Possible candidates for a supersymmetric SO(10) model with an intermediate scale

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We study the possibility of an intermediate scale existing in supersymmetric SO(10) grand unified theories. The intermediate scale is required to be around 10^{12} GeV so that neutrinos can obtain masses suitable for explaining the experimental data on the deficit of solar neutrinos with the Mikheyev-Smirnov-Wolfenstein solution and the existence of hot dark matter. We show that any Pati-Salam-type intermediate symmetries are excluded by requiring reasonable conditions and only $\mathrm{SU}(2)_L\times \mathrm{SU}(2)_R\times \mathrm{SU}(3)_C\times\mathrm{U}(1)_{B-L}$ is likely to be realized as an intermediate symmetry.

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In constructing a realistic unified theory of matter and fields, it is inevitable to answer a question about neutrino masses. There seems to exist experiments indicating the neutrino masses and their mixing [1]: Some experiments show a deficit of the solar neutrino, which may be explained by the Mikheyev-Smirnov-Wolfenstein (MSW) solution [2]. For example, according to one of the MSW solutions, the mass of the muon neutrino seems to be $m_{\nu_\mu} \simeq 10^{-3}$ eV. Those small masses may be explained by the seesaw mechanism [3]: A muon neutrino can acquire such a small mass if the Majorana mass of the right-handed muon neutrino is about 10^{12} GeV. Furthermore, if all Majorana masses of right-handed neutrinos are about 10^{12} GeV, the seesaw mechanism leads to a mass of the τ neutrino of $m_{\nu_\tau} \simeq 10$ eV, which is desirable for the interpretation that the tau neutrino may be hot dark matter. In the framework of the SO(10) grand unified theory (GUT) [4], we can naturally incorporate right-handed neutrinos into ordinary fermions.

On the other hand it is well known that in the minimal supersymmetric standard model (MSSM) the present experimental values of gauge couplings are successfully unified at a unification scale $M_U \simeq 10^{16}$ GeV [5].

Then how can the right-handed neutrinos acquire masses of about 10^{12} GeV when we have no scale other than M_U? There are several possibilities for the right-handed neutrinos to obtain masses of the intermediate scale, $M_R \simeq 10^{12}$ GeV. First, the radiative correction of GUT scale physics, what we call the Witten mechanism [6], can induce M_R. In a supersymmetric model, however, this mechanism cannot work because the nonrenormalization theorem [7] protects the inducement of terms via radiative corrections which are not contained in the original Lagrangian. The second possibility is that the Yukawa coupling of the right-handed Majorana neutrino is so small that the mass may be the intermediate scale even if it originates at the GUT scale. Third, singlet Higgs particles develop a vacuum expectation value at the intermediate scale mass to supply the mass of M_R to ν_R. In unrenormalizable models such as supergravity those latter two possibilities may be realized.

Our point of view is, however, that it is more natural to consider that one energy scale corresponds to a dynamical phenomenon, for instance, symmetry breaking. Thus we are led to another possibility that a certain group breaks down to the standard group at the intermediate scale at which right-handed neutrinos gain mass. This idea is consistent with the survival hypothesis. There are some papers which indicate that intermediate groups can enter in breaking chains of groups from the GUT to the standard model consistently with the analyses of the MSSM such as coupling unification [8], but models presented in these papers do not involve Higgs contents which are able to provide mass to right-handed neutrinos. We would like to explain right-handed neutrino mass according to symmetry breaking.

This paper is devoted to an investigation of the possibility that right-handed neutrinos acquire mass of order 10^{12} GeV through symmetry breaking within the SUSY SO(10) GUT with one intermediate scale below which the MSSM is realized. First we show assumptions about the models and constraints on our analysis. Next we explain our analysis. Then we show results. Finally we give a summary.

First of all we show the breaking patterns allowed in our scenario:

$$\begin{aligned}
&\text{(I)}\ \ \mathrm{SO}(10) \longrightarrow \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_{\mathrm{PS}}\ (G_{224}) \longrightarrow \mathrm{MSSM},\\
&\text{(II)}\ \ \mathrm{SO}(10) \longrightarrow \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{SU}(4)_{\mathrm{PS}}\ (G_{214}) \longrightarrow \mathrm{MSSM},\\
&\text{(III)}\ \ \mathrm{SO}(10) \longrightarrow \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(3)_C \times \mathrm{U}(1)_{B-L}\ (G_{2231}) \longrightarrow \mathrm{MSSM}.
\end{aligned} \tag{1}$$

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The first assumption is that there is at least one Higgs multiplet which is able to supply intermediate scale mass to right-handed neutrinos in addition to ordinary matter (ordinary fermions, right-handed neutrinos, and two Higgs doublets).

The second is that candidates for the matter content in the intermediate region are multiplets included in representations **10, 16, 45, 54, 120, 126**, and **210** of SO(10).[1]

Phenomenologically we impose further constraints to our models.

(1) The unified scale M_U is larger than 10^{16} GeV. This is necessary for suppression of proton decay [9].

(2) The intermediate scale is taken at 10^{10}, 10^{11}, 10^{12}, 10^{13}, or 10^{14} GeV because of the right-handed neutrino mass. The reason why an intermediate scale is an input will be made clear after Eq. (4).

(3) Any colored Higgs multiplet is not contained in the intermediate physics. This is needed also for suppression of proton decay [9].

Under these conditions, we specify combinations of matter mentioned above which realize the unification of gauge couplings to achieve the GUT with a simple group.

Next we show the condition on the β functions in the intermediate scale in order to achieve the unification of gauge couplings. In the following we make an analysis based on renormalization group equations (RGE) up to one loop. The conditions of the unification are described by

$$\begin{aligned}\alpha_Y^{-1}(M_S) &= \alpha_U^{-1}(M_U) + \frac{1}{2\pi}b_Y R + \frac{1}{2\pi}b'_Y(U-R),\\ \alpha_L^{-1}(M_S) &= \alpha_U^{-1}(M_U) + \frac{1}{2\pi}b_L R + \frac{1}{2\pi}b'_L(U-R),\\ \alpha_C^{-1}(M_S) &= \alpha_U^{-1}(M_U) + \frac{1}{2\pi}b_C R + \frac{1}{2\pi}b'_C(U-R).\end{aligned} \tag{2}$$

$b_i(i = Y, L, C)$'s with and without a prime denote the β function in the lower scale and higher scale than the intermediate scale M_R, respectively. R and U are defined by

$$R = \ln\frac{M_R}{M_S}, \qquad U = \ln\frac{M_U}{M_S}. \tag{3}$$

These equations lead to the relation which R and U must satisfy

$$\begin{aligned}(b_Y - b_L)R + (b'_Y - b'_L)(U-R) &= 2\pi\left[\alpha_Y^{-1}(M_S) - \alpha_L^{-1}(M_S)\right],\\ (b_C - b_L)R + (b'_C - b'_L)(U-R) &= 2\pi\left[\alpha_C^{-1}(M_S) - \alpha_L^{-1}(M_S)\right].\end{aligned} \tag{4}$$

Here we have assumed that in the lower scale the MSSM is realized, and so Eq. (4) has always a solution $U = R \simeq 10^{16}$ GeV, which corresponds to the case that there is no intermediate scale physics. Therefore if there is a nontrivial intermediate scale R, the β functions must satisfy the condition

$$(b_Y - b_L)(b'_C - b'_L) - (b_C - b_L)(b'_Y - b'_L) = 0. \tag{5}$$

Since the β functions in the MSSM are given by

$$b_Y = \frac{33}{5}, \quad b_L = 1, \quad b_C = -3, \tag{6}$$

the β functions between the intermediate scale and GUT scale must satisfy the equation

$$5b'_Y - 12b'_L + 7b'_C = 0, \tag{7}$$

which we call "the unification condition."[2] This is a sufficient condition on the gauge coupling unification under the assumption that the MSSM is realized in the lower scale. When Eq. (7) is satisfied, R becomes an arbitrary parameter. Therefore we introduce an intermediate scale M_R as an input parameter.

Using the unification condition for the β functions in addition to the above restrictions, we make an analysis as follows: Taking one combination of matter content on the intermediate physics, we see whether or not the unification condition is satisfied. If so, we can calculate the unified scale M_U and the gauge coupling $\alpha_U(M_U)$ at the unified scale using the equations

$$\begin{aligned}M_U &= M_R \exp\left(2\pi\frac{\alpha_Y^{-1}(M_R) - \alpha_L^{-1}(M_R)}{b'_Y - b'_L}\right),\\ \alpha_U(M_U) &= \left(\alpha_L^{-1}(M_R) - \frac{1}{2\pi}b'_L(U-R)\right)^{-1},\end{aligned} \tag{8}$$

once M_R and $\alpha_i^{-1}(M_R)$'s are given.

In principle we can calculate $\alpha_i^{-1}(M_R)$'s from low-energy experimental values of α_i^{-1}'s according to the RGE. We choose, however, another way to calculate $\alpha_i^{-1}(M_R)$ in order to avoid ambiguities such as the SUSY-breaking scale M_S, strong coupling α_C, and so on. Because we already know the unification scale $M_U^{\rm MSSM}$ and $\alpha_U^{{\rm MSSM}^{-1}}(M_U^{\rm MSSM})$ in the MSSM GUT and above the intermediate scale considered in this paper, all couplings α_i's are small enough for the one-loop approximation of the RGE to work well, we calculate $\alpha_i^{-1}(M_R)$'s from the input parameter $\alpha_U^{{\rm MSSM}^{-1}}(M_U^{\rm MSSM})$ at the GUT scale $M_U^{\rm MSSM}$. We choose the input parameters from Ref. [5] as,

[1]This is just an assumption. In general, however, the models mentioned above are difficult to be accompanied by a representation which is contained in a higher representation of SO(10) only. See the statement below Eq. (9) and the summary for an example.

[2]Though $b'_Y = b'_L = b'_C$ satisfies the unification condition, in this case the condition that all couplings are unified is not fulfilled. Therefore this case is excluded.

$$M_U^{\rm MSSM} = 10^{16.3}\,{\rm GeV}, \quad \alpha_U^{{\rm MSSM}^{-1}}(M_U^{\rm MSSM}) = 25.7. \quad (9)$$

Then we select the matter content which satisfies the criteria. A search was made for all possible combinations of matter content by using a computer. This is actually possible because the number of each matter multiplet which we can take into account simultaneously cannot be very large due to the conditions we have already mentioned. Generally, the larger the number of matter content, the bigger its contribution to the β functions is and the stronger the corresponding couplings α_i's become. As a result $\alpha_U^{-1}(M_U)$ becomes negative below the unification scale M_U.

Now we present our results.

Case (I). There is no solution for the breaking chain of (I), in which SO(10) breaks down to the MSSM through so-called Pati-Salam symmetry [11]. The reason is as follows: We need at least one Higgs multiplet which gives mass to the right-handed neutrinos. This is a representation (1,3,10). This is rather a large representation. Its contribution to the β function of the strong coupling above the intermediate scale, $b'_{C\ (1,3,10)}$,

$$b'_{C\ (1,3,10)} = 18, \quad (10)$$

is so large that it makes the inverse of the unification coupling, $\alpha_U^{-1}(M_U)$, much smaller.

To see this, let us denote the contribution of (1,3,10) to the inverse of the strong coupling at the GUT scale $\alpha_C^{-1}(M_U)$ by $\Delta\alpha_C^{-1}$:

$$\Delta\alpha_C^{-1} \equiv \frac{1}{2\pi} b'_{C\ (1,3,10)} \ln\frac{M_R}{M_U} = \frac{18}{2\pi}\ln\frac{M_R}{M_U}. \quad (11)$$

At the intermediate scale the inverse of the strong coupling, $\alpha_C^{-1}(M_R)$, is

$$\alpha_C^{-1}(M_R) = \alpha_U^{{\rm MSSM}^{-1}}(M_U^{\rm MSSM}) + \frac{1}{2\pi} b_C \ln\frac{M_U^{\rm MSSM}}{M_R}. \quad (12)$$

Because $\alpha_U^{-1}(M_U)$ must be positive,

$$\alpha_U^{-1}(M_U) = \alpha_C^{-1}(M_R) - \frac{1}{2\pi} b_C \ln\frac{M_U}{M_R} + \Delta\alpha_C^{-1} > 0, \quad (13)$$

and therefore, by substituting Eqs. (11) and (13),

$$\alpha_U^{{\rm MSSM}^{-1}}(M_U^{\rm MSSM}) + \frac{1}{2\pi} b_C \ln\frac{M_U^{\rm MSSM}}{M_R} + \frac{15}{2\pi}\ln\frac{M_R}{M_U} > 0,$$

$$\alpha_U^{{\rm MSSM}^{-1}}(M_U^{\rm MSSM}) > \frac{15}{2\pi}\ln\frac{M_U}{M_R} - \frac{1}{2\pi} b_C \ln\frac{M_U^{\rm MSSM}}{M_R}. \quad (14)$$

As we required $M_U > 10^{16}$ GeV [see constraint (I)],

$$\begin{aligned} &\alpha_U^{{\rm MSSM}^{-1}}(M_U^{\rm MSSM}) > \frac{18}{2\pi}\ln\frac{10^{16}}{M_R}, \\ &8.97 > \ln\frac{10^{16}}{M_R}, \\ &M_R > 10^{12}\ {\rm GeV}. \end{aligned} \quad (15)$$

In fact, because we need other multiplets to satisfy the unification condition, in the region $M_R \le 10^{13}$ GeV we have no solution even if we allow $\alpha_U^{-1}(M_U) = 0$ ($\alpha_U \to \infty$). For the case $M_R = 10^{14}$ GeV (though this scale is very high as an intermediate one), the minimal matter content gives $\alpha_U^{-1}(M_U) = 4.19$. Though from a perturbative view this coupling value is very large, as an example we list one of the combinations below.

Solution $\alpha_U^{-1}(M_U) = 4.19$, $M_U = 10^{16.3}$ GeV,

Higgs content

(1,3,10)	1	$(1,3,\overline{10})$	1
(2,2,1)	2		
(2,1,4)	7	$(2,1,\overline{4})$	7

In this list, for example, (1,3,10) 1 stands for the fact that the representation of the Higgs boson under G_{224} is (1,3,10) and its number is 1.

Therefore if we consider both the supersymmetric GUT and the scenario described above, in which right-handed neutrino mass is given by symmetry breaking at the intermediate scale, it is impossible to use the group G_{224} as an intermediate group.

Case (II). This case is a little better than case (I), since the intermediate group is a little smaller, $SU(2)_R$ being replaced by $U(1)_R$. We cannot, however, find any solution so far as $M_R \le 10^{11}$ GeV. In the case $M_R = 10^{12}$ GeV we meet several solutions though in every case the unified coupling is calculated to be vary large. As an illustration we list one of the solutions which gives the smallest value of the unified coupling.

Solution $\alpha_U^{-1}(M_U) = 4.03$, $M_U = 10^{16.3}$ GeV,

Higgs content

(1,2,10)	1	$(1,-2,\overline{10})$	1
(2,1,1)	2	(2,−1,1)	2
(2,0,4)	2	$(2,0,\overline{4})$	2
(3,0,1)	1		
(1,0,20′)	1		

In this table, we adopt the normalization for $U(1)_R$, $T_{3R} = {\rm diag}(1,-1)$. For example, (1,2,10) stands for the fact that the representation of the Higgs boson under G_{214} is (1,2,10). Again in this case the coupling value is very large for a perturbative view. Even if we take $M_R = 10^{14}$ GeV, this combination of matter content gives $\alpha_U^{-1}(M_U) \sim 14$ while other solutions give larger values of the unified coupling.

So it is still a bit difficult for the group G_{214} to be realized at the high energy region in this scenario.

Case (III). Case (III) is the most preferable group for us accepting it as the intermediate one. In Table I we list the number of possible candidates versus $\alpha_U^{-1}(M_U)$ up to $\alpha_U(M_U) = 1/15$ in the case of $M_R = 10^{12}$ GeV. Because the matter content N increases with α_U, and the number

TABLE I. The number of possible candidates in the breaking chain (III). The left column corresponds to the inverse of the unified coupling. The right column gives the number of possible candidates in the case of $M_R = 10^{12}$ GeV.

$\alpha_U^{-1}(M_U)$	No. of combinations
21.2	2
19.7	1
18.3	25
16.8	69
15.3	108

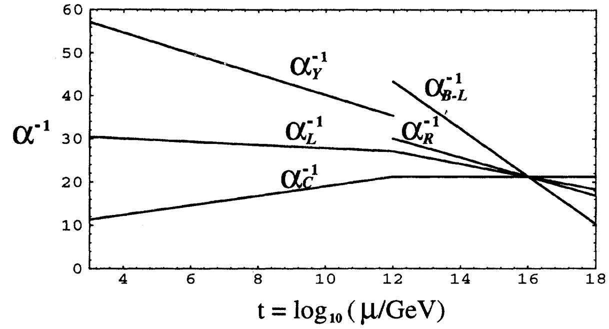

FIG. 1. Evolution of coupling constants under the breaking chain given in (II) and the Higgs content in solution (i). M_R is taken to be 10^{12} GeV. The calculation is based on a one-loop approximation.

of the combinations increases almost $N!$ with N matter content, there are too many combinations of matter multiplets satisfying the conditions to list all the cases here. Thus we see from Table I that the minimal value of the unified coupling is found to be $\sim 1/21$, which of course is larger than $\alpha_U^{\rm MSSM}(M_U^{\rm MSSM}) = 1/25.7$. This value corresponds to the cases of the minimal matter content. Actually most combinations may lead a gauge coupling α_U beyond the acceptable region to apply the perturbation theory.

We list the following two combinations as cases of minimal matter content which result in the smallest value of $\alpha_U \simeq 1/20$.

$$\text{Solution (i)}: \quad \alpha_U^{-1}(M_U) = 21.2, \quad M_U = 10^{16.3}\ \text{GeV},$$
$$\begin{array}{llll} \multicolumn{4}{l}{\text{Higgs content}} \\ (1,3,1)(6) & 1 & (1,3,1)(-6) & 1 \\ (2,2,1)(0) & 2 & & \\ (3,1,1)(0) & 1 & & \\ (1,1,8)(0) & 1 & & \end{array}$$

$$\text{Solution (ii)}: \quad \alpha_U^{-1}(M_U) = 21.2, \quad M_U = 10^{16.3}\ \text{GeV},$$
$$\begin{array}{llll} \multicolumn{4}{l}{\text{Higgs content}} \\ (1,3,1)(6) & 1 & (1,3,1)(-6) & 1 \\ (2,2,1)(0) & 1 & & \\ (2,1,1)(3) & 1 & (2,1,1)(-3) & 1 \\ (3,1,1)(0) & 1 & & \\ (1,1,8)(0) & 1 & & \end{array}$$

with an input parameter $M_R = 10^{12}$ GeV. In the case of another M_R, $\alpha_U^{-1}(M_U)$ is slightly varied, though M_U does not change. In this table, we adopt the normalization for $U(1)_{B-L}$, $T_4^{15} = \text{diag}(1,1,1,-3)$. For example, (1,3,1)(6) stands for the fact that the representation of the Higgs boson under G_{2231} is (1,3,1)(6). In solution (i), (1,3,1)($\pm$6) are contained in **126** of SO(10) and possibly give Majorana mass to the right-handed neutrino. (2,2,1)(0) can be regarded as the standard Higgs boson in the MSSM, belonging to **10** or **126** of SO(10). (3,1,1)(0) and (1,1,8)(0) are involved in **45** of SO(10) [10]. The unification of the gauge couplings of solution (i) is represented in Fig. 1.

Finally we give a summary. When we construct a SUSY SO(10) GUT model under the assumption that the right-handed neutrinos acquire their mass of the intermediate scale by the renormalizable coupling (Yukawa coupling) and there is only one intermediate scale. Among the three cases G_{2231} is the most favorable group to be built in. The reason why neither G_{224} nor G_{214} can be used is as follows. In case (I), to give right-handed neutrinos mass, a (1,3,10) Higgs representation is needed. This representation, however, makes too large a contribution to the β functions to achieve a small coupling.

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