# Signature of the minimal supersymmetric standard model with right-handed neutrinos in long baseline experiments 

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#### Abstract

The effective interactions which violate lepton flavor accompanied with neutrinos (nLFV) are considered. Such new physics effects are expected to be measured in future neutrino oscillation experiments with a long baseline. They are induced by radiative corrections in the framework of the minimal supersymmetric standard model with right-handed neutrinos. We numerically evaluate the size of nLFV interactions in this framework. The slepton mixing is not only the origin of lepton flavor violation in the charged lepton sector (cLFV) but also that of the nLFV. We find that the nLFV couplings are strongly correlated with the corresponding cLFV processes, and they are constrained to be $\mathcal{O}\left(10^{-5}\right)$ times smaller than the standard four-Fermi couplings.


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## I. INTRODUCTION

Numerous observations of neutrinos from the Sun [1-6], the atmosphere [7-9], and from reactor [10] and accelerator [11] experiments suggest that neutrinos are massive and, hence, there are mixings in the lepton sector. This fact means that the standard model (SM) has to be extended so that the neutrino masses and the lepton mixings are introduced into the model. Lots of models to explain those experimental results have been proposed. Among them, a model with the seesaw mechanism [12] has a promising attribute, in which tiny neutrino masses are naturally induced. Neutrino experiments have also revealed that the mixings in the lepton sector are much larger than those in the quark sector. This fact may imply that the lepton flavor number is strongly violated in the physics beyond the SM. Therefore, we can expect that nature might exhibit sizable lepton flavor violation (LFV) and, hence, that we could observe the remains of physics at high energy scales. In the minimal supersymmetric standard model with heavy right-handed neutrinos (MSSMRN), in which the seesaw mechanism is realized, the LFV with charged leptons (cLFV) is expected to become large [13-15]. In this class of models, the renormalization effect due to the neutrino Yukawa couplings induces a significant size of the off-diagonal elements of the slepton mass matrix, $\left(m_{\tilde{L}}^{2}\right)_{\alpha}{ }^{\beta}$ $(\alpha \neq \beta, \alpha, \beta=e, \mu, \tau)$, which are the seeds for the cLFV. Here the flavor indices $\alpha$ and $\beta$ should be understood to indicate the mass eigenstates of the charged lepton fields. Concretely, the superpotential, with the neutrino Yukawa couplings $\left(f_{\nu}\right)_{i}^{\alpha}$ and the Majorana masses for the righthanded neutrinos $M^{i j}$, includes the following terms:

$$
\begin{equation*}
W \supset\left(f_{\nu}\right)_{i}^{\alpha} \bar{N}^{i} H_{u} \epsilon L_{\alpha}+\frac{1}{2} M^{i j} N_{i} N_{j} \tag{1}
\end{equation*}
$$

[^0]where $N_{i}, L_{\alpha}$, and $H_{u}$ denote the chiral supermultiplet for the right-handed neutrinos, the lepton doublets, and the uptype Higgs field, respectively, and $\epsilon$ is the antisymmetric tensor for the $S U(2)_{L}$ fundamental representation. The indices $i$ and $j$ stand for the generation of right-handed neutrinos, which do not necessarily indicate the mass eigenstates. The renormalization group equation for the off-diagonal elements of the slepton mass matrix is given as [13]
\[

$$
\begin{align*}
\mu \frac{d\left(m_{\tilde{L}}^{2}\right)_{\alpha}^{\beta}}{d \mu}= & \frac{1}{16 \pi^{2}}\left[m_{\tilde{L}}^{2} f_{\nu}^{\dagger} f_{\nu}+f_{\nu}^{\dagger} f_{\nu} m_{\tilde{L}}^{2}+2\left(f_{\nu}^{\dagger} m_{\tilde{\nu}}^{2} f_{\nu}\right.\right. \\
& \left.\left.+\tilde{m}_{H_{u}}^{2} f_{\nu}^{\dagger} f_{\nu}+A_{\nu}^{\dagger} A_{\nu}\right)\right]_{\alpha}^{\beta}, \quad(\alpha \neq \beta) \tag{2}
\end{align*}
$$
\]

where $m_{\tilde{\nu}}^{2}$ is the soft supersymmetry (SUSY) breaking mass matrix for the right-handed sneutrino and $\tilde{m}_{H_{u}}^{2}$ is that for the up-type Higgs doublet. The matrix $A_{\nu}$ denotes the trilinear scalar couplings corresponding to the first term in Eq. (1). Note that if the neutrino Yukawa couplings do not exist, there is no LFV effect. We can approximately solve Eq. (2) as

$$
\begin{equation*}
\left(\Delta m_{\tilde{L}}^{2}\right)_{\alpha}^{\beta} \simeq-\frac{\left(6+2 a_{0}^{2}\right) m_{0}^{2}}{16 \pi^{2}}\left(f_{\nu}^{\dagger} f_{\nu}\right)_{\alpha}^{\beta} \ln \frac{M_{G}}{M_{R}} \tag{3}
\end{equation*}
$$

with a cutoff scale $M_{G}$ and a typical mass scale for the right-handed neutrinos $M_{R}$. Here universal soft SUSY breaking at $M_{G}$ is assumed, and $m_{0}$ is the parameter for sfermion masses and $a_{0}$ is that for scalar trilinear couplings. In terms of the mass insertion method, we can see that the off-diagonal elements of the slepton mass matrix are the origin of cLFV. This is diagrammatically shown in Fig. 1. From this diagram, it is obvious that the element $\left(\Delta m_{\tilde{L}}^{2}\right)_{\alpha}^{\beta}$ is relevant to the process $l_{\beta} \rightarrow l_{\alpha} \gamma$. In this class of models, the off-diagonal elements can become large, so that the typical values of predicted branching ratios are within a sensitivity reach of near future experiments [16-


FIG. 1. One of the diagrams which contribute to the cLFV process $l_{\beta} \rightarrow l_{\alpha} \gamma$. This effect is approximately understood by insertion of the LFV mass term $\left(\Delta m_{\tilde{L}}^{2}\right)_{\alpha}{ }^{\beta}(\alpha \neq \beta)$.

18]. Therefore, the search for cLFV processes is one of the promising ways to inspect new physics effects beyond the SM.

In this article, however, we investigate an alternative approach to explore the LFV, the search for the processes of the LFV with neutrinos (nLFV) at a long baseline (LBL) neutrino oscillation experiment. In the forthcoming experiments, the oscillation parameters such as the mixing angles and the squared mass differences are expected to be determined with high precision [19-21]. Therefore, the measurement of nLFV effects might be possible. The feasibility studies on the nLFV interaction search at future LBL experiments, without assuming a specific model, have been made by Refs. [22-31]. The current experimental bounds on nLFV interactions are given in Ref. [32]. The sensitivity of solar neutrinos [33], atmospheric neutrinos [34], the Liquid Scintillator Neutrino Detector experiment at Los Alamos results [35], and supernova neutrinos [36] to nLFV effects have also been considered. It was pointed out that the nLFV signal is enhanced by the interference effect between the amplitude including nLFV interactions and that of the standard oscillation (SO, which means the neutrino oscillation with SM interactions).

Here we investigate the nLFV interactions in the MSSMRN. ${ }^{1}$ The origins of nLFV processes are the same as those of cLFV processes, which are the off-diagonal elements of the slepton mass matrix. They can become significant in this framework. In addition, there is an enhancement mechanism due to the interference effect. It can be expected that a detectable magnitude of nLFV effects is induced. We make numerical calculations of the size of the nLFV couplings and show a correlation between nLFV and cLFV.

[^1]In Sec. II, we recapitulate the model-independent approach in the detection of nLFV effects at LBL neutrino oscillation experiments. We also show the way to parametrize the nLFV interactions. In Sec. III, we calculate these nLFV couplings in the MSSMRN and numerically evaluate the size of them under the universal soft SUSY breaking scenario, the so-called constrained MSSMRN. Here we concentrate our attention on the nLFV interactions which are relevant in the oscillation channel $\nu_{\mu} \rightarrow \nu_{\tau}$. Finally, in Sec. IV we will give a summary. In Appendix A, we describe the model in order to make our notation clear, and in Appendix $B$ we give formulas of the nLFV interactions.

## II. NLFV INTERACTION IN NEUTRINO OSCILLATION

In this section, we explain how to parametrize the new physics effects in a model-independent way [26]. First, we note that in neutrino oscillation experiments we do not observe neutrinos themselves but do observe their products, i.e., their corresponding charged leptons. Propagating neutrinos appear only in intermediate states. Therefore, the existence of nLFV interactions suggests that there are some amplitudes whose initial and final states are the same as those of the SO.

To make the argument clear, we show an example. When we assume the measurement of $\nu_{\mu} \rightarrow \nu_{\tau}$ oscillations at a neutrino factory experiment, all we can measure is the decay of muons at a muon storage ring and the appearance of tau leptons in a detector, which is located at a length $L$ away from the source of the neutrino beam, just after the time $L / c$, where $c$ is the speed of light. We interpret these events as the evidence of the $\mathrm{SO}, \nu_{\mu} \rightarrow \nu_{\tau}$. The amplitude for this process $A_{\text {SO }}$ can be expressed by the product of the amplitudes for the subprocesses:

$$
\begin{align*}
& A_{\mathrm{SO}}\left(\mu^{-}+I \rightarrow \tau^{-}+F\right) \\
& \quad=A_{s}\left(\mu^{-} \rightarrow \nu_{\mu} \bar{\nu}_{e} e^{-}\right) A_{p}\left(\nu_{\mu} \xrightarrow{\text { osc. }} \nu_{\tau}\right) A_{d}\left(\nu_{\tau} d \rightarrow \tau^{-} u\right), \tag{4}
\end{align*}
$$

where $I(F)$ denotes all the other particles besides the muon (tau lepton) in the initial (final) state, which can be measured in principle. In this example, $I$ is $d$ (a down-type quark) in $A_{d}$, and $F$ means $\bar{\nu}_{e}^{2}$ and $e^{-}$in $A_{s}$ and $u$ (an uptype quark) in $A_{d}$. The subscripts $s, p$, and $d$ indicate "at the source of the neutrino beam," "at the propagation process," and "at the detection process," respectively. Suppose that there is an effective four-Fermi nLFV interaction,

$$
\begin{equation*}
\mathcal{L}_{\mathrm{eff}}=\lambda\left(\bar{e} \gamma^{\rho} \mathrm{P}_{L} \mu\right)\left(\bar{\nu}_{\tau} \gamma_{\rho} \mathrm{P}_{L} \nu_{e}\right)+\text { h.c. } \tag{5}
\end{equation*}
$$

[^2]then we have the same signal through another process than Eq. (4):
\[

$$
\begin{align*}
& A_{\mathrm{nLFV}}\left(\mu^{-}+I \rightarrow \tau^{-}+F\right) \\
& \quad=A_{s}\left(\mu^{-} \rightarrow \nu_{\tau} \bar{\nu}_{e} e^{-}\right) A_{p}\left(\nu_{\tau} \xrightarrow{\text { no osc. }} \nu_{\tau}\right) A_{d}\left(\nu_{\tau} d \rightarrow \tau^{-} u\right) . \tag{6}
\end{align*}
$$
\]

The external particles in Eq. (6) are completely the same as those in Eq. (4). Therefore, we cannot distinguish the contributions from these two amplitudes. In quantum mechanics, we first sum up these amplitudes and next square the summation in order to obtain the transition rate. Therefore, an interference term arises between these two amplitudes for this process ${ }^{3}$ :

$$
\begin{align*}
P\left(\mu^{-}+I\right. & \left.\rightarrow \tau^{-}+F\right) \\
& =\left|A_{\mathrm{SO}}\right|^{2}+2 \operatorname{Re}\left[A_{\mathrm{SO}}^{*} A_{\mathrm{nLFV}}\right]+\left|A_{\mathrm{nLFV}}\right|^{2} . \tag{7}
\end{align*}
$$

The term of the SO, the first term on the right-hand side, gives the leading contribution, and it is described by using the muon decay width $\Gamma\left[=\left|A_{s}\left(\mu^{-} \rightarrow \nu_{\mu} \bar{\nu}_{e} e^{-}\right)\right|^{2}\right]$ and the cross section for the charged current interaction $\sigma$ [= $\left.\left|A_{d}\left(\nu_{\tau} d \rightarrow \tau^{-} u\right)\right|^{2}\right]$ as

$$
\begin{equation*}
\left|A_{\mathrm{SO}}\right|^{2}=\Gamma \times P_{\nu_{\mu} \rightarrow \nu_{\tau}} \times \sigma, \tag{8}
\end{equation*}
$$

where $P_{\nu_{\mu} \rightarrow \nu_{\tau}}$ is the oscillation probability for $\nu_{\mu} \rightarrow \nu_{\tau}$ in the SO, which is defined as $\left|A_{p}\left(\nu_{\mu} \xrightarrow{\text { osc. }} \nu_{\tau}\right)\right|^{2}$. The second term which represents the interference between the amplitude of the SO Eq. (4) and that including the nLFV interaction Eq. (6) is

$$
\begin{align*}
2 \operatorname{Re}\left[A_{\mathrm{SO}}^{*} A_{\mathrm{nLFV}}\right]= & \Gamma \times 2 \operatorname{Re}\left[\frac{\lambda}{2 \sqrt{2} G_{F}} A_{p}^{*}\left(\nu_{\mu} \xrightarrow{\text { osc. }} \nu_{\tau}\right)\right. \\
& \left.\times A_{p}\left(\nu_{\tau} \xrightarrow{\text { no osc. }} \nu_{\tau}\right)\right] \times \sigma, \tag{9}
\end{align*}
$$

where $G_{F}$ is the Fermi constant. Note that the nLFV effect contributes to the oscillation probability not quadratically but linearly. Thus, the effect can be enhanced, and, hence, even if the nLFV coupling is small, it can contribute to the oscillation probability significantly [26-28].

We now turn to the parametrization of effective couplings for nLFV interactions. As we have already shown, the amplitude for the neutrino oscillation process can be divided into three pieces, $A_{s}, A_{p}$, and $A_{d}$. First, we consider the decay process of parent particles, $A_{s}$. Since all final states must be the same, nLFV interactions with $(V-A) \times$ ( $V-A$ ) type are important for the neutrino factory experiment [26]. We can introduce the interference effect by treating the initial state of a propagating neutrino as the

[^3]superposition of all flavor eigenstates. For the case of Eq. (5), we can take the initial neutrino state $|\nu\rangle$ as
\[

$$
\begin{equation*}
|\nu\rangle=\left|\nu_{\mu}\right\rangle+\epsilon_{\mu \tau}^{s}\left|\nu_{\tau}\right\rangle, \tag{10}
\end{equation*}
$$

\]

where $\epsilon_{\mu \tau}^{s}=\lambda /\left(2 \sqrt{2} G_{F}\right)$. It can be generalized to the case of an initial neutrino with any flavor by using the source state notation which is introduced in Ref. [22] as ${ }^{4}$

$$
\left|\nu_{\beta}^{s}\right\rangle=\left(U^{s}\right)_{\beta}{ }^{\alpha}\left|\nu_{\alpha}\right\rangle, \quad U^{s} \equiv\left(\begin{array}{ccc}
1 & \epsilon_{e \mu}^{s} & \epsilon_{e \tau}^{s}  \tag{11}\\
\epsilon_{\mu e}^{s} & 1 & \epsilon_{\mu \tau}^{s} \\
\epsilon_{\tau e}^{s} & \epsilon_{\tau \mu}^{s} & 1
\end{array}\right) .
$$

We can include the total nLFV effect into the oscillation probability as

$$
\begin{equation*}
\left.P_{\nu_{\alpha}^{s} \rightarrow \nu_{\beta}}=\left|\left\langle\nu^{\beta}\right|\left(e^{-i H_{\mathrm{s} 0} L}\right)_{\beta}{ }^{\alpha}\left(U^{s}\right)_{\alpha}{ }^{\gamma}\right| \nu_{\gamma}\right\rangle\left.\right|^{2}, \tag{12}
\end{equation*}
$$

with the propagation Hamiltonian for the $\mathrm{SO}, H_{\mathrm{SO}}$ :

$$
\begin{align*}
\left(H_{\mathrm{SO}}\right)_{\beta}{ }^{\alpha}= & \frac{1}{2 E_{\nu}}\left\{\left(U_{\mathrm{MNS}}\right)_{\beta}^{i}\left(\begin{array}{lll}
0 & & \\
& \Delta m_{21}^{2} & \\
& & \Delta m_{31}^{2}
\end{array}\right)\right. \\
& \times\left(U_{\mathrm{MNS}}^{\dagger} i_{i}^{\alpha}+\left(\begin{array}{lll}
\bar{a} & & \\
& 0 & \\
& & 0
\end{array}\right)_{\beta}^{\alpha}\right\}, \tag{13}
\end{align*}
$$

where $E_{\nu}$ is the neutrino energy, $\bar{a}$ is the usual matter effect which is given as $2 \sqrt{2} G_{F} n_{e} E_{\nu}$ by using the electron number density $n_{e}, U_{\mathrm{MNS}}$ is the mixing matrix for the lepton sector, and $\Delta m_{21}^{2}\left(\Delta m_{31}^{2}\right)$ is the mass squared difference for the solar (atmospheric) neutrino oscillation.

Next, we consider the propagation process, $A_{p}$. The nLFV interactions modify the Hamiltonian for neutrino propagation as [24]

$$
\left(H_{\mathrm{nLFV}}\right)_{\beta}^{\alpha}=\left(H_{\mathrm{SO}}\right)_{\beta}^{\alpha}+\frac{1}{2 E_{\nu}}\left(\begin{array}{ccc}
a_{e e} & a_{e \mu} & a_{e \tau}  \tag{14}\\
a_{e \mu}^{*} & a_{\mu \mu} & a_{\mu \tau} \\
a_{e \tau}^{*} & a_{\mu \tau}^{*} & a_{\tau \tau}
\end{array}\right)_{\beta}^{\alpha}
$$

where $a_{\alpha \beta}$ is the extra matter effect due to nLFV interactions, which is defined as $a_{\alpha \beta}=\sum_{p=e, d, u} 2 \sqrt{2} \times$ $\epsilon_{\alpha \beta}^{m, p} G_{F} n_{p} E_{\nu}$, where $n_{p}$ is the number density for the particle $p$. Assuming matter which consists of the same number of electrons, neutrons, and protons, we can reduce it to

$$
\begin{equation*}
a_{\alpha \beta}=2 \sqrt{2} \epsilon_{\alpha \beta}^{m} G_{F} n_{e} E_{\nu}, \tag{15}
\end{equation*}
$$

with

$$
\begin{equation*}
\epsilon_{\alpha \beta}^{m} \equiv \epsilon_{\alpha \beta}^{m, e}+3 \epsilon_{\alpha \beta}^{m, d}+3 \epsilon_{\alpha \beta}^{m, u} . \tag{16}
\end{equation*}
$$

Note that to consider the magnitude of the matter effect, the

[^4]type of interaction is irrelevant since matter particles are at rest, and, hence, the dependence on their chirality is averaged out [40].

Finally, we make a comment on nLFV interactions which affect a detection process, $A_{d}$. We can adopt a quite similar treatment compared to that at the source of the neutrino beam. In this article, we consider the case in which the nLFV interactions do not depend on the flavor of the target quarks, which is almost the case for the socalled constrained MSSMRN. Therefore, we have the neutrino state for the detection process in the following form:

$$
\left|\nu_{\beta}^{d}\right\rangle=\left(U^{d}\right)_{\beta}^{\alpha}\left|\nu_{\alpha}\right\rangle, \quad U^{d} \equiv\left(\begin{array}{ccc}
1 & \boldsymbol{\epsilon}_{e \mu}^{d} & \boldsymbol{\epsilon}_{e \tau}^{d}  \tag{17}\\
\boldsymbol{\epsilon}_{\mu e}^{d} & 1 & \boldsymbol{\epsilon}_{\mu \tau}^{d} \\
\boldsymbol{\epsilon}_{\tau e}^{d} & \boldsymbol{\epsilon}_{\tau \mu}^{d} & 1
\end{array}\right)
$$

Finally, the transition probability including the whole nLFV effects is given by

$$
\begin{equation*}
P\left(\mu^{-}+I \rightarrow \tau^{-}+F\right) \simeq \Gamma \times P_{\nu_{\alpha}^{s} \rightarrow \nu_{\beta}^{d}} \times \sigma, \tag{18}
\end{equation*}
$$

where

$$
\begin{equation*}
\left.P_{\nu_{\alpha}^{s} \rightarrow \nu_{\beta}^{d}}=\left|\left\langle\nu^{\delta}\right|\left(U^{d \dagger}\right)_{\delta}{ }^{\beta}\left(e^{-i H_{\mathrm{nLFV}} L}\right)_{\beta}^{\alpha}\left(U^{s}\right)_{\alpha}^{\gamma}\right| \nu_{\gamma}\right\rangle\left.\right|^{2} . \tag{19}
\end{equation*}
$$

## III. NLFV INTERACTIONS IN THE MSSMRN

In this section, we evaluate the effective couplings for the nLFV interactions in the MSSMRN which are induced by the off-diagonal elements of the slepton mass matrix. They are similar to the cLFV interactions. We can naively estimate the size of the nLFV parameter $\epsilon_{\alpha \beta}^{s}$ from the diagram in Fig. 2 to be [41]

$$
\begin{equation*}
\epsilon_{\alpha \beta}^{s}\left(\sim \epsilon_{\beta \alpha}^{s}\right) \sim \frac{g_{2}^{2}}{16 \pi^{2}} \frac{\left(\Delta m_{\tilde{L}}^{2}\right)_{\beta}^{\alpha}}{m_{\mathrm{S}}^{2}} \sim m_{\mathrm{S}}^{2} \sqrt{\operatorname{Br}\left(l_{\beta} \rightarrow l_{\alpha} \gamma\right)} \tag{20}
\end{equation*}
$$

Here $g_{2}$ is the gauge coupling for $S U(2)_{L}$, and $m_{\mathrm{S}}$ is the


FIG. 2. One of the diagrams which contribute to $\epsilon_{\beta \alpha}^{s}$.
typical SUSY breaking scale. This relation means that there is a correlation between the two processes $l_{\beta}^{-} \rightarrow$ $\nu_{\alpha} \bar{\nu}_{e} e^{-}$and $l_{\beta} \rightarrow l_{\alpha} \gamma$. In the following, we concentrate on the nLFV associated with the tau lepton because the $n L F V$ in the $\mu-e$ sector is strongly constrained by the corresponding cLFV processes. The current experimental bound on the branching ratio of $\tau \rightarrow \mu \gamma$ is $3.1 \times 10^{-7}$ at $90 \%$ confidence level [42], and then this experimental bound constrains the nLFV coupling parameter $\epsilon_{\mu \tau}^{S}$. According to this naive estimate, the value of $\epsilon_{\mu \tau}^{s}$ may become as large as $\mathcal{O}\left(10^{-4}\right)$, which would be detected in a future LBL experiment such as the neutrino factory [26].

## A. Analytic calculation of $\boldsymbol{\epsilon}_{\boldsymbol{\alpha} \boldsymbol{\beta}}^{\boldsymbol{s}, \boldsymbol{m}, \boldsymbol{d}}$

In this subsection, we explain the calculation to obtain $\epsilon_{\alpha \beta}^{s, m, d}$ in detail and compare them with that of cLFV processes. The thorough results are given in Appendix B.

For example, the effective Lagrangian relevant for the nLFV interactions giving a potentially significant contribution to the oscillation $\nu_{\mu} \rightarrow \nu_{\tau}$ in a neutrino factory is given by [26]

$$
\begin{align*}
\mathcal{L}_{\mathrm{eff}}^{\mathrm{nLFV}}= & 2 \sqrt{2} G_{F} \epsilon_{\mu \tau}^{s}\left(\bar{\nu}_{\tau} \gamma^{\rho} \mathrm{P}_{L} \nu_{e}\right)\left(\bar{e} \gamma_{\rho} \mathrm{P}_{L} \mu\right) \\
& +2 \sqrt{2} G_{F} \sum_{p=e, d, u} \epsilon_{\mu \tau}^{m, p}\left(\bar{\nu}_{\tau} \gamma^{\rho} \mathrm{P}_{L} \nu_{\mu}\right)\left(\bar{p} \gamma_{\rho} p\right) \\
& +2 \sqrt{2} G_{F} \epsilon_{\mu \tau}^{d}\left(\bar{\tau} \gamma^{\rho} \mathrm{P}_{L} \nu_{\mu}\right)\left(\bar{u} \gamma_{\rho} \mathrm{P}_{L} d\right) \tag{21}
\end{align*}
$$

These effective couplings arise from penguin-type diagrams and from box-type diagrams as they are shown in Appendix B. The calculation of box diagrams is straightforward. It is almost the same as for cLFV processes such as $\mu$-e conversion [14] except for the fact that only the $(V-A)(V-A)$ type interactions are taken into account in the calculation of $\epsilon_{\alpha \beta}^{s}$ and $\epsilon_{\alpha \beta}^{d}$. However, it is necessary to make a careful calculation of the penguin-type diagrams. In general, while the neutral and electromagnetic currents corresponding to Fig. 1 take the form

$$
\begin{align*}
& \bar{f}^{\alpha}(p-q)\left[i\left\{\mathfrak{A H}\left(q^{2}\right)+\mathfrak{B}\left(q^{2}\right) \gamma_{5}\right\} q_{\nu} \sigma^{\mu \nu}+\left\{\mathscr{C}\left(q^{2}\right)\right.\right. \\
& \left.\left.\quad+\mathfrak{D}\left(q^{2}\right) \gamma_{5}\right\} \gamma^{\mu}+\left\{\mathfrak{H}\left(q^{2}\right)+\mathfrak{F}\left(q^{2}\right) \gamma_{5}\right\} q^{\mu}\right] f_{\beta}(p)+\text { h.c. } \tag{22}
\end{align*}
$$

the charged current corresponding to Fig. 2 is decomposed to

$$
\begin{align*}
& \bar{\nu}^{\alpha}(p-q) \mathrm{P}_{R}\left\{i \mathfrak{U l}^{\prime}\left(q^{2}\right) q_{\nu} \sigma^{\mu \nu}+\mathfrak{5}^{\prime}\left(q^{2}\right) \gamma^{\mu}\right. \\
& \left.\quad+\mathfrak{K}^{\prime}\left(q^{2}\right) q^{\mu}\right\} l_{\beta}(p)+\text { h.c. } \tag{23}
\end{align*}
$$

Here $f$ denotes the charged lepton field $l$ or the neutrino field $\nu, p$ is the momentum of the incoming particle, and $q$ is that of the gauge boson. All the coefficients, $\mathfrak{H}\left(\mathfrak{A}^{\prime}\right), \mathfrak{B}$, $\mathfrak{C}\left(\mathfrak{C}^{\prime}\right), \mathfrak{D}, \mathfrak{C}\left(\mathfrak{C}^{\prime}\right)$, and $\mathfrak{F}$, are the functions of $q^{2}$. In the limit $q^{2} \rightarrow 0$, $\mathfrak{C}$ and $\mathfrak{D}$ for the electromagnetic current must vanish due to the gauge symmetry $U(1)_{\mathrm{em}}$. On the
other hand, those for the neutral and the charged current do not vanish because the corresponding gauge symmetry $S U(2)_{L}$ is broken. In other words, the Lorentz structure of the nLFV interaction which is shown in Fig. 2 is different from that of the cLFV process $l_{\beta} \rightarrow l_{\alpha} \gamma$ with a real photon emission. The former one is dominated by the vector exchange interaction $\left[\mathbb{C}^{\prime}(0)\right],{ }^{5}$ while the latter one is due to a dipole type interaction $\left[\mathfrak{H}(0)+\mathfrak{B}(0) \gamma_{5}\right]$. Therefore, they are definitely correlated with each other but are not the same functions.

In the calculation of nLFV diagrams, we have to pay attention to the following two facts: (i) we regard neutrinos as highly off-shell particles, and (ii) we must avoid counting one contribution twice. First, we explain the reason why neutrinos behave as highly off-shell particles, and, hence, it is necessary to treat them as massless particles in diagrams for effective nLFV interactions. In Fig. 3, the condition for neutrino oscillation to occur is described as [43]

$$
\begin{equation*}
\delta(p-q) \sim \frac{1}{\delta x} \gg \frac{1}{L} \sim \frac{\Delta m^{2}}{p-q} \tag{24}
\end{equation*}
$$

from the viewpoint of the uncertainty principle. Here $\delta x$ is the uncertainty in the position and in the time in which neutrinos are produced. $\delta(p-q)$ is the uncertainty of the energy momentum of the outgoing neutrino, and $\Delta m^{2}$ is the neutrino mass squared difference corresponding to an LBL oscillation experiment. The inequality results from the fact that the neutrino production position must be determined much more accurately than the baseline length $L$. The observation of neutrino oscillation phenomena in an LBL experiment requires that the equality on the righthand side should be satisfied. Thus, the uncertainty of the squared momentum must conform to the following relation:

$$
\frac{(p-q)^{2}}{(p-q)^{2}-m_{\nu_{\beta}}^{2}} \Rightarrow\left\{\begin{array}{l}
1 \\
\frac{m_{\nu_{\alpha}}^{2}}{m_{\nu_{\alpha}}^{2}-m_{\nu_{\beta}}^{2}}
\end{array}\right.
$$

Next we turn to the problem of double counting and show the way to solve it. First, we note that we must calculate the process $\mu^{-}+I \rightarrow \tau^{-}+F$ with the methods of field theory. Hence, we must not calculate the nLFV effect for each stage because we cannot observe each stage separately. If we calculate the diagrams shown in Fig. 4(a) for $\epsilon_{\mu \tau}^{s}$ and Fig. 4(b) for $\epsilon_{\mu \tau}^{m}$, then it will mean that we count twice the diagram of Fig. 4(c) in the calculation of the process $\mu^{-}+I \rightarrow \tau^{-}+F$. In order to avoid double counting, we have to get rid of one of them. For example, we should not include the contribution of the diagram of Fig. 4(b) in $\epsilon_{\mu \tau}^{m}$. A similar situation occurs between $\epsilon_{\mu \tau}^{d}$

[^5]

FIG. 3 (color online). One example of the diagrams in which we have to take into account the off-shellness for the neutrino propagation. In the neutrino propagator, which is pointed out by the oval, we must treat the neutrino as if it were a massless particle.

$$
\begin{equation*}
\delta\left\{(p-q)^{2}\right\} \gg \Delta m^{2} \quad \text { then }\left\langle(p-q)^{2}\right\rangle \gg m_{\nu}^{2} \tag{25}
\end{equation*}
$$

where $\left\langle(p-q)^{2}\right\rangle$ denotes the average of $(p-q)^{2}$. This inequality shows that the average of the neutrino momentum is much larger than its mass, and it follows from this that neutrinos are generally highly off-shell fields in oscillation experiments. Furthermore, this means that all diagrams for the nLFV interactions include off-shell neutrinos as external lines. In the exact meaning of field theory, we do not have a method to calculate diagrams with off-shell external legs. However, we can evaluate these diagrams by treating the neutrinos as massless. For practical purposes, we make the following replacement which we refer to as the off-shell prescription in calculations of the type of diagram shown in Fig. $3^{6}$ :
(off-shell prescription),
(for usual on-shell particle case).
and $\epsilon_{\mu \tau}^{m}$. The penguin contribution to $\epsilon_{\mu \tau}^{d}$ is essentially given by the complex conjugate of the corresponding one to $\epsilon_{\tau \mu}^{s}$. However, we must eliminate the contribution from diagrams similar to Fig. 4(a) from $\epsilon_{\mu \tau}^{d}$.

Finally, we should notice another important aspect of the double counting problem; to which stage (source or matter) does the contribution of Fig. 4(c) belong? For example, instead of removing the contribution in Fig. 4(b) in $\epsilon_{\mu \tau}^{m}$, we can eliminate the contribution in Fig. 4(a) in $\epsilon_{\mu \tau}^{s}$. For this ambiguity, we adopt the way to divide the diagram into each stage so that the epsilon parameters in each stage will disappear in the limit where $m_{\mathrm{S}} \rightarrow \infty$ after using the off-

[^6]

FIG. 4. Schematic explanation of the double counting problem. If we count diagram (a) into $\boldsymbol{\epsilon}_{\mu \tau}^{s}$ and we also do diagram (b) into $\boldsymbol{\epsilon}_{\mu \tau}^{m}$, then we count diagram (c) twice.
shell prescription. The $S U(2)_{L}$ symmetry is recovered and then $\epsilon_{\mu \tau}^{s, m, d}$ should disappear in the large SUSY scale limit. This is shown analytically. Note that each diagram gives a rather large contribution and stays almost constant in the large SUSY scale limit. The cancellation among the diagrams is highly nontrivial. Therefore, we can be confident about the legitimacy of our treatment for the internal neutrino lines by checking the cancellation among the diagrams.

## B. Numerical study

A numerical study to evaluate the epsilon parameters is necessary in order to make it clear whether our naive estimate Eq. (20) is correct. We here assume the universal soft SUSY breaking at $M_{G}$ and adopt the scenario of the
radiative electroweak symmetry breaking [44]. The details of the calculations are shown in the appendices. We scan the values for the soft SUSY breaking parameters $m_{0}$ and $M_{1 / 2}$ in the range of $100-1000 \mathrm{GeV}$ and also scan the elements of the neutrino Yukawa matrix. In addition, we take the normal hierarchy for the neutrino mass matrix. The scatter plots for the size of $\epsilon_{\mu \tau}^{s}, \epsilon_{\mu \tau}^{m}$, and $\boldsymbol{\epsilon}_{\mu \tau}^{d}$ are presented in Fig. 5. All of them show a similar behavior. As expected from Eq. (20), the nLFV parameters are correlated with the branching ratio of the process $\tau \rightarrow$ $\mu \gamma$. However, the size of the parameters is much smaller than that of the naive estimate. This can be understood by the cancellation among the diagrams contributing to the nLFV interaction. In the $S U(2)_{L}$ symmetric limit, the diagrams for the penguin contribution to the nLFV interaction must cancel each other out because of the gauge symmetry.


FIG. 5 (color online). Scatter plots for size of nLFV parameters $\epsilon_{\mu \tau}^{s}, \epsilon_{\mu \tau}^{m}$, and $\epsilon_{\mu \tau}^{d}$. Dots are for $\tan \beta=10$ and triangles are for $\tan \beta=3$. We here fix the soft SUSY breaking parameters as $a_{0}=0$ and $\mu>0$.

Since the diagrams for nLFV interactions are induced at the scale $m_{\mathrm{S}}$, in the limit where $m_{\mathrm{S}} \gg m_{Z}$, the $S U(2)_{L}$ symmetry is assumed to be recovered. Even in the case where $m_{\mathrm{S}} \sim \mathcal{O}(100) \mathrm{GeV}$, the cancellation is rather significant. Thus, our naive estimate must be modified so that the additional suppression factor $m_{Z} / m_{\mathrm{S}}$ is introduced. It may be worth pointing out that the epsilon parameters do not strongly depend on the value of $\tan \beta$ unlike the branching ratio of the cLFV process. This arises from the difference in the structure of the chirality in each process. Furthermore, the process $\tau \rightarrow \mu \gamma$ is dominated by the diagrams including the left-right mixing of the slepton which is proportional to $\tan \beta$. The nLFV processes do not pick up such a left-right mixing term because a chirality flip is not necessary. Therefore, the search for nLFV may be advantageous for relatively small $\tan \beta$. We also note that it is obvious from Eq. (3) that both nLFV and cLFV are enhanced when $a_{0}$ takes a large value.

## IV. SUMMARY

We summarize our study and give some discussions. It is known that the magnitude of nLFV couplings can become large enough to be detected at future LBL experiments within a model-independent approach. Moreover, we considered the nLFV interactions in the MSSMRN in the universal soft SUSY breaking scenario, which is one of the most promising candidates for the physics beyond the SM.

We find that in this scenario the nLFV couplings cannot be significant, and, hence, it is quite difficult to observe these effects in future oscillation experiments. The reason they are strongly suppressed is that the $S U(2)_{L}$ gauge symmetry is approximately maintained. All the particles and the interactions which can generate nLFV interactions respect the $S U(2)_{L}$ symmetry in the limit $m_{\mathrm{S}} \rightarrow \infty$. Although each diagram contributing to nLFV interactions can become large, a brilliant cancellation among those diagrams occurs. Therefore, the penguin contributions are strongly suppressed.

We adopt the approximation for the calculation of penguin diagrams which is explained in Sec. III A. In order to confirm the validity of this approximation, we need to
make the calculation for the process shown in Fig. 4(c) by using the methods of field theory. However, the calculation which we adopt here must be reliable in the sense of field theory because in the limit $m_{\mathrm{S}} \rightarrow \infty$ the recovery of $S U(2)_{L}$ gauge symmetry is obviously maintained in our calculation.

Finally, we mention our future work. Since the decay process of a muon differs from that of a pion, we might expect that a sizable new physics effect exists only in the decay of a pion. Thus, it is necessary to investigate nLFV effects in the MSSMRN in superbeam experiments individually. Furthermore, as the epsilon parameters can be still significant within a model-independent approach and there are lots of other models than the constrained MSSMRN which can explain the neutrino masses and the lepton mixings, we need to examine such possibilities.

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## APPENDIX A: MODEL

We basically follow the notation which is adopted in Ref. [14]. However, we pay attention to the fact that the mixing matrices to diagonalize the mass matrices for sfermions, chargino, and neutralino are in general complex matrices.

The mass matrices for charged sleptons, down-type squarks, and up-type squarks are

$$
\begin{align*}
-\mathcal{L}_{\tilde{f}} & =\left(\begin{array}{ll}
\tilde{f}_{L}^{*} & \tilde{f}_{R}^{*}
\end{array}\right)^{\alpha}\left(M_{\tilde{f}}^{2}\right)_{\alpha}^{\beta}\binom{\tilde{f}_{L}}{\tilde{f}_{R}}_{\beta} \\
& =\left(\begin{array}{ll}
\tilde{f}_{L}^{*} & \tilde{f}_{R}^{*}
\end{array}\right)\left(\begin{array}{cc}
M_{\tilde{f}_{L L}}^{2} & \left(M_{\tilde{f}_{L R}}^{2}\right)^{\dagger} \\
M_{\tilde{f}_{L R}}^{2} & M_{\tilde{f}_{R R}}
\end{array}\right)\binom{\tilde{f}_{L}}{\tilde{f}_{R}}, \tag{A1}
\end{align*}
$$

where

$$
\begin{align*}
& M_{\tilde{f}_{L L}}^{2}= \begin{cases}m_{\tilde{f}_{L}}^{2}+\left(f_{f}^{\dagger} f_{f}\right) \frac{v^{2}}{\sqrt{2}} \sin ^{2} \beta+m_{Z}^{2} \cos 2 \beta\left\{\left(\frac{1}{2}\right)-Q_{\mathrm{em}}^{f} \sin ^{2} \theta_{W}\right\}, & (f=u), \\
m_{\tilde{f}_{L}}^{2}+\left(f_{f}^{\dagger} f_{f}\right) \frac{v^{2}}{\sqrt{2}} \cos ^{2} \beta+m_{Z}^{2} \cos 2 \beta\left\{\left(-\frac{1}{2}\right)-Q_{\mathrm{em}}^{f} \sin ^{2} \theta_{W}\right\}, & (f=l, d),\end{cases}  \tag{A2}\\
& M_{\tilde{f}_{R R}}^{2}= \begin{cases}m_{\tilde{f}_{R}}^{2}+\left(f_{f} f_{f}^{\dagger}\right) \frac{v^{2}}{\sqrt{2}} \sin ^{2} \beta-m_{Z}^{2} \cos 2 \beta\left\{-Q_{\mathrm{em}}^{f} \sin ^{2} \theta_{W}\right\}, & (f=u), \\
m_{\tilde{f}_{R}}^{2}+\left(f_{f} f_{f}^{\dagger}\right) \frac{v^{2}}{\sqrt{2}} \cos ^{2} \beta-m_{Z}^{2} \cos 2 \beta\left\{-Q_{\mathrm{em}}^{f} \sin ^{2} \theta_{W}\right\}, & (f=l, d),\end{cases}  \tag{A3}\\
& M_{\tilde{f}_{L R}}= \begin{cases}-A_{f} \frac{v}{\sqrt{2}} \sin \beta-\mu f_{f} \frac{v}{\sqrt{2}} \sin \beta \cot \beta, & (f=u), \\
A_{f} \frac{v}{\sqrt{2}} \cos \beta-\mu f_{f} \frac{v}{\sqrt{2}} \cos \beta \tan \beta, & (f=l, d) .\end{cases} \tag{A4}
\end{align*}
$$

Here the indices $\alpha$ and $\beta$ are for interaction eigenstates for their superpartner fermion fields. We take the basis where the mass matrix for the charged lepton field is diagonal, so that the index for charged leptons indicates its interaction eigenstate and its mass eigenstate, simultaneously. The unitary matrix $U_{\tilde{f}}$ is defined as

$$
\begin{equation*}
\left(U_{\tilde{f}}\right)_{X}^{\alpha}\left(M_{\tilde{f}}^{2}\right)_{\alpha}^{\beta}\left(U_{\tilde{f}}^{\dagger}\right)_{\beta}^{Y}=\operatorname{diag}\left(m_{\tilde{f}_{X}}^{2}\right) \delta_{X}^{Y} . \tag{A5}
\end{equation*}
$$

The relations between the mass eigenstates and the interaction eigenstates are

$$
\begin{gather*}
\tilde{f}_{X}=\left(U_{\tilde{f}}\right)_{X}{ }^{\alpha} \tilde{f}_{L \alpha}+\left(U_{\tilde{f}}\right)_{X}^{\alpha+3} \tilde{f}_{R \alpha},  \tag{A6}\\
\tilde{f}_{L \alpha}=\left(U_{\tilde{f}}^{\dagger}\right)_{\alpha}^{X} \tilde{f}_{X}, \quad \tilde{f}_{R \alpha}=\left(U_{\tilde{f}}^{\dagger}\right)_{\alpha+3}{ }_{\alpha}^{X} \tilde{f}_{X} . \tag{A7}
\end{gather*}
$$

The sneutrino mass term is also given as

$$
\begin{equation*}
-\mathcal{L}_{\tilde{\nu}}=\tilde{\nu}^{* \alpha}\left(M_{\tilde{\nu}}^{2}\right)_{\alpha}^{\beta} \tilde{\nu}_{\beta} \tag{A8}
\end{equation*}
$$

where

$$
\begin{equation*}
\left(M_{\tilde{\nu}}^{2}\right)=m_{\tilde{L}}^{2}+m_{\nu}^{\dagger} m_{\nu}+m_{Z}^{2} \cos 2 \beta\left(\frac{1}{2}\right), \tag{A9}
\end{equation*}
$$

where $m_{\nu}$ is the neutrino mass matrix which is induced by the seesaw mechanism;

$$
\begin{equation*}
m_{\nu}=f_{\nu}^{\top} M^{-1} f_{\nu} \frac{v^{2}}{2} \sin \beta \tag{A10}
\end{equation*}
$$

The unitary matrix $U_{\tilde{v}}$ is defined as

$$
\begin{equation*}
\left(U_{\tilde{\nu}}\right)_{X}^{\alpha}\left(M_{\tilde{\nu}}^{2}\right)_{\alpha}^{\beta}\left(U_{\tilde{\nu}}^{\dagger}\right)_{\beta}{ }^{Y}=\operatorname{diag}\left(m_{\tilde{\nu}_{X}}^{2}\right) \delta_{X}{ }^{Y} \tag{A11}
\end{equation*}
$$

The relations between the mass eigenstates and the interaction eigenstates are

$$
\begin{equation*}
\tilde{\nu}_{X}=\left(U_{\tilde{\nu}}\right)_{X}^{\alpha} \tilde{\nu}_{L \alpha}, \quad \tilde{\nu}_{L \alpha}=\left(U_{\tilde{\nu}}^{\dagger}\right)_{\alpha}^{X} \tilde{\nu}_{X} \tag{A12}
\end{equation*}
$$

The chargino mass term in the 2 -spinor representation is

$$
\begin{align*}
-\mathcal{L}_{\tilde{\chi}^{-}}= & \left(\begin{array}{ll}
\tilde{w}^{+} & \tilde{h}_{u}^{+}
\end{array}\right)^{i}\left(M_{C}\right)_{i}^{j}\binom{\tilde{w}^{-}}{\tilde{h}_{d}^{-}}_{j} \\
= & \left(\begin{array}{cc}
\tilde{w}^{+} & \tilde{h}_{u}^{+}
\end{array}\right)\left(\begin{array}{cc}
M_{2} & \sqrt{2} m_{W} \cos \beta \\
\sqrt{2} m_{W} \sin \beta & \mu
\end{array}\right) \\
& \times\binom{\tilde{w}^{-}}{\tilde{h}_{d}^{-}} \tag{A13}
\end{align*}
$$

The diagonalization is done by unitary matrices $U_{R}$ and $U_{L}$ as

$$
\begin{equation*}
\left(U_{R}\right)_{A}^{i}\left(M_{C}\right)_{i}^{j}\left(U_{L}^{\dagger}\right)_{j}^{B}=\operatorname{diag}\left(M_{\tilde{\chi}_{A}^{-}}\right) \delta_{A}^{B} \tag{A14}
\end{equation*}
$$

The relations between the interaction eigenstates and the mass eigenstates are

$$
\begin{align*}
\left(\tilde{x}_{A}^{-}\right)_{a} & =\left(U_{L}\right)_{A}{ }^{i}\binom{\left(\tilde{w}^{-}\right)_{a}}{\left(\tilde{h}_{d}^{-}\right)_{a}}, \\
\left(\tilde{x}^{+A}\right)_{a} & =\left(U_{R}^{*}\right)^{A}{ }_{i}\binom{\left(\tilde{w}^{+}\right)_{a}}{\left(\tilde{h}_{u}^{+}\right)_{a}}^{i}, \tag{A15}
\end{align*}
$$

where $a(\dot{a})$ denotes indices for the Lorentz spinor for $2(\overline{\mathbf{2}})$ under $S L(2, C)$. We here adopt the same rule as in Ref. [45]. The 4 -spinors for mass eigenstates are constructed as

$$
\begin{equation*}
\tilde{\chi}_{A}^{-}=\binom{\left(\tilde{x}_{A}^{-}\right)_{a}}{\left(\tilde{\tilde{x}}_{A}^{+}\right)^{\dot{a}}}, \quad \tilde{\chi}^{+A}=\binom{\left(\tilde{x}^{+A}\right)_{a}}{\left(\tilde{\tilde{x}}^{-A}\right)^{\dot{a}}}, \tag{A16}
\end{equation*}
$$

and then those for interaction eigenstates are

$$
\begin{align*}
& \tilde{W}^{-}=\binom{\left(\tilde{w}^{-}\right)_{a}}{\left(\tilde{w}^{+}\right)^{\dot{a}}}=\binom{\left(U_{L}^{\dagger}\right)_{1}{ }^{A}\left(\tilde{x}_{A}^{-}\right)_{a}}{\left(U_{R}^{\dagger}\right)_{1}{ }^{A}\left(\tilde{\tilde{x}}^{+}{ }_{A}\right)^{\dot{a}}}, \\
& \tilde{W}^{+}=\binom{\left(\tilde{w}^{+}\right)_{a}}{\left(\tilde{w}^{-}\right)^{\dot{a}}}=\binom{\left(U_{R}^{\top}\right)^{1}{ }_{A}\left(\tilde{x}^{+A}\right)_{a}}{\left(U_{L}^{\top}\right)^{1}\left(\tilde{\tilde{x}}^{-A}\right)^{\dot{a}}}, \\
& \tilde{H}^{-}=\binom{\left(\tilde{h}_{d}^{-}\right)_{a}}{\left(\tilde{h}_{u}^{+}\right)^{\dot{a}}}=\binom{\left(U_{L}^{\dagger}\right)_{2}{ }^{A}\left(\tilde{x}_{A}^{-}\right)_{a}}{\left(U_{R}^{\dagger}\right)_{2}{ }^{A}\left(\tilde{\tilde{x}}^{+}{ }_{A}\right)^{\dot{a}}},  \tag{A17}\\
& \tilde{H}^{+}=\binom{\left(\tilde{h}_{u}^{+}\right)_{a}}{\left(\tilde{h}_{d}^{-}\right)^{\dot{a}}}=\binom{\left(U_{R}^{\top}\right)^{2}{ }_{A}\left(\tilde{x}^{+A}\right)_{a}}{\left(U_{L}^{\top}\right)^{2}{ }_{A}\left(\tilde{x}^{-A}\right)^{\dot{a}}} .
\end{align*}
$$

The neutralino mass term in 2-spinor representation is

$$
\begin{align*}
-\mathcal{L}_{\tilde{\chi}^{0}} & =\frac{1}{2}\left(\begin{array}{llll}
\tilde{b}^{0} & \tilde{w}^{0} & \tilde{h}_{d}^{0} & \tilde{h}_{u}^{0}
\end{array}\right)_{i}\left(M_{N}\right)^{i j}\left(\begin{array}{c}
\tilde{b}^{0} \\
\tilde{w}^{0} \\
\tilde{h}_{d}^{0} \\
\tilde{h}_{u}^{0}
\end{array}\right)_{j} \\
& =\frac{1}{2}\left(\begin{array}{llll}
\tilde{b}^{0} & \tilde{w}^{0} & \tilde{h}_{d}^{0} & \tilde{h}_{u}^{0}
\end{array}\right)\left(\begin{array}{cccc}
M_{1} & 0 & -m_{Z} \sin \theta_{W} \cos \beta & m_{Z} \sin \theta_{W} \sin \beta \\
0 & M_{2} & m_{Z} \cos \theta_{W} \cos \beta & -m_{Z} \cos \theta_{W} \sin \beta \\
-m_{Z} \sin \theta_{W} \cos \beta & m_{Z} \cos \theta_{W} \cos \beta & 0 & -\mu \\
m_{Z} \sin \theta_{W} \sin \beta & -m_{Z} \cos \theta_{W} \sin \beta & -\mu & 0
\end{array}\right)\left(\begin{array}{l}
\tilde{b}^{0} \\
\tilde{w}^{0} \\
\tilde{h}_{d}^{0} \\
\tilde{h}_{u}^{0}
\end{array}\right) . \tag{A18}
\end{align*}
$$

The unitary matrix $U_{N}$ is defined as

$$
\begin{equation*}
\left(U_{N}^{*}\right)^{A}{ }_{i}\left(M_{N}\right)^{i j}\left(U_{N}^{\dagger}\right)_{j}^{B}=\operatorname{diag}\left(M_{\tilde{\chi}_{A}^{0}}\right) \delta^{A B} . \tag{A19}
\end{equation*}
$$

The relations between the interaction eigenstates and the mass eigenstates are

$$
\tilde{x}_{A}^{0}=\left(U_{N}\right)_{A}{ }^{i}\left(\begin{array}{c}
\tilde{b}^{0}  \tag{A20}\\
\tilde{w}^{0} \\
\tilde{h}_{d}^{0} \\
\tilde{h}_{u}^{0}
\end{array}\right)_{i} .
$$

Using this 2 -spinor $\tilde{x}^{0}$, the 4-Majorana spinor can be constructed as

$$
\begin{equation*}
\mathrm{P}_{L} \tilde{\chi}_{A}^{0}+\mathrm{P}_{R} \tilde{\chi}^{0 A}=\binom{\left(x_{A}^{0}\right)_{a}}{\left(\bar{x}^{0 A}\right)^{\dot{a}}} . \tag{A21}
\end{equation*}
$$

The interaction eigenstates are

$$
\begin{align*}
& \tilde{B}^{0}=\binom{\left(\tilde{b}^{0}\right)_{a}}{\left(\tilde{b}^{0}\right)^{\dot{a}}}=\binom{\left(U_{N}^{\dagger}\right)_{1}{ }^{A}\left(\tilde{x}_{A}^{0}\right)_{a}}{\left(U_{N}^{\top}\right)^{1}{ }_{A}\left(\overline{\tilde{x}}^{0 A}\right)^{\dot{a}}}, \\
& \tilde{W}^{0}=\binom{\left(\tilde{w}^{0}\right)_{a}}{\left(\tilde{w}^{0}\right)^{\dot{a}}}=\binom{\left(U_{N}^{\dagger}\right)_{2}{ }^{A}\left(\tilde{x}_{A}^{0}\right)_{a}}{\left(U_{N}^{\top}\right)^{2}{ }_{A}\left(\tilde{\tilde{x}}^{0 A}\right)^{\dot{a}}}, \\
& \tilde{H}_{d}^{0}=\binom{\left(\tilde{h}_{d}^{0}\right)_{a}}{\left(\tilde{\tilde{h}}_{d}^{0}\right)^{\dot{a}}}=\binom{\left(U_{N}^{\dagger}\right)_{3}{ }^{A}\left(\tilde{x}_{A}^{0}\right)_{a}}{\left(U_{N}^{\top}\right)^{3}{ }_{A}\left(\tilde{\tilde{x}}^{0 A}\right)^{\dot{a}}},  \tag{A22}\\
& \tilde{H}_{u}^{0}=\binom{\left(\tilde{h}_{u}^{0}\right)_{a}}{\left(\tilde{\tilde{h}}_{u}^{0}\right)^{\dot{a}}}=\binom{\left(U_{N}^{\dagger}\right)_{4}{ }^{A}\left(\tilde{x}_{A}^{0}\right)_{a}}{\left(U_{N}^{\top}\right)^{4}{ }_{A}\left(\tilde{\tilde{x}}^{0 A}\right)^{\dot{a}}},
\end{align*}
$$

The Lagrangian for gaugino-sfermion-fermion interactions is described as

$$
\begin{align*}
\mathcal{L}_{\mathrm{int}}= & \bar{l}^{\alpha}\left\{\left(C_{R}^{(l)}\right)_{\alpha}^{A X} \mathrm{P}_{R}+\left(C_{L}^{(l)}\right)_{\alpha}^{A X} \mathrm{P}_{L}\right\} \tilde{\chi}_{A}^{-} \tilde{\nu}_{X}+\bar{\nu}^{\alpha}\left(C_{R}^{(\nu)}\right)_{\alpha A}^{X} \mathrm{P}_{R} \tilde{\chi}^{+A} \tilde{l}_{X}+\bar{l}^{\alpha}\left(N_{R}^{(l)}\right)_{\alpha A}^{X} \mathrm{P}_{R} \tilde{\chi}^{0 A} \tilde{l}_{X}+\bar{l}^{\alpha}\left(N_{L}^{(l)}\right)_{\alpha}^{A X} \mathrm{P}_{L} \tilde{\chi}_{A}^{0} \tilde{l}_{X} \\
& +\bar{\nu}^{\alpha}\left(N_{R}^{(\nu)}\right)_{\alpha A}^{X} \mathrm{P}_{R} \tilde{\chi}^{0 A} \tilde{\nu}_{X}+\bar{d}^{\alpha}\left\{\left(C_{R}^{(d)}\right)_{\alpha}^{A X} \mathrm{P}_{R}+\left(C_{L}^{(d)}\right)_{\alpha}^{A X} \mathrm{P}_{L}\right\} \tilde{\chi}_{A}^{-} \tilde{u}_{X}+\bar{u}^{\alpha}\left\{\left(C_{R}^{(u)}\right)_{\alpha A}^{X} \mathrm{P}_{R}+\left(C_{L}^{(u)}\right)_{\alpha A}^{X} \mathrm{P}_{L}\right\} \tilde{\chi}^{+A} \tilde{d}_{X} \\
& +\bar{d}^{\alpha}\left(N_{R}^{(d)}\right)_{\alpha A}^{X} \mathrm{P}_{R} \tilde{\chi}^{0 A} \tilde{d}_{X}+\bar{d}^{\alpha}\left(N_{L}^{(d)}\right)_{\alpha}^{A X} \mathrm{P}_{L} \tilde{\chi}_{A}^{0} \tilde{d}_{X}+\bar{u}^{\alpha}\left(N_{R}^{(u)}\right)_{\alpha A}^{X} \mathrm{P}_{R} \tilde{\chi}^{0 A} \tilde{u}_{X}+\bar{u}^{\alpha}\left(N_{L}^{(u)}\right)_{\alpha}^{A X} \mathrm{P}_{L} \tilde{\chi}_{A}^{0} \tilde{u}_{X}+\text { h.c. } \tag{A23}
\end{align*}
$$

where the coefficients are

$$
\begin{gather*}
\left(C_{R}^{(l)}\right)_{\alpha}^{A X}=-g_{2}\left(U_{R}^{*}\right)_{1}^{A}\left(U_{\tilde{\nu}}^{*}\right)_{\alpha}^{X},  \tag{A24}\\
\left(C_{L}^{(l)}\right)_{\alpha}^{A X}=g_{2} \frac{m_{l_{\alpha}}}{\sqrt{2} m_{W} \cos \beta}\left(U_{L}^{*}\right)_{2}^{A}\left(U_{\tilde{\nu}}^{*}\right)^{X}{ }_{\alpha},  \tag{A25}\\
\left.\left(C_{R}^{(\nu)}\right)_{\alpha A}^{X}=-g_{2}\left(U_{L}\right)_{A}{ }^{1}\left(U_{\tilde{l}}^{*}\right)^{X}{ }_{\alpha}^{(l)}\right)_{\alpha A}^{X}=-\frac{g_{2}}{\sqrt{2}}\left[\left\{-\left(U_{N}\right)_{A}^{2}-\left(U_{N}\right)_{A}^{1} \tan \theta_{W}\right\}\left(U_{\tilde{l}}^{*}\right)^{X}{ }_{\alpha}\right.  \tag{A26}\\
\left.+\frac{m_{l_{\alpha}}}{m_{W} \cos \beta}\left(U_{N}\right)_{A}^{3}\left(U_{\tilde{l}}^{*}\right)^{X}{ }_{\alpha+3}\right], \\
\left(N_{L}^{(l)}\right)_{\alpha}^{A X}=-\frac{g_{2}}{\sqrt{2}}\left\{\frac{m_{l_{\alpha}}}{m_{W} \cos \beta}\left(U_{N}^{*}\right)^{A}{ }_{3}\left(U_{\tilde{l}}^{*}\right)_{\alpha}^{X}\right.  \tag{A27}\\
\left.+2\left(U_{N}^{*}\right)^{A}{ }_{1} \tan \theta_{W}\left(U_{\tilde{l}}^{*}\right)^{X}{ }_{\alpha+3}\right\}, \\
\left(N_{R}^{(\nu)}\right)_{\alpha A}^{X}=-\frac{g_{2}}{\sqrt{2}}\left\{\left(U_{N}\right)_{A}^{2}-\left(U_{N}\right)_{A}^{1} \tan ^{2} \theta_{W}\right\}\left(U_{\tilde{\nu}}^{*}\right)^{X}{ }_{\alpha},  \tag{A28}\\
\left(C_{R}^{(d)}\right)_{\alpha}^{A X}=g_{2}\left\{-\left(U_{R}^{*}\right)_{1}^{A}\left(U_{\tilde{u}}^{*}\right)^{X}{ }_{\alpha}+\frac{m_{u_{\alpha}}}{\sqrt{2} m_{W} \sin \beta}\right.  \tag{A29}\\
\left(C_{L}^{(d)}\right)_{\alpha}^{A X}=g_{2} \frac{m_{d_{\alpha}}}{\sqrt{2} m_{W} \cos \beta}\left(U_{L}^{*}\right)^{A}{ }_{2}^{*}\left(U_{\tilde{u}}^{*}\right)^{X}{ }_{\alpha},
\end{gather*}
$$

$$
\begin{align*}
\left(C_{R}^{(u)}\right)_{\alpha A}^{X}= & g_{2}\left\{-\left(U_{L}\right)_{A}{ }^{1}\left(U_{\tilde{d}}^{*}\right)^{X}{ }_{\alpha}+\frac{m_{d_{\alpha}}}{\sqrt{2} m_{W} \cos \beta}\right. \\
& \left.\times\left(U_{L}\right)_{A}{ }^{2}\left(U_{\tilde{d}}^{*}\right)^{X}{ }_{\alpha+3}\right\},  \tag{A32}\\
\left(C_{L}^{(u)}\right)_{\alpha A}^{X}= & g_{2} \frac{m_{u_{\alpha}}}{\sqrt{2} m_{W} \sin \beta}\left(U_{R}\right)_{A}{ }^{2}\left(U_{\tilde{d}}^{*}\right)^{X}{ }_{\alpha},  \tag{A33}\\
\left(N_{R}^{(d)}\right)_{\alpha A}^{X}= & -\frac{g_{2}}{\sqrt{2}}\left[\left\{-\left(U_{N}\right)_{A}{ }^{2}+\frac{1}{3}\left(U_{N}\right)_{A}{ }^{1} \tan \theta_{W}\right\}\left(U_{\tilde{d}}^{*}\right)^{X}{ }_{\alpha}\right. \\
+ & \left.\frac{m_{d_{\alpha}}}{m_{W} \cos \beta}\left(U_{N}\right)_{A}{ }^{3}\left(U_{\tilde{d}}^{*}\right)^{X}{ }_{\alpha+3}\right],  \tag{A34}\\
\left(N_{L}^{(d)}\right)_{\alpha}^{A X}= & -\frac{g_{2}}{\sqrt{2}\left\{\frac{m_{d_{\alpha}}}{m_{W} \cos \beta}\left(U_{N}^{*}\right)^{A}{ }_{3}\left(U_{\tilde{d}}^{*}\right)^{X}{ }_{\alpha}+\frac{2}{3}\right.} \begin{aligned}
&\left.\times\left(U_{N}^{*}\right)^{A}{ }_{1} \tan \theta_{W}\left(U_{\tilde{d}}^{*}\right)^{X}{ }_{\alpha+3}\right\}, \\
&\left(N_{R}^{(u)}\right)_{\alpha A}^{X}=-\frac{g_{2}}{\sqrt{2}}\left[\left\{\left(U_{N}\right)_{A}{ }^{2}+\frac{1}{3}\left(U_{N}\right)_{A}{ }^{1} \tan \theta_{W}\right\}\left(U_{\tilde{u}}^{*}\right)^{X}{ }_{\alpha}\right. \\
& m_{W} \sin \beta \\
&\left.\left(U_{N}\right)_{A}{ }^{4}\left(U_{\tilde{u}}^{*}\right)^{X}{ }_{\alpha+3}\right], \\
&\left.\times\left(U_{N}^{*}\right)^{A}{ }_{1} \tan \theta_{W}\left(U_{\tilde{u}}^{*}\right)^{X}{ }_{\alpha+3}\right\} . \\
&\left(N_{L}^{(u)}\right)_{\alpha}^{A X}=-\frac{g_{2}}{\sqrt{2}\left\{\frac{m_{u_{\alpha}}}{m_{W} \sin \beta}\left(U_{N}^{*}\right)^{A}{ }_{4}\left(U_{\tilde{u}}^{*}\right)^{X}{ }_{\alpha}-\frac{4}{3}\right.}
\end{aligned},
\end{align*}
$$

## APPENDIX B: DETAILS FOR $\boldsymbol{\epsilon}$ 'S

We here show the explicit form of the nLFV parameters $\epsilon_{\mu \tau}^{s}, \epsilon_{\mu \tau}^{m}$, and $\epsilon_{\mu \tau}^{d}$ in the MSSMRN. They are calculated from 1-loop diagrams.

## 1. For $\epsilon_{\mu \tau}^{s}$

The effective coupling comprises two kinds of contributions; one comes from a penguin-type diagram associated with the $W$ boson and the other is a box-type diagram:

$$
\begin{equation*}
\epsilon_{\mu \tau}^{s}=\left(\epsilon_{\mu \tau}^{s}\right)_{W \text {-penguin }}+\left(\epsilon_{\mu \tau}^{s}\right)_{\text {box }} . \tag{B1}
\end{equation*}
$$

The penguin part is represented as

$$
\begin{equation*}
\left(\epsilon_{\mu \tau}^{s}\right)_{W \text {-penguin }}=\sum_{i} \mathcal{A}_{\mu \tau}^{(s-i)}, \tag{B2}
\end{equation*}
$$

and each contribution which is shown in Fig. 6 is calculated to be

$$
\begin{gather*}
\mathcal{A}_{\beta \alpha}^{(s-1)}=\frac{1}{(4 \pi)^{2}}\left(N_{R}^{(\nu)}\right)_{\alpha A}^{X}\left(N_{R}^{(\nu) *}\right)_{X}^{\beta A} D\left(m_{\tilde{\nu}_{X}}, M_{\tilde{\chi}_{A}^{0}}\right),  \tag{B3}\\
\mathcal{A}_{\beta \alpha}^{(s-2)}=\frac{1}{(4 \pi)^{2}}\left(C_{R}^{(\nu)}\right)_{\alpha A}^{X}\left(C_{R}^{(\nu) *}\right)_{X}^{\beta A} D\left(m_{\tilde{l}_{X}}, M_{\tilde{\chi}_{A}^{+}}\right),  \tag{B4}\\
\mathcal{A}_{\beta \alpha}^{(s-3)}=\frac{1}{(4 \pi)^{2}} \frac{m_{l_{\beta}}^{2}}{m_{l_{\beta}}^{2}-m_{l_{\alpha}}^{2}}\left(N_{R}^{(l)}\right)_{\alpha A}^{X}\left(N_{R}^{(l) *}\right)_{X}^{\beta A} D\left(m_{\tilde{l}_{X}}, M_{\tilde{\chi}_{A}^{0}}\right),
\end{gather*}
$$

$\mathcal{A}_{\beta \alpha}^{(s-4)}=\frac{1}{(4 \pi)^{2}} \frac{m_{l_{\beta}}^{2}}{m_{l_{\beta}}^{2}-m_{l_{\alpha}}^{2}}\left(C_{R}^{(l)}\right)_{\alpha}^{A X}\left(C_{R}^{(l) *}\right)_{A X}^{\beta} D\left(m_{\tilde{\nu}_{X}}, M_{\tilde{\chi}_{A}^{-}}\right)$,
$E\left(m_{X}, m_{Y}, M_{A}\right) \equiv \frac{1}{2} \frac{1}{x_{A Y}-x_{A X}}\left(\frac{x_{A Y} \ln x_{A X}}{1-x_{A X}}-\frac{x_{A X} \ln x_{A Y}}{1-x_{A Y}}\right)$,

$$
\begin{align*}
D\left(m_{X}, M_{A}\right) \equiv & \frac{1}{4} \frac{1}{\left(1-x_{A X}\right)^{2}}\left\{1-4 x_{A X}+3 x_{A X}^{2}\right. \\
& \left.-2 x_{A X}^{2} \ln x_{A X}-2\left(1-x_{A X}\right)^{2} \ln m_{X}^{2}\right\} \tag{B5}
\end{align*}
$$

$$
\begin{align*}
\mathcal{A}_{\beta \alpha}^{(s-6)}= & \sqrt{2} \frac{1}{(4 \pi)^{2}}\left(N_{R}^{(\nu)}\right)_{\alpha A}^{X}\left(C_{R}^{(l) *}\right)_{B X}^{\beta} \\
\times & {\left[\delta^{A A}\left(O^{R *}\right)_{A}^{B} G\left(m_{\tilde{\nu}_{X}}, M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{-}}\right)\right.} \\
- & \left.\left(O^{L *}\right)^{A B} \frac{1}{2} F\left(m_{\tilde{\nu}_{X}}, M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{-}}\right)\right] \\
\mathcal{A}_{\beta \alpha}^{(s-7)}= & -\sqrt{2} \frac{1}{(4 \pi)^{2}}\left(C_{R}^{(\nu)}\right)_{\alpha A}^{X}\left(N_{R}^{(l) *}\right)_{X}^{\beta B} \\
& \times\left[\delta_{B B}\left(O^{L *}\right)^{B A} G\left(m_{\tilde{l}_{X}}, M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{0}}\right)\right. \\
& \left.-\left(O^{R *}\right)_{B}^{A} \frac{1}{2} F\left(m_{\tilde{l}_{X}}, M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{0}}\right)\right], \tag{B8}
\end{align*}
$$

where the functions $D, E, F$, and $G$ are defined as
-

$$
\begin{equation*}
E\left(m_{X}, m_{Y}, M_{A}\right) \equiv \frac{1}{2} \frac{1}{x_{A Y}-x_{A X}}\left(\frac{x_{A Y} \ln x_{A X}}{1-x_{A X}}-\frac{x_{A X} \ln x_{A Y}}{1-x_{A Y}}\right) \tag{B6}
\end{equation*}
$$



FIG. 6. Diagrams which contribute to $\left(\epsilon_{\mu \tau}^{s}\right)_{W \text {-penguin }}$.

$$
\begin{align*}
F\left(m_{X}, M_{A}, M_{B}\right) \equiv & \ln x_{A X}+\frac{1}{x_{A X}-x_{B X}}  \tag{B18}\\
& \times\left(\frac{x_{A X}^{2} \ln x_{A X}}{1-x_{A X}}-\frac{x_{B X}^{2} \ln x_{B X}}{1-x_{B X}}\right)  \tag{B12}\\
G\left(m_{X}, M_{A}, M_{B}\right) \equiv & \sqrt{x_{A X} x_{B X}} \frac{1}{x_{A X}-x_{B X}}  \tag{B19}\\
& \left(\frac{x_{A X} \ln x_{A X}}{1-x_{A X}}-\frac{x_{B X} \ln x_{B X}}{1-x_{B X}}\right) \tag{B13}
\end{align*}
$$

with $x_{A X} \equiv M_{A}^{2} / m_{X}^{2}$. The couplings for the chargino-neutralino- $W$-boson interaction, $O^{L}$ and $O^{R}$, are given as [45]

$$
\begin{align*}
\left(O^{L}\right)_{A B} & =-\frac{1}{\sqrt{2}}\left(U_{N}\right)_{A}^{4}\left(U_{R}\right)_{B}^{2}+\left(U_{N}\right)_{A}^{2}\left(U_{R}\right)_{B}^{1}  \tag{B14}\\
\left(O^{R}\right)_{B}^{A} & =\frac{1}{\sqrt{2}}\left(U_{N}^{*}\right)_{3}^{A}\left(U_{L}\right)_{B}^{2}+\left(U_{N}^{*}\right)^{A}{ }_{2}\left(U_{L}\right)_{B}^{1} \tag{B15}
\end{align*}
$$

The box part is represented as

$$
\begin{equation*}
\left(\epsilon_{\mu \tau}^{s}\right)_{\mathrm{box}}=\sum_{i} \mathcal{B}_{\mu \tau}^{(s-i)} \tag{B16}
\end{equation*}
$$

$$
\begin{aligned}
\mathcal{B}_{\beta \alpha}^{(s-2)}= & \frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{-}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{\nu}_{X}}, m_{\tilde{\nu}_{Y}}\right)\left(C_{R}^{(l) *}\right)_{A X}^{\beta}\left(C_{R}^{(l)}\right)_{e}^{A Y} \\
& \times\left(N_{R}^{(\nu) *}\right)_{Y}^{e B}\left(N_{R}^{(\nu)}\right)_{\alpha B}^{X},
\end{aligned}
$$

$$
\begin{align*}
\mathcal{B}_{\beta \alpha}^{(s-3)}= & \frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{+}}, m_{\tilde{l}_{X}}, m_{\tilde{\nu}_{Y}}\right) M_{\tilde{\chi}_{A}^{0}} M_{\tilde{\chi}_{B}^{+}}\left(N_{R}^{(l) *}\right)_{X}^{\beta A} \\
& \times\left(N_{R}^{(\nu) *}\right)_{Y}^{e A}\left(C_{R}^{(\nu)}\right)_{\alpha B}^{X}\left(C_{R}^{(l)}\right)_{e}^{B Y}, \\
\mathcal{B}_{\beta \alpha}^{(s-4)}= & \frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{-}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{\nu}_{X}}, m_{\tilde{l}_{Y}}\right) M_{\tilde{\chi}_{A}^{-}} M_{\tilde{\chi}_{B}^{0}}\left(C_{R}^{(l) *}\right)_{A X}^{\beta} \\
& \times\left(C_{R}^{(\nu) *}\right)_{X}^{e A}\left(N_{R}^{(\nu)}\right)_{\alpha B}^{X}\left(N_{R}^{(l)}\right)_{e B}^{Y}, \tag{B20}
\end{align*}
$$

where $I_{4}$ and $J_{4}$ are the functions which are given as

$$
\begin{aligned}
& I_{4}\left(M_{A}, M_{B}, m_{X}, m_{Y}\right) \\
& \quad \equiv \int \frac{d^{4} k}{(2 \pi)^{4} i} \frac{1}{\left(k^{2}-M_{A}^{2}\right)\left(k^{2}-M_{B}^{2}\right)\left(k^{2}-m_{X}^{2}\right)\left(k^{2}-m_{Y}^{2}\right)}
\end{aligned}
$$

$$
\begin{align*}
& J_{4}\left(M_{A}, M_{B}, m_{X}, m_{Y}\right)  \tag{B21}\\
& \quad \equiv \int \frac{d^{4} k}{(2 \pi)^{4} i} \frac{k^{2}}{\left(k^{2}-M_{A}^{2}\right)\left(k^{2}-M_{B}^{2}\right)\left(k^{2}-m_{X}^{2}\right)\left(k^{2}-m_{Y}^{2}\right)} \tag{B22}
\end{align*}
$$

and each contribution, shown in Fig. 7, is calculated to be

$$
\begin{align*}
\mathcal{B}_{\beta \alpha}^{(s-1)}= & \frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{+}}, m_{\tilde{l}_{X}}, m_{\tilde{l}_{Y}}\right)\left(N_{R}^{(l) *}\right)_{X}^{\beta A}\left(N_{R}^{(l)}\right)_{e A}^{Y} \\
& \times\left(C_{R}^{(\nu) *}\right)_{Y}^{e B}\left(C_{R}^{(\nu)}\right)_{\alpha B}^{X}, \tag{B17}
\end{align*}
$$

## 2. For $\boldsymbol{\epsilon}_{\boldsymbol{\mu} \boldsymbol{m}}$

Since the matter of the Earth is neutral for $U(1)_{\mathrm{em}}$, there is no contribution to $\epsilon_{\mu \tau}^{m}$ from photon-penguin diagrams.


FIG. 7. Diagrams which contribute to $\left(\epsilon_{\mu \tau}^{s}\right)_{\text {box }}$.

The $Z$-penguin contribution associated with a proton and that with a neutron cancel each other out. The contributions which need to be taken into account are the Z-penguin contribution associated with an electron and the box contributions:

$$
\begin{gather*}
\epsilon_{\mu \tau}^{m, e}=\left(\epsilon_{\mu \tau}^{m, e}\right)_{Z-\text { penguin }}+\left(\epsilon_{\mu \tau}^{m, e}\right)_{\mathrm{box}}  \tag{B23}\\
\epsilon_{\mu \tau}^{m, u}=\left(\epsilon_{\mu \tau}^{m, u}\right)_{\mathrm{box}}  \tag{B24}\\
\epsilon_{\mu \tau}^{m, d}=\left(\epsilon_{\mu \tau}^{m, d}\right)_{\mathrm{box}} \tag{B25}
\end{gather*}
$$

The penguin contribution consists of diagrams which are drawn in Fig. 8,

$$
\begin{equation*}
\left(\epsilon_{\mu \tau}^{m, e}\right)_{Z-\text { penguin }}=\sum_{i} \mathcal{A}_{\mu \tau}^{(m-i)} . \tag{B26}
\end{equation*}
$$

Each diagram is calculated to be

$$
\begin{align*}
& \mathcal{A}_{\alpha \beta}^{(m-1)}= \frac{1}{(4 \pi)^{2}} \frac{1}{2}\left(2 \sin ^{2} \theta_{W}-1\right)\left(C_{R}^{(\nu)}\right)_{\beta A}^{X} \\
& \times\left(C_{R}^{(\nu) * *}\right)_{X}^{\alpha A} D\left(m_{\tilde{l}_{X}}, M_{\tilde{\chi}_{A}^{+}}\right), \quad(\mathrm{B} 27)  \tag{B27}\\
& \mathcal{A}_{\alpha \beta}^{(m-2)}=- \frac{1}{(4 \pi)^{2}}\left(2 \sin ^{2} \theta_{W}-1\right)\left(C_{R}^{(\nu)}\right)_{\beta A}^{X}\left(C_{R}^{(\nu) *}\right)_{Y}^{\alpha A}\left\{\left(-\frac{1}{2}\right.\right. \\
&+\left.\sin ^{2} \theta_{W}\right)\left(U_{\tilde{l}}\right)_{X}^{\gamma=1-3}\left(U_{\tilde{l}}^{\dagger}\right)_{\gamma=1-3}{ }^{Y}+\left(\sin ^{2} \theta_{W}\right) \\
&\left.\times\left(U_{\tilde{l}}\right)_{X}^{\gamma=4-6}\left(U_{\tilde{l}}^{\dagger}\right)_{\gamma=4-6}{ }^{Y}\right\} E\left(m_{\tilde{l}_{X}}, m_{\tilde{l}_{Y}}, M_{\tilde{\chi}_{A}^{+}}\right), \tag{B28}
\end{align*}
$$

$$
\begin{align*}
\mathcal{A}_{\alpha \beta}^{(m-3)}= & -\frac{1}{(4 \pi)^{2}}\left(2 \sin ^{2} \theta_{W}-1\right)\left(N_{R}^{(\nu)}\right)_{\beta A}^{X}\left(N_{R}^{(\nu) *}\right)_{X}^{\alpha B} \\
& \times\left[\left(O^{\prime \prime L}\right)_{A}^{B} G\left(m_{\tilde{\nu}_{X}}, M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{0}}\right)\right. \\
& \left.-\frac{1}{2}\left(O^{\prime \prime R}\right)^{A}{ }_{B} F\left(m_{\tilde{\nu}_{X}}, M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{0}}\right)\right],  \tag{B29}\\
\mathcal{A}_{\alpha \beta}^{(m-4)}= & -\frac{1}{(4 \pi)^{2}}\left(2 \sin ^{2} \theta_{W}-1\right)\left(C_{R}^{(\nu)}\right)_{\beta A}^{X}\left(C_{R}^{(\nu) *}\right)_{X}^{\alpha B} \\
& \times\left[\left(O^{\prime L}\right)^{A}{ }_{B} G\left(m_{\tilde{l}_{X}}, M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{+}}\right)\right. \\
& \left.-\frac{1}{2}\left(O^{\prime R}\right)_{B}^{A} F\left(m_{\tilde{l}_{X}}, M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{+}}\right)\right] . \tag{B30}
\end{align*}
$$

The couplings for the chargino-chargino- $Z$ boson and neutralino-neutralino- $Z$ boson are [45]

$$
\begin{align*}
\left(O^{\prime L}\right)_{B}^{A}= & -\left(U_{R}^{*}\right)_{1}^{A}\left(U_{R}\right)_{B}^{1}-\frac{1}{2}\left(U_{R}^{*}\right)_{2}^{A}\left(U_{R}\right)_{B}{ }^{2} \\
& +\delta_{B}^{A} \sin ^{2} \theta_{W}  \tag{B31}\\
\left(O^{\prime R}\right)_{B}^{A}= & -\left(U_{L}^{*}\right)^{A}{ }_{1}\left(U_{L}\right)_{B}^{1}-\frac{1}{2}\left(U_{L}^{*}\right)^{A}{ }_{2}\left(U_{L}\right)_{B}{ }^{2} \\
& +\delta_{B}^{A} \sin ^{2} \theta_{W}  \tag{B32}\\
\left(O^{\prime \prime L}\right)_{A}^{B}=- & \frac{1}{2}\left(U_{N}\right)_{A}^{3}\left(U_{N}^{*}\right)_{3}^{B}+\frac{1}{2}\left(U_{N}\right)_{A}{ }^{4}\left(U_{N}^{*}\right)^{B}  \tag{B33}\\
& \left(O^{\prime \prime R}\right)_{B}^{A}=-\left(O^{\prime \prime L^{*}}\right)_{B}^{A} \tag{B34}
\end{align*}
$$

Here we take into account the procedure to resolve the double counting problem which is explained in Sec. III A.


FIG. 8. Diagrams which contribute to $\left(\epsilon_{\mu \tau}^{m, e}\right)_{Z \text {-penguin }}$. The diagrams ( $m-\mathrm{A}$ ) and ( $m-\mathrm{D}$ ) are not counted into $\epsilon_{\mu \tau}^{m}$, which are included in $\epsilon_{\mu \tau}^{s}$ [cf. ( $s-2$ ) and ( $s-1$ ) in Fig. 6]. The diagrams ( $m-\mathrm{B}$ ) and ( $m$-C) cancel each other out after the off-shell prescription [Eq. (26)].

The box contribution associated with the electron in the Earth's matter is

$$
\begin{equation*}
\left(\epsilon_{\mu \tau}^{m, e}\right)_{\mathrm{box}}=\sum_{i} \mathcal{B}_{\mu \tau}^{(m, e-i)} \tag{B35}
\end{equation*}
$$

where

$$
\begin{align*}
\mathcal{B}_{\alpha \beta}^{(m, e-1)}= & \frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{\nu}_{X}}, m_{\tilde{l}_{Y}}\right)\left(N_{R}^{(\nu)}\right)_{\beta B}^{X} \\
& \times\left(N_{R}^{(l) *}\right)_{Y}^{e B}\left(N_{R}^{(l)}\right)_{e A}^{Y}\left(N_{R}^{(\nu) *}\right)_{X}^{\alpha A},  \tag{B36}\\
\mathcal{B}_{\alpha \beta}^{(m, e-2)}= & -\frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{\nu}_{X}}, m_{\tilde{l}_{Y}}\right) M_{\tilde{\chi}_{A}^{0}} M_{\tilde{\chi}_{B}^{0}} \\
& \times\left(N_{R}^{(\nu)}\right)_{\beta B}^{X}\left(N_{L}^{(l) *}\right)_{B Y}^{e}\left(N_{L}^{(l)}\right)_{e}^{A Y}\left(N_{R}^{(\nu) *}\right)_{X}^{\alpha A},  \tag{B37}\\
\mathcal{B}_{\alpha \beta}^{(m, e-3)}= & \frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{\nu}_{X}}, m_{\tilde{l}_{Y}}\right) M_{\tilde{\chi}_{A}^{0}} M_{\tilde{\chi}_{B}^{0}} \\
& \times\left(N_{R}^{(\nu)}\right)_{\beta B}^{X}\left(N_{R}^{(l)}\right)_{e B}^{Y}\left(N_{R}^{(\nu) *}\right)_{X}^{\alpha A}\left(N_{R}^{(l) *}\right)_{Y}^{e A}, \tag{B38}
\end{align*}
$$

$\mathcal{B}_{\alpha \beta}^{(m, e-4)}=-\frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{\nu}_{X}}, m_{\tilde{l}_{Y}}\right)\left(N_{R}^{(\nu)}\right)_{\beta B}^{X}$

$$
\begin{equation*}
\times\left(N_{R}^{(l)}\right)_{e B}^{Y}\left(N_{R}^{(\nu) *}\right)_{X}^{\alpha A}\left(N_{R}^{(l) *}\right)_{Y}^{e A} \tag{B39}
\end{equation*}
$$

$$
\begin{align*}
\mathcal{B}_{\alpha \beta}^{(m, e-5)}= & \frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{+}}, m_{\tilde{l}_{X}}, m_{\tilde{\nu}_{Y}}\right) M_{\tilde{\chi}_{A}^{+}} M_{\tilde{\chi}_{B}^{+}} \\
& \times\left(C_{R}^{(\nu)}\right)_{\beta B}^{X}\left(C_{R}^{(l)}\right)_{e}^{B Y}\left(C_{R}^{(l) *}\right)_{A Y}^{e}\left(C_{R}^{(\nu) *}\right)_{X}^{\alpha A}, \tag{B40}
\end{align*}
$$

$\mathcal{B}_{\alpha \beta}^{(m, e-6)}=-\frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{+}}, m_{\tilde{l}_{X}}, m_{\tilde{\nu}_{Y}}\right)\left(C_{R}^{(\nu)}\right)_{\beta B}^{X}$

$$
\begin{equation*}
\times\left(C_{L}^{(l)}\right)_{e}^{B Y}\left(C_{L}^{(l) *}\right)_{A Y}^{e}\left(C_{R}^{(\nu) *}\right)_{X}^{\alpha A} \tag{B41}
\end{equation*}
$$

$\mathcal{B}_{\alpha \beta}^{(m, e-7)}=\frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{l}_{X}}, m_{\tilde{\nu}_{Y}}\right) M_{\tilde{\chi}_{A}^{+}} M_{\tilde{\chi}_{B}^{0}}$

$$
\begin{equation*}
\times\left(N_{R}^{(l)}\right)_{e B}^{X}\left(N_{R}^{(\nu)}\right)_{\beta B}^{Y}\left(C_{R}^{(l) *}\right)_{A Y}^{e}\left(C_{R}^{(\nu) *}\right)_{X}^{\alpha A} \tag{B42}
\end{equation*}
$$

$$
\begin{align*}
\mathcal{B}_{\alpha \beta}^{(m, e-8)}= & \frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{l}_{X}}, m_{\tilde{\nu}_{Y}}\right)\left(N_{L}^{(l)}\right)_{e}^{B X}  \tag{B54}\\
& \times\left(N_{R}^{(\nu)}\right)_{\beta B}^{Y}\left(C_{L}^{(l) *}\right)_{A Y}^{e}\left(C_{R}^{(\nu) *}\right)_{X}^{\alpha A}, \tag{B43}
\end{align*}
$$

$$
\mathcal{B}_{\alpha \beta}^{(m, e-9)}=\frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{-}}, m_{\tilde{\nu}_{X}}, m_{\tilde{l}_{Y}}\right) M_{\tilde{\chi}_{A}^{0}} M_{\tilde{\chi}_{B}^{-}}
$$

$$
\begin{equation*}
\times\left(C_{R}^{(l)}\right)_{e}^{B X}\left(C_{R}^{(\nu)}\right)_{\beta B}^{Y}\left(N_{R}^{(l) *}\right)_{Y}^{e A}\left(N_{R}^{(\nu) *}\right)_{X}^{\alpha A} \tag{B44}
\end{equation*}
$$

$$
\mathcal{B}_{\alpha \beta}^{(m, e-10)}=\frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{-}}, m_{\tilde{\nu}_{X}}, m_{\tilde{l}_{Y}}\right)\left(C_{L}^{(l)}\right)_{e}^{B X}
$$

$$
\begin{equation*}
\times\left(C_{R}^{(\nu)}\right)_{\beta B}^{Y}\left(N_{L}^{(l) *}\right)_{Y}^{e A}\left(N_{R}^{(\nu) *}\right)_{X}^{\alpha A} \tag{B45}
\end{equation*}
$$

The box contribution associated with the down quark in the matter of the Earth is

$$
\begin{equation*}
\left(\epsilon_{\mu \tau}^{m, d}\right)_{\mathrm{box}}=\sum_{i} \mathcal{B}_{\mu \tau}^{(m, d-i)} \tag{B46}
\end{equation*}
$$

where

$$
\begin{align*}
\mathcal{B}_{\alpha \beta}^{(m, u-1)}= & \frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{\nu}_{X}}, m_{\tilde{u}_{Y}}\right)\left(N_{R}^{(\nu)}\right)_{\beta B}^{X}  \tag{B55}\\
& \times\left(N_{R}^{(u) *}\right)_{Y}^{u B}\left(N_{R}^{(u)}\right)_{u A}^{Y}\left(N_{R}^{(\nu) *}\right)_{X}^{\alpha A},  \tag{1}\\
\mathcal{B}_{\alpha \beta}^{(m, u-2)}= & -\frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{\nu}_{X}}, m_{\tilde{u}_{Y}}\right) M_{\tilde{\chi}_{A}^{0}} M_{\tilde{\chi}_{B}^{0}}  \tag{DJכ}\\
& \times\left(N_{R}^{(\nu)}\right)_{\beta B}^{X}\left(N_{L}^{(u) *}\right)_{B Y}^{u}\left(N_{L}^{(u)}\right)_{u}^{A Y}\left(N_{R}^{(\nu) *}\right)_{X}^{\alpha A}, \tag{B56}
\end{align*} \mathrm{~B}_{\alpha \beta}^{(m, u-3)}=\frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{\nu}_{X}}, m_{\tilde{u}_{Y}}\right) M_{\tilde{\chi}_{A}^{0}} M_{\tilde{\chi}_{B}^{0}}, ~(\mathrm{E}
$$

The box contribution associated with the up quark in the matter of the Earth is

$$
\begin{equation*}
\left(\epsilon_{\mu \tau}^{m, u}\right)_{\mathrm{box}}=\sum_{i} \mathcal{B}_{\mu \tau}^{(m, u-i)} \tag{B53}
\end{equation*}
$$

where

$$
\begin{align*}
\mathcal{B}_{\alpha \beta}^{(m, u-4)}= & -\frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{\nu}_{X}}, m_{\tilde{u}_{Y}}\right)\left(N_{R}^{(\nu)}\right)_{\beta B}^{X} \\
& \times\left(N_{R}^{(u)}\right)_{u B}^{Y}\left(N_{R}^{(\nu) *}\right)_{X}^{\alpha A}\left(N_{R}^{(u) *}\right)_{Y}^{u A}, \tag{B57}
\end{align*}
$$

$$
\begin{align*}
\mathcal{B}_{\alpha \beta}^{(m, u-5)}= & \frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{+}}, m_{\tilde{l}_{X}}, m_{\tilde{d}_{Y}}\right)\left(C_{R}^{(\nu)}\right)_{\beta B}^{X} \\
& \times\left(C_{R}^{(u) *}\right)_{Y}^{u B}\left(C_{R}^{(u)}\right)_{u A}^{Y}\left(C_{R}^{(\nu) *}\right)_{X}^{\alpha A}, \tag{B58}
\end{align*}
$$

$$
\begin{align*}
\mathcal{B}_{\alpha \beta}^{(m, u-6)}= & -\frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{+}}, m_{\tilde{l}_{X}}, m_{\tilde{d}_{Y}}\right) M_{\tilde{\chi}_{A}^{+}} M_{\tilde{\chi}_{B}^{+}} \\
& \times\left(C_{R}^{(\nu)}\right)_{\beta B}^{X}\left(C_{L}^{(u) *}\right)_{Y}^{u B}\left(C_{L}^{(u)}\right)_{u A}^{Y}\left(C_{R}^{(\nu) *}\right)_{X}^{\alpha A} . \tag{B59}
\end{align*}
$$

## 3. For $\boldsymbol{\epsilon}_{\boldsymbol{\mu} \tau}^{\boldsymbol{d}}$

We consider the charged current interaction between a neutrino and a nucleon in the detector as a detection process. It consists of the penguin contribution and the box contribution:

$$
\begin{equation*}
\epsilon_{\mu \tau}^{d}=\left(\epsilon_{\mu \tau}^{d}\right)_{W \text {-penguin }}+\left(\epsilon_{\mu \tau}^{d}\right)_{\text {box }} . \tag{B60}
\end{equation*}
$$

The penguin contribution can be represented as the complex conjugate of that for $\epsilon_{\tau \mu}^{s}$. However, we must eliminate the diagrams which are already counted in the calculation of $\epsilon_{\mu \tau}^{m}$. The detail is shown in Sec. III A.

$$
\begin{equation*}
\left(\epsilon_{\mu \tau}^{d}\right)_{W \text {-penguin }}=\sum_{i=3}^{7} \mathcal{A}_{\tau \mu}^{(s-i) *} \tag{B61}
\end{equation*}
$$

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The box contribution is calculated to be

$$
\begin{equation*}
\left(\epsilon_{\mu \tau}^{d}\right)_{\mathrm{box}}=\sum_{i} \mathcal{B}_{\mu \tau}^{(d-i)} \tag{B62}
\end{equation*}
$$

where

$$
\begin{align*}
\mathcal{B}_{\alpha \beta}^{(d-1)}= & \frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{-}}, m_{\tilde{\nu}_{X}}, m_{\tilde{u}_{Y}}\right)\left(C_{R}^{(l)}\right)_{\beta}^{B X} \\
& \times\left(C_{R}^{(d) *}\right)_{B Y}^{d}\left(N_{R}^{(u)}\right)_{u A}^{Y}\left(N^{(\nu) *}\right)_{X}^{\alpha A},  \tag{B63}\\
\mathcal{B}_{\alpha \beta}^{(d-2)}= & \frac{1}{8 \sqrt{2} G_{F}} J_{4}\left(M_{\tilde{\chi}_{A}^{+}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{l}_{X}}, m_{\tilde{d}_{Y}}\right)\left(N_{R}^{(l)}\right)_{\beta B}^{X}\left(N_{R}^{(d) *}\right)_{Y}^{d B} \\
& \times\left(C_{R}^{(u)}\right)_{u A}^{Y}\left(C_{R}^{(\nu) *}\right)_{X}^{\alpha A}, \tag{B64}
\end{align*}
$$

$$
\begin{align*}
\mathcal{B}_{\alpha \beta}^{(d-3)}= & \frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{0}}, M_{\tilde{\chi}_{B}^{+}}, m_{\tilde{\nu}_{X}}, m_{\tilde{d}_{Y}}\right) M_{\tilde{\chi}_{A}^{0}} M_{\tilde{\chi}_{B}^{+}} \\
& \times\left(C_{R}^{(l)}\right)_{\beta}^{B X}\left(C_{R}^{(u)}\right)_{u B}^{Y}\left(N_{R}^{(d) *}\right)_{Y}^{d A}\left(N_{R}^{(\nu)}\right)_{X}^{\alpha A} \tag{B65}
\end{align*}
$$

$$
\begin{align*}
\mathcal{B}_{\alpha \beta}^{(d-4)}= & \frac{1}{4 \sqrt{2} G_{F}} I_{4}\left(M_{\tilde{\chi}_{A}^{-}}, M_{\tilde{\chi}_{B}^{0}}, m_{\tilde{l}_{X}}, m_{\tilde{u}_{Y}}\right) M_{\tilde{\chi}_{A}^{-}} M_{\tilde{\chi}_{B}^{0}} \\
& \times\left(N_{R}^{(l)}\right)_{\beta B}^{X}\left(N_{R}^{(u)}\right)_{u B}^{Y}\left(C_{R}^{(d) *}\right)_{A Y}^{d}\left(C_{R}^{(\nu) *}\right)_{X}^{\alpha A} . \tag{B66}
\end{align*}
$$

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[^1]:    ${ }^{1}$ Studies for other models are done in, e.g., Ref. [37].

[^2]:    ${ }^{2}$ Precisely, it is one of the mass eigenstates of the neutrino [38].

[^3]:    ${ }^{3}$ It is necessary to treat the neutrino as a wave packet [39] in the discussion on the coherence between these two amplitudes. Here we adopt the usual treatment for the neutrino propagation, so that the neutrino propagation is described by a plane wave.

[^4]:    ${ }^{4}$ The matrix $U^{s}$ is not necessarily unitary.

[^5]:    ${ }^{5}$ Since the effective four-Fermi couplings are induced by the exchange of the massive gauge bosons $Z$ and $W$ and $q^{2}$ is of $\mathcal{O}\left(m_{f}\right)$, essentially we can put $q^{2}=0$.

[^6]:    ${ }^{6}$ The mass parameter $m_{\nu_{\alpha}}$ for the flavor eigenstate [38] appears in Eq. (26). However, we finally neglect it in our off-shell prescription.

