	110 m	61	4.65 d	3.4:96.6
${}^{48}_{24}$Cr, 6	${}^{48}_{23}$V*[420], 5	1239	21.56 h	82	74 d	98.0:2.0
${}^{111}_{50}$Sn, 29	${}^{111}_{49}$In, 28	2445	35.3 m	42	24.7 h	40.5:59.5
${}^{113}_{50}$Sn*[77], 29	${}^{113}_{49}$In, 28	1113	4.01 h	93	15.4 d	100:0

neutrinos with a line spectrum and a continuous spectrum simultaneously, we may get another good feature for this kind of beams.

In this kind of a beta-capture beam, we can produce only a ν_e beam. To study *CP* violation, we need a $\bar{\nu}_e$ [18] or ν_μ [19] beam. Contrary to the e^- capture case, since e^+ cannot be bound by nuclei, it is almost impossible to have a sufficiently strong $\bar{\nu}_e$ beam. Instead, we can make use of μ capture to get a monoenergetic ν_μ beam, though, since the mass of μ is very high, the emitted neutrinos have a very high energy. We must find a nucleus whose daughter has a mass higher than that of the mother by $O(\mu$ mass) so that the energy of ν_μ in the rest frame of the mother nucleus is sufficiently low.

Apart from the idea to make use of e^- capture, the nucleus $^{18}_{9}$F should be considered more seriously as the β beam source. Note that under ideal circumstances, with a large storage ring with strong permanent magnets, etc., we do not need any power supply to maintain nucleus current. Therefore, the mother nuclei do not have to decay "immediately." Since Δ_F is 1655.5 keV while Δ_{Ne} is 4446 keV, we have much a better "quality factor" than for $^{18}_{10}$Ne. $^{18}_{9}$F is used for medical examination, positron emission tomography (PET). There, one makes samples of $O(10^{10})$ Bq within 1 h, i.e., about 10^{14} nuclei per hour $\simeq 10^{18}$ nuclei per year even for a medical examination. We can use a much larger amount of such nuclei much more easily than $^{18}_{10}$Ne. Similarly, we need to reconsider a candidate nucleus for a $\bar{\nu}_e$ source with lower Q than $^{6}_{2}$He, e.g., $^{31}_{14}$Si.

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