Progress of Theoretical Physics, Vol. 96, No. 3, September 1996

# Possible Candidates for SUSY $E_6$ GUT with an Intermediate Scale

Joe SATO*)

*Department of Physics, Kyoto University, Kyoto 606-01*

(Received February 2, 1996)

We study the possibility of an intermediate scale existing in supersymmetric $E_6$ grand unified theories. The intermediate scale is demanded to be around $10^{12}$ GeV so that neutrinos can obtain masses suitable for explaining the experimental data on the deficit of solar neutrinos with the Mikheev-Smirnov-Wolfenstein solution and the existence of hot dark matter. We require that at the intermediate scale, there exists a certain symmetry breakdown to the Standard Model symmetry. We show that only a few $E_6$ subgroups are likely to be realized as the intermediate symmetry, though there are many candidates for the intermediate symmetry in $E_6$ GUT.

## § 1. Introduction

When we construct a Grand Unified Theory (GUT) based on $SO(10)$[1] and $E_6$,[2] in general, we have many extra fields which are contained in the same multiplets as those of the quarks and leptons. Under the Standard Model (SM) these fields can have mass terms because they belong to a real representation under the SM symmetry; some of them are singlet fermions under the SM, and the others appear with their complex conjugate representations. Singlet fermions may play the role of right-handed neutrinos. Then the scale of the extra fields is expected to be a scale below which the SM is realized.**)

It is well known that in the Minimal Supersymmetric Standard Model (MSSM) the present experimental values of gauge couplings are successfully unified at a unification scale $M_U \simeq 10^{16}$ GeV.[3] This fact implies that if we would like to consider the gauge unification, it is favourable that the symmetry of the GUT breaks down to that of the SM at the unification scale. In this case the scale of the right-handed neutrinos $M_{\nu R}$ and that of the other extra fields are expected to be the unification scale $M_U$. This implies also that there is no intermediate scale between the supersymmetry (SUSY) breaking scale and the unification scale.

On the other hand it is believed that $M_{\nu R} \sim 10^{10-12}$ GeV.[4] The experimental data on the deficit of the solar neutrino can be explained by the Mikheev-Smirnov-Wolfenstein (MSW) solution.[5] According to one of the MSW solutions, the mass of the muon neutrino seems to be $m_{\nu_\mu} \simeq 10^{-3}$ eV. Such a small mass can occur as a result of the seesaw mechanism:[6] a muon neutrino can acquire a mass of $O(10^{-3})$ eV if the Majorana mass of the right-handed muon neutrino is about $10^{12}$ GeV.

How can the right-handed neutrinos acquire masses of approximately $10^{12}$ GeV when we have no scale other than $M_U$? There are several possibilities for the right-handed neutrinos to obtain masses of the intermediate scale, $M_{\nu R} \simeq 10^{12}$ GeV. First, radiative correction of GUT scale physics, the Witten mechanism,[7] can induce

*) Current address: Institute for Cosmic Ray Research, University of Tokyo, Midoricho, Tanashi, Tokyo 188.

**) In the case that $E_6$ breaks down to the SM symmetry with several scales, the scale of the extra fields may be higher than the scale below which the SM is realized.

$M_{\nu R}$. In a supersymmetric model, however, this mechanism cannot work because the non-renormalization theorem[8] protects inducement of terms via radiative corrections which are not contained in the original Lagrangian. The second possibility is that the Yukawa coupling of right-handed Majorana neutrino is so small that the mass may be the intermediate scale even if it originates at the GUT scale. Third, singlet Higgs particles develop a vacuum expectation value at the intermediate scale to supply the mass of $M_{\nu R}$ to $\nu_R$. In unrenormalizable models such as supergravity, these latter two possibilities may be realized.

Our point of view is, however, that it is more natural to consider that one energy scale corresponds to a dynamical phenomenon, for instance a symmetry breaking. Thus we are led to another possibility: a certain intermediate group breaks down to the standard group at the intermediate scale at which right-handed neutrinos gain mass. This idea is consistent with the survival hypothesis. In previous papers, we examined whether it is possible to have an intermediate symmetry in a SUSY $SO(10)$ GUT.[9),10)] We saw that there is a possibility of having a SUSY $SO(10)$ GUT with an intermediate symmetry $SU(2)_L \times SU(2)_R \times SU(3)_C \times U(1)_{B-L}$,[9)] and actually we can construct such a SUSY $SO(10)$ GUT.[10)]

In this paper we consider $E_6$ GUT as an extension of $SO(10)$ GUT. In $E_6$ GUT, one family is embedded into one irreducible multiplet. We examine whether we can have an intermediate symmetry in a SUSY $E_6$ GUT using the same method as that in Ref.9).

First we show what groups can be the intermediate symmetry. Next we give a brief review of the method of Ref. 9). Then we give the result. Finally we give a summary and discussion.

## § 2. Intermediate group

### 2.1. *Matter content*

One quark and lepton family is embedded in $E_6$ **27**. Then there are 12 extra matter fields in **27**. According to their quantum number, we denote these fields as follows. There are two SM singlets. We label them $\nu_R$. When we need to distinguish these singlet from each other, we use a subscript 1 or 2. Other fields have the same quantum numbers as those of the right-handed down type quark $d^c$ and its charge conjugate $\bar{d}^c$, and those of lepton doublet $l$ and its charge conjugate $\bar{l}$. The definition of notation for quarks and leptons is as follows.

| $E_6$ | $SU(2)_L \times SU(3)_C \times U(1)_Y$ | |
|---|---|---|
| 27 | $(2, 3, 1/6)$ | $q$ |
| | $(1, \bar{3}, -2/3)$ | $u^c$ |
| | $2\times(1, \bar{3}, 1/3)$ | $d_1{}^c$, $d_2{}^c$ |
| | $2\times(2, 1, -1/2)$ | $l_1$, $l_2$ |
| | $(1, 1, 1)$ | $e^c$ |

| | |
|---|---|
| $(1, 3, -1/3)$ | $\bar{d}^c$ |
| $(2, 1, 1/2)$ | $\bar{l}$ |
| $2\times(1, 1, 0)$ | $\nu_{R1}$, $\nu_{R2}$ |

### 2.2. *Intermediate group*

We have many kinds of $G_{\rm Intermediate}$. We list all of these and show the assignment of quantum numbers. We refer to the regular maximal subgroup of $E_6$ to find $G_{\rm Intermediate}$. To know the decomposition of representations and subgroups, see Ref. 11). There are three regular maximal subgroups in $E_6$:

(A) $SO(10)\times U(1)$,

(B) $SU(3)\times SU(3)\times SU(3)$,

(C) $SU(2)\times SU(6)$.

$G_{\rm Intermediate}$ must consist of at least three simple groups, because at the intermediate scale none of the three gauge couplings coincide. Moreover, one of the simple groups must contain $SU(3)_C$, the color group, and one of the others must contain $SU(2)_L$, the weak group.

(A) Subgroup contained in $SO(10)\times U(1)_X$

Under $SO(10)\times U(1)_X$, $E_6$ **27** becomes $1(4)+10(-2)+16(1)$.

For the reason mentioned adove, we must consider the subgroups of $SO(10)$, $SO(10)\times U(1)\supset SU(2)\times SU(2)\times SU(4)\times U(1)$. Under this subgroup, $E_6$ **27** is decomposed to be $(1,1,1)(4)+(2,2,1)(-2)+(1,1,6)(-2)+(2,1,4)(1)+(1,2,\bar{4})(1)$. One of the two appearances of $SU(2)$ must be identified with $SU(2)_L$. We can give two ways of assigning meaning to the other $SU(2)$ according to the definition of the hypercharge: i) One is referred to as $SU(2)_R$. ii) The other is the diagonal group to the SM group.

Thus we see subgroups as follows:

i) 1) $SU(2)_L\times SU(2)_R\times SU(4)_{PS}\times U(1)_X$,

2) $SU(2)_L\times U(1)_R\times SU(4)_{PS}\times U(1)_X$,

3) $SU(2)_L\times SU(2)_R\times SU(3)_C\times U(1)_{B-L}\times U(1)_X$,

ii) 4) $SU(2)_L\times SU(4)_{PS'}\times U(1)_X\times G_4$, $G_4=SU(2)$ and its subgroups.

In the case i), $U(1)_X$ is irrelevant in the SM symmetry. Quarks and leptons are contained in $SO(10)$ **16**. The subgroup is recognized as a direct product of the Pati-Salam group $(SU(2)_L\times SU(2)_R\times SU(4)_{PS})$[13] and $U(1)_X$. There are also subgroups without $U(1)_X$. Such subgroups are $SO(10)$ subgroups, and hence when one of those subgroups is realized as the intermediate symmetry, we cannot see any difference from $SO(10)$ GUT.[9] We will not consider these three groups in this paper. The groups 1)~3) have $U(1)_X$, a reflection of $E_6$ GUT.

In the case ii), the hypercharge $Y$ is obtained by $1/4X-1/12PS'$, where $PS'$ is $SU(4)$ $T^{15}$ component charge. $T_4^{15}={\rm diag}\{1,1,1,-3\}$. The way of embedding quarks and leptons is as follows:

| $SU(2)_L\times SU(4)_{PS'}\times U(1)_X$ | $SU(2)_L\times SU(3)_C\times U(1)_Y$ | |
|---|---|---|
| $(1,1,4)$ | $(1,1,1)$ | $e^c$ |
| $(2,1,-2)$ | $(2,1,-1/2)$ | $l$ |
| $(1,6,-2)$ | $(1,\bar{3},-2/3)+(1,3,-1/3)$ | $u^c,\ \bar{d}^c$ |
| $(2,4,1)$ | $(2,3,1/6)+(2,1,1/2)$ | $q,\ \bar{l}$ |
| $(1,\bar{4},1)$ | $(1,\bar{3},-2/3)+(1,1,0)$ | $d^c,\ \nu_R$ |

There are other *ii*-type subgroups with a form of "the SM symmetry" $\times G$, where $G$ is $SU(2)\times U(1)$ and its subgroups. In the case that one of these groups is realized as the intermediate group, we do not have any constraint from the unification condition. The reason is that though there are extra multiplets contained in **27** of $E_6$ in the intermediate region, these multiplets are identified with **10** of $SO(10)$, an irreducible representation of a simple group. When we add a full multiplet of a simple group like $SO(10)$, the prediction of the gauge unification by MSSM is not spoiled. Thus we do not consider these subgroups here.

(B) Subgroups contained in $SU(3)\times SU(3)\times SU(3)$

Under $SU(3)\times SU(3)$, $E_6$ **27** becomes $(\bar{3},3,1)+(3,1,3)+(1,\bar{3},\bar{3})$.

In this case, one of the appearances of $SU(3)$ is identified with $SU(3)_C$, and one of the others is identified with $SU(3)_L$, the group containing $SU(2)_L$. We have the following subgroups here.*)

5) $SU(3)_L\times SU(3)_R\times SU(3)_C$ ,

6) $SU(2)_L\times SU(3)_R\times SU(3)_C\times U(1)_Z$ ,

7) $SU(3)_L\times SU(2)_R\times SU(3)_C\times U(1)_{Z'}$ ,

8) $SU(3)_L\times U(1)_{R'}\times SU(3)_C\times G_8$ ,  $G_8=SU(2)$ and its subgroups.

The hypercharge $Y$ is obtained by $1/6Z\ -1/2\,T_R{}^3+1/6Z'$, where $Z$ and $Z'$ are $SU(3)$ $T^8$ component charge. $T_3{}^8=\mathrm{diag}\{1,1,-2\}$. $T_R{}^3$ is an $SU(2)_R$ generator, defined by $\mathrm{diag}\{1,-1\}$. $R'$ is obtained by $3/2\,T_R{}^3-1/2Z$. The way of embedding quarks and leptons is as follows:

| $SU(3)_L\times SU(3)\times SU(3)_C$ | $SU(2)_L\times SU(3)_C\times U(1)_Y$ | |
|---|---|---|
| $(\bar{3},3,1)$ | $2\times(2,1,-1/2)+(2,1,1/2)+2\times(1,1,0)+(1,1,1)$ | $l,\bar{l},\nu_R,e^c$ |
| $(3,1,3)$ | $(2,3,1/6)+(1,3,-1/3)$ | $q,\ \bar{d}^c$ |
| $(1,\bar{3},\bar{3})$ | $(1,\bar{3},-2/3)+2\times(1,3,1/3)$ | $u^c,\ d^c$ |

(C) Subgroups contained in $SU(2)\times SU(6)$

First we decompose $SU(6)$ into its subgroup because, as mentioned above, there

*) Exactly, we have the subgroup 3) as the subgroup of $SU(3)\times SU(3)\times SU(3)$. We omit such a duplication in the following.

must exist at least three simple groups.

$$SU(6)\supset\begin{cases}SU(5)\times U(1)\,,\\ SU(2)\times SU(4)\times U(1)\,,\\ SU(3)\times SU(3)\times U(1)\,.\end{cases}$$

The second group and its subgroups are the same as those appearing in (A) (the $SO(10)$ case). The third one coincides with case (B).

As a subgroup of $SU(6)$, we consider only $SU(5)\times U(1)$. As mentioned above, $SU(2)$ is identified with $SU(2)_L$. $SU(5)$ is identified with $SU(5)_C$, a group which contains the $SU(3)$ color group. Under $SU(2)_L\times SU(5)_C\times U(1)$, **27** is decomposed into $(2,\bar{5},-1)+(2,1,5)+(1,5,-4)+(1,10,2)$. We can give two ways of assigning meaning to $SU(5)_C$ according to the definition of the hypercharge.

9) $SU(2)_L\times SU(5)_C\times U(1)_W$,

10) $SU(2)_L\times SU(5)_{C'}\times U(1)_W$.

To give hypercharge, first we consider $SU(2)_R\times SU(3)_C\times U(1)_X$, the subgroup of $SU(5)$. Under this subgroup $SU(5)$ **5** is $(2,1,3)+(1,3,-2)$. Thus the hypercharge $Y$ is equal to $2/15X+1/10W$ in 9) and $1/30X+1/2T_R^3-1/10W'$ in 10). The way of embedding the quarks and leptons is as follows:*)

9)

| $SU(2)_L\times SU(5)_C\times U(1)_W$ | $SU(2)_L\times SU(3)_C\times U(1)_Y$ | |
|---|---|---|
| $(2,\bar{5},-1)$ | $2\times(2,1,-1/2)+(2,\bar{3},1/6)$ | $l, q$ |
| $(2,1,5)$ | $(2,1,1/2)$ | $\bar{l}$ |
| $(1,5,-4)$ | $2\times(1,1,0)+(1,3,-2/3)$ | $\nu_R, u^c$ |
| $(1,10,2)$ | $(1,1,1)+(1,\bar{3},-1/3)+2\times(1,3,1/3)$ | $e^c, \bar{d}^c, d^c$ |

10)

| $SU(2)_L\times SU(5)_C\times U(1)_W$ | $SU(2)_L\times SU(3)_C\times U(1)_Y$ | |
|---|---|---|
| $(2,\bar{5},-1)$ | $(2,1,\pm1/2)+(2,\bar{3},1/6)$ | $\bar{l}, l, q$ |
| $(2,1,5)$ | $(2,1,1/2)$ | $l$ |
| $(1,5,-4)$ | $(1,1,1)+(1,1,0)+(1,3,1/3)$ | $e^c, \nu_R, d^c$ |
| $(1,10,2)$ | $(1,1,0)+(1,\bar{3},-1/3)+(1,3,-2/3)+(1,3,1/3)$ | $\nu_R, \bar{d}^c, u^c, d^c$ |

Their subgroups are the same as the groups appearing in (A) and are omitted here. Thus as the intermediate group, we must consider these 10 subgroups.

*) In the following table, the definition of **3** and $\bar{\mathbf{3}}$ is opposite to the standard one.

## § 3. Unification condition

Here we give a brief review of Ref. 9).

The outline of our scenario is

$$E_6 \rightarrow G_{\text{intermediate}} \rightarrow \text{MSSM},$$

where $G_{\text{intermediate}}$ is a subgroup of $E_6$ and contains the SM symmetry. One quark and lepton family is contained in $E_6$ **27**.

We require that there exist multiplets which can give mass to the right-handed neutrino and also, if they exist, to the other extra fields at the intermediate scale, where the intermediate symmetry breaks down to the SM. In general, by introducing such multiplets, the gauge unification by MSSM is spoiled. We cannot achieve the gauge unification without introducing several multiplets at the intermediate region between the GUT scale and the intermediate scale, in addition to ordinary matter, three generations of quarks and leptons and a pair of so-called Higgs doublets.

What condition is required of the multiplets at the intermediate scale to recover the gauge unification? In the following we make an analysis based on the RGE up to one loop. We obtain the condition on the beta functions at the intermediate scale in order to achieve the unification of gauge couplings. The conditions of the unification are described by

$$\begin{aligned}
\alpha_Y^{-1}(M_S) &= \alpha_U^{-1}(M_U) + \frac{1}{2\pi} b_Y R + \frac{1}{2\pi} b'_Y (U-R),\\
\alpha_L^{-1}(M_S) &= \alpha_U^{-1}(M_U) + \frac{1}{2\pi} b_L R + \frac{1}{2\pi} b'_L (U-R),\\
\alpha_C^{-1}(M_S) &= \alpha_U^{-1}(M_U) + \frac{1}{2\pi} b_C R + \frac{1}{2\pi} b'_C (U-R),
\end{aligned} \tag{1}$$

where the $b_i$ $(i=Y, L, C)$ with dash and without dash denote the beta function in the lower scale and the higher scale than the intermediate scale $M_{\nu R}$, respectively. $M_S$ is a certain scale which is usually taken to be the SUSY breaking scale. $R$ and $U$ are defined by

$$R = \ln\frac{M_{\nu R}}{M_S}, \quad U = \ln\frac{M_U}{M_S}. \tag{2}$$

These equations lead to the relations which $R$ and $U$ must satisfy:

$$\begin{aligned}
(b_Y - b_L)R + (b'_Y - b'_L)(U-R) &= 2\pi(\alpha_Y^{-1}(M_S) - \alpha_L^{-1}(M_S)),\\
(b_C - b_L)R + (b'_C - b'_L)(U-R) &= 2\pi(\alpha_C^{-1}(M_S) - \alpha_L^{-1}(M_S)).
\end{aligned} \tag{3}$$

Here we have assumed that in the lower scale, MSSM is realized, so Eq. (3) always has the solution $U=R$, which corresponds to the case that there is no intermediate scale physics. Therefore if there is a nontrivial intermediate scale $R$, the beta functions must satisfy the condition

$$(b_Y - b_L)(b'_C - b'_L) - (b_C - b_L)(b'_Y - b'_L) = 0 . \tag{4}$$

Since the beta functions in the MSSM are given by

$$b_Y = \frac{33}{5}, \quad b_L = 1, \quad b_C = -3, \tag{5}$$

the beta functions between the intermediate scale $M_{\nu R}$ and GUT scale $M_U$ must satisfy the equation

$$5b'_Y - 12b'_L + 7b'_C = 0, \tag{6}$$

which we call "the unification condition".*) This is a necessary condition on the gauge coupling unification under the assumption that MSSM is realized in the lower scale. When Eq. (6) is satisfied, $R$ becomes an arbitrary parameter. Therefore we introduce an intermediate scale $M_{\nu R}$ as an input parameter.

Using the unification condition for the beta functions, we make an analysis as follows: Taking one combination of matter content on the intermediate physics, we determine whether the unification condition is fulfilled.**) If this is the case, we can calculate the unified scale $M_U$ and the gauge coupling $\alpha_U(M_U)$ at the unified scale using equations

$$M_U = M_{\nu R} \exp\left(2\pi \frac{\alpha_Y{}^{-1}(M_{\nu R}) - \alpha_L{}^{-1}(M_{\nu R})}{b'_Y - b'_L}\right),$$

$$\alpha_U(M_U) = \left(\alpha_L{}^{-1}(M_{\nu R}) - \frac{1}{2\pi} b'_L (U - R)\right)^{-1}, \tag{7}$$

once $M_R$ and the $\alpha_i{}^{-1}(M_{\nu R})$ are given.

In principle we can calculate the $\alpha_i{}^{-1}(M_{\nu R})$ from low-energy experimental values of the $\alpha_i{}^{-1}$ according to the RGE. We choose, however, another way to calculate the $\alpha_i{}^{-1}(M_{\nu R})$ in order to avoid ambiguities such as the SUSY breaking scale $M_S$, the strong coupling $\alpha_C$, and so on. Because we already know the unification scale $M_U{}^{\mathrm{MSSM}}$ and $\alpha_U{}^{\mathrm{MSSM}-1}(M_U{}^{\mathrm{MSSM}})$ in MSSM GUT and above the intermediate scale considered in this paper all coupling $\alpha_i$ are small enough for one-loop approximation of RGE to work well, we calculate the $\alpha_i{}^{-1}(M_{\nu R})$ from the input parameter $\alpha_U{}^{\mathrm{MSSM}-1}(M_U{}^{\mathrm{MSSM}})$ at the GUT scale $M_U{}^{\mathrm{MSSM}}$. We choose input parameters from Ref. 3) as

$$M_U{}^{\mathrm{MSSM}} = 10^{16.3}\,\mathrm{GeV}, \quad \alpha_U{}^{\mathrm{MSSM}-1}(M_U{}^{\mathrm{MSSM}}) = 25.7 . \tag{8}$$

Then we select the matter content which satisfies the following phenomenological criteria:

---

*) Though $b'_Y = b'_L = b'_C$ satisfies the unification condition, in this case the condition that all couplings are unified is not fulfilled. Therefore this case is excluded.

**) Candidates for the matter content in the intermediate region are multiplets included in representations **27**, **78**, **351**, **351'** and **650** of $E_6$. Using these representations, all of the $G_{\mathrm{intermediate}}$ can be realized if these have appropriate vacuum expectation values. This is just an assumption. In general, however, in the models mentioned above it is difficult to include a representation which is contained in a higher representation of $E_6$ only (see the statement at the end of this section).

1. The unified scale $M_U$ is larger than $10^{16}$ GeV. This is necessary for suppression of proton decay.[12)]
2. The intermediate scale is taken at $10^{10}$, $10^{11}$, $10^{12}$, $10^{13}$ or $10^{14}$ GeV because of right-handed neutrino masses.
3. No colored Higgs is contained in the intermediate physics. This is needed also for suppression of proton decay.[12)]

A search was made for all possible combinations of matter content by numerical computation. This is actually possible because the number of each matter multiplet which we can take into account simultaneously cannot be very large due to the conditions we have already mentioned; generally, the larger the number of the matter content is, the larger their contributions to beta functions are, and the stronger the corresponding couplings $\alpha_i$ become. As a result $\alpha_U^{-1}(M_U)$ becomes negative below the unification scale $M_U$.

## § 4. Result

Here we list the results in order. As matter content which satisfies the unification condition Eq. (6) we show matter content which leads to the smallest unified coupling.

(A) Subgroups of $SO(10)\times U(1)$

In the case of i), the results are identical to those given in Ref. 9) ($SO(10)$ GUT case), because the extra $U(1)_X$ is orthogonal to $SO(10)$. Of course, since there are extra multiplets in the intermediate region as a result of $E_6$ GUT, the unified couplings are calculated to be smaller.

When $SU(4)$ is realized as the intermediate symmetry, as in case 1) and 2), it is difficult to have a solution. To give mass to the right-handed neutrino, we must introduce **10** of $SU(4)$. Its contribution to the $\beta$ function is so large that it makes the inverse of the unification coupling, $\alpha_U^{-1}(M_U)$, much smaller. Details are given in Ref. 9).

In case 3) we have many solutions. As an example we show a result which gives the smallest value of $\alpha_U \simeq 1/16.8$.*)

$$\alpha_U^{-1}(M_U)=16.8\,, \quad M_U=10^{16.3}\,\mathrm{GeV}\,, \quad M_{\nu R}=10^{12}\,\mathrm{GeV}\,,$$

Higgs contents

| | |
|---|---|
| $(1, 3, 1, 6, -2)$+h.c. | 1 |
| $(2, 2, 1, 0, -2)$+h.c. | 1 |
| $(3, 1, 1, 0, 0)$ | 1 |
| $(1, 1, 8, 0, 0)$ | 1 . |

*) We can add an arbitrary $(1, 1, 1, 0, x)$+h.c., where $x$ is a charge of $U(1)_X$. For example $(1, 1, 1, 0, 4)$ is contained in $E_6$ **27**. Such a multiplet is a singlet under the SM, and hence it does not contribute to the running of the gauge couplings. Actually we need $(1, 1, 1, 0, 4)$+h.c. in addition to $(1, 3, 1, 6, -2)$+h.c. to break the intermediate symmetry down to the SM symmetry.

In the case of another $M_{\nu R}$, $\alpha_U{}^{-1}(M_U)$ is slightly varied, though $M_U$ does not change. The notation $(1, 3, 1, 6, -2)$+h.c. 1 indicates that the representation of the Higgs under the subgroup 3) is $(1, 3, 1, 6, -2)$+h.c., and there is a single field. $(1, 3, 1, 6, -2)$+h.c. is responsible for the right-handed neutrino mass. $(2, 2, 1, 0, \mp 2)$ can be regarded as the standard Higgs in the MSSM.

In the case of ii), as mentioned above, since $SU(4)$ is contained in the intermediate symmetry, generally it is difficult to have a solution. In other words the unified coupling is, in general, calculated to be larger. When $G_4$ is $SU(2)$,*) for example,

$$\alpha_U{}^{-1}(M_U)=10.9\,, \quad M_U=10^{16.3}\,\mathrm{GeV}\,, \quad M_{\nu R}=10^{12}\,\mathrm{GeV}\,,$$

<u>Higgs contents</u>

| | |
|---|---|
| $(2, 1, -2, 2)$+h.c. | 1 |
| $(2, 4, -1, 1)$+h.c. | 1 |
| $(1, \bar{4}, 1, 2)$+h.c. | 1 |
| $(1, \overline{10}, 2, 1)$+h.c. | 1 |
| $(1, 1, 4, 1)$+h.c. | 2 |
| $(3, 1, 0, 1)$ | 1 |

is a solution.**) $(1, \overline{10}, 2, 1)$+h.c. gives mass to right-handed neutrino. $(1, \bar{4}, 1, 2)$+h.c. is responsible for mass terms of $d^c\bar{d}^c$ and $l\bar{l}$. $(2, 1, -2, 2)$+h.c. becomes the Higgs doublet for down type quarks and leptons. $(2, 4, -1, 1)$+h.c. becomes the Higgs doublet for up type quarks.

In the case that $G_4$ is $U(1)$ or null,

$$\alpha_U{}^{-1}(M_U)=12.4\,, \quad M_U=10^{16.3}\,\mathrm{GeV}\,, \quad M_{\nu R}=10^{12}\,\mathrm{GeV}\,,$$

<u>Higgs contents</u>

| | |
|---|---|
| $(2, 1, -2, 1)$+h.c. | 3 |
| $(2, 4, -1, 0)$+h.c. | 1 |
| $(1, \bar{4}, 1, 1)$+h.c. | 1 |
| $(1, \overline{10}, 2, 0)$+h.c. | 1 |
| $(1, 1, 4, 0)$+h.c. | 1 |

is a solution.***)

(B) Subgroups of $SU(3)\times SU(3)\times SU(3)$

*) We can add any number of $(1, 1, 0, n)$, where $n$ is an arbitrary representation of $G_4$, since it is a singlet under the SM gauge group.

**) $(2, 1, -2, 2)$+h.c. and $(1, 1, 4, 1)$+h.c.+$(3, 1, 0, 1)$ have the same contribution to the unification condition, and hence we can replace them with each other.

***) The two $(2, 1, -2, 1)$+h.c. can be replaced by $(1, 1, 4, 0)$+h.c.+$(3, 1, 0, 0)$.

There is no solution in chain 5).

In the case that 6) is realized as the intermediate symmetry, there are many solutions:

$$\alpha_U^{-1}(M_U)=18.3\,,\quad M_U=10^{16.3}\,\mathrm{GeV}\,,\quad M_{\nu R}=10^{12}\,\mathrm{GeV}\,,$$

<u>Higgs contents</u>

| | |
|---|---|
| (1, 3, 1, 2)+h.c. | 2 |
| (2, 3, 1, −1)+h.c. | 1 |
| (1, 3, 1, −4)+h.c. | 1 |
| (1, 1, 3, 4)+h.c. | 2 . |

(2, 3, 1, −1)+h.c. becomes Higgs doublets under the SM. (1, 3, 1, 2)+h.c. is needed for the mass term of $d^c$ and $\bar{d}^c$. It is also necessary for $l\bar{l}$. (1, 3, 1, −4)+h.c. gives mass to the $\nu_R$.

There are nine solutions which lead to the unified gauge coupling to 1/16.8, and twelve solutions corresponding to the unified gauge coupling 1/15.3. Thus this breaking chain seems to be hopeful.

Next we consider 7). There are many solutions, though the unified coupling becomes rather high:*)

$$\alpha_U^{-1}(M_U)=7.96\,,\quad M_U=10^{16.3}\,\mathrm{GeV}\,,\quad M_{\nu R}=10^{12}\,\mathrm{GeV}\,,$$

<u>Higgs contents</u>

| | |
|---|---|
| ($\bar{3}$, 2, 1, 1)+h.c. | 1 |
| ($\bar{3}$, 1, 1, −2)+h.c. | 1 |
| (6, 2, 1, 1)+h.c. | 1 |
| (1, 3, 1, 0) | 1 |
| (1, 1, 8, 0) | 1 |
| (1, 1, $\bar{3}$, −4)+h.c. | 1 |
| (1, 1, 6, −4)+h.c. | 1 . |

($\bar{3}$, 2, 1, 1)+h.c. and ($\bar{3}$, 1, 1, −2)+h.c. become Higgs doublets under the SM. They are also necessary for the mass term of $d^c\bar{d}^c$ and $l\bar{l}$. (6, 2, 1, 1)+h.c. gives mass to the $\nu_R$.

Finally we give a result of 8). When $G_8$ is $SU(2)$,**)

$$\alpha_U^{-1}(M_U)=12.4\,,\quad M_U=10^{16.3}\,\mathrm{GeV}\,,\quad M_{\nu R}=10^{12}\,\mathrm{GeV}\,,$$

*) We can replace (1, 3, 1, 0) with (1, 2, 1, 3)+h.c., (1, 1, 6, −4)+h.c. with 2×(1, 3, 1, 0)+(1, 1, 6, 2)+h.c., and so on.

**) We can add any number of (1, 1, 0, $n$), where $n$ is an arbitrary representation of $G_8$, since it is singlet under the SM gauge group.

Higgs contents

| | |
|---|---|
| $(\bar{3}, 1, 1, 2)$+h.c. | 2 |
| $(\bar{3}, 1, -2, 1)$+h.c. | 1 |
| $(6, 1, -2, 1)$+h.c. | 1 |
| $(1, 8, 0, 1)$ | 2 |

is a solution. $(\bar{3}, 1, 1, 2)$+h.c. and $(\bar{3}, 1, -2, 1)$+h.c. become Higgs doublets under the SM. $(\bar{3}, 1, 1, 2)$+h.c. plays the role of giving the mass terms $d^c\bar{d}^c$ and $l\bar{l}$. $(6, 1, -2, 1)$+h.c. gives mass to the $\nu_R$.

If $G_8$ is $U(1)$ or null, there are other solutions:*)

Higgs contents

| | |
|---|---|
| $(\bar{3}, 1, 1, 1)$+h.c. | 1 |
| $(\bar{3}, 1, -2, 0)$+h.c. | 1 |
| $(6, 1, -2, 0)$+h.c. | 1 |
| $(1, 8, 0, 0)$ | 1 |
| $(3, \bar{3}, 1, 1)$+h.c. | 1 . |

(C) Subgroups of $SU(2)\times SU(6)$

In the case 9) we have solutions, though the unified coupling is too large to believe that the subgroup 9) $SU(2)_L\times SU(5)_C\times U(1)_W$ is realized as the intermediate symmetry.

$$\alpha_U{}^{-1}(M_U)=6.48\,,\quad M_U=10^{16.3}\,\text{GeV}\,,\quad M_{\nu R}=10^{12}\,\text{GeV}\,,$$

Higgs contents

| | |
|---|---|
| $(1, 5, -4)$+h.c. | 1 |
| $(2, 1, 5)$+h.c. | 2 |
| $(2, \bar{5}, -1)$+h.c. | 1 |
| $(1, \overline{10}, 8)$+h.c. | 1 |
| $(3, 1, 0)$ | 2 |
| $(1, 24, 0)$ | 2 . |

$(1, 5, -4)$+h.c. plays the role of giving the mass term of $d^c\bar{d}^c$ and $l\bar{l}$. $(2, \bar{5}, -1)$+h.c. becomes the down type Higgs doublet under the SM. $(2, 1, 5)$ is the up type Higgs doublet. $(1, \overline{10}, 8)$+h.c. gives mass to the $\nu_R$.

On the other hand, there is no solution in 10). The reason is that in this case the representation of the necessary Higgs multiplets becomes very high.

*) $(3, \bar{3}, 1, 1)$+h.c. can be replaced with $(1, 1, 3, 1)$+h.c.+$(3, 3, 0, 0)$+h.c.

## § 5. Summary

In most cases we have solutions which satisfy the unification condition Eq. (6). Among them, however, only four intermediate groups can lead to a small unified coupling. 3) $SU(2)_L \times SU(2)_R \times SU(3)_C \times U(1)_{B-L} \times U(1)_X$ and 6) $SU(2)_L \times SU(3)_R \times SU(3)_C \times U(1)_Z$ seem possible to be realized as the intermediate group. 4) $SU(2)_L \times SU(4)_{PS'} \times U(1)_X \times G_4$ and 8) $SU(3)_L \times SU(3)_C \times U(1)_Z \times G_8$ may also be possible.

The reason why the unified coupling, in general, becomes rather large is that there are extra multiplets in the intermediate region which are contained in $E_6$ **27**. By the intermediate symmetry, these multiplets cannot acquire mass. They contribute to the running of the gauge couplings. The extra multiplets lead the unified gauge coupling to a larger value.

This may be related to the fact that in all cases the unification scale is $M_U^{MSSM}$. There is no solution for which $M_U = M_{\text{Planck}}$. We required that there must be Higgs fields responsible for masses of extra fields in the intermediate region. These also contribute to the running of the gauge couplings. Therefore it is very unlikely that gauge couplings are small at $M_{\text{Planck}}$.

Thus in the case of $E_6$ GUT, it is difficult to have a solution. We can pick up favourable subgroups for the intermediate symmetry, though there are many $E_6$ subgroups.

We have candidates for the intermediate symmetry and the matter content in the intermediate scale. This does not imply that we can construct an $E_6$ GUT with an intermediate scale. To construct an $E_6$ GUT with an intermediate scale, there are other requirements similar to those we met in constructing an $SO(10)$ GUT.[10] In the $SO(10)$ case, a certain multiplet was needed in addition to multiplets responsible for quark and lepton masses. In general, such a requirement makes it difficult to construct a concrete model. Nevertheless, we believe that it is possible to construct an $E_6$ GUT with an intermediate scale, because we could construct an $SO(10)$ GUT. Even in $E_6$ GUT we can consider the right-handed neutrino mass to be a reflection of symmetry breaking.

## Acknowledgements

The author wish to acknowledge M. Bando and T. Takahashi for valuable comments and discussion and thanks A. Bordner for proof reading.

### References

1) H. Georgi, in *Particle and Fields*, ed. C. E. Carlson (American Institute of Physics, 1975).
H. Fritzsch and P. Minkowski, Ann. of Phys. **93** (1975), 193.
2) F. Gürsey, P. Ramond and P. Sikivie, Phys. Lett. **B60** (1976), 177.
Y. Achiman and B. Stech, Phys. Lett. **B77** (1978), 389; Ann. of Phys. **93** (1975), 193.
3) U. Amaldi, W. Boer and H. Fürstenau, Phys. Lett. **B260** (1991), 447.
P. Langacker and M. Luo, Phys. Rev. **D44** (1991), 817.
4) For a review, M. Fukugita and T. Yanagida, in *Physics and Astrophysics of Neutrinos*, ed. M. Fukugita and A. Suzuki (Springer-Verlag, Tokyo, 1994); in *Neutrinos*, ed. H. V. Klapdor (Springer-Verlag, Berlin, Heidelberg, 1988).

5) L. Wolfenstein, Phys. Rev. **D17** (1978), 2369.
S. P. Mikheev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42** (1985), 913.
6) T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe,* ed. A. Sawada and H. Sugawara (KEK, Tsukuba, Japan, 1979).
M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, ed. F. van Nieuwenhuizen and D. Freedman (North Holland, 1979).
7) E. Witten, Phys. Lett. **91B** (1980), 81.
8) M. T. Grisaru, M. Roček and W. Siegel, Nucl. Phys. **B159** (1979), 429.
9) M. Bando, J. Sato and T. Takahashi, Phys. Rev. **D52** (1995), 3076.
10) J. Sato, Phys. Rev. **D53** (1996), 3884.
11) R. Slansky, Phys. Rep. **79** (1981), 1.
12) For a review, W. Lucha, Comments Nucl. Part. Phys. **16** (1986), 155.
P. Langacker, Phys. Rep. **72** (1981), 185.
13) J. C. Pati and A. Salam, Phys. Rev. **D10** (1974), 275.