Predictions of m_b/m_τ and m_t in an asymptotically nonfree theory

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We discuss an extension of the minimal supersymmetric standard model with five generations of matter superfields. The extra generations are assumed to form a generation–mirror-generation pair (the fourth and antifourth generations) enabling the extra fermions to have $SU(2)_L \times U(1)_Y$-invariant masses. Because of the contribution of the extra generations, all three running gauge couplings of $SU(3)_C \times SU(2)_L \times U(1)_Y$ become asymptotically nonfree while preserving gauge coupling unification at the grand unified theory (GUT) scale. We show that, due to the asymptotically nonfree character of the gauge couplings, (1) the top and bottom Yukawa couplings are strongly focused onto infrared fixed points as they are evolved down in scale making their values at $\mu = M_Z$ insensitive to their initial values at $\mu = M_{GUT}$, (2) the model predicts $R_{b\tau}(M_Z) \equiv Y_b/Y_\tau|_{\mu=M_Z} \approx 1.8$, which is consistent with the experimental value provided we take the ratio of Yukawa couplings at the GUT scale to be $R_{b\tau}(M_{GUT}) = Y_b/Y_\tau|_{\mu=M_{GUT}} = 1/3$, and (3) the t mass prediction comes out to be $m_t \approx 180$ GeV which is also consistent with experiment. [S0556-2821(97)05815-3]

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I. INTRODUCTION

The popularity of the minimal supersymmetric standard model (MSSM) in recent years is mainly due to its success in attaining gauge coupling unification: given the particle content of the MSSM, the three coupling constants of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge groups converge to a common value at a common scale [the grand unified theory (GUT) scale] when evolved up to higher energies using the renormalization group equations (RGE) [1]. This unification of the gauge coupling constants is crucial if one wishes to construct a GUT which unifies the three gauge groups of the standard model (SM) into a larger simple group at a single scale. However, it should be noted that the particle content which achieves such unification is not unique [2]. In particular, as pointed out in Ref. [3], one always has the freedom to add complete generations of matter superfields to the MSSM without destroying the unification condition.[1]

Another attractive feature of the MSSM is the possibility of unifying the b and τ Yukawa couplings: if one assumes

$$R_{b\tau}(M_{GUT}) = Y_b(M_{GUT})/Y_\tau(M_{GUT}) = 1 \qquad (1.1)$$

at the GUT scale,[2] then one finds that the MSSM can reproduce the experimental value of $R_{b\tau}(M_Z) = Y_b(M_Z)/Y_\tau(M_Z) \approx 1.8$ if the Yukawa couplings of the top and the bottom were such that $Y_t(M_{GUT}) \gtrsim 2 \gg Y_b(M_{GUT})$ ($1 \lesssim \tan\beta \lesssim 3$), or $Y_t(M_{GUT}) \approx Y_b(M_{GUT}) \approx 1$ ($40 \lesssim \tan\beta \lesssim 60$) [5].[3]

The reason why the experimental value of $R_{b\tau}$ can only be reproduced for either small or large $\tan\beta$ is easy to understand:[4] QCD interactions will enhance $Y_b(\mu)$ over $Y_\tau(\mu)$ as they are evolved down from M_{GUT} to M_Z so that $R_{b\tau}(M_Z)$ will end up well above the experimental value if only running due to gauge interactions were taken into account. This QCD effect must be countered by strong Yukawa interactions which will slow down the running. A smaller value of $R_{b\tau}$ consistent with experiment can be obtained when Y_t is large enough to counter the QCD enhancement

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[1]Of course, if one adds too many generations, the gauge couplings will reach the Landau pole before reaching the GUT scale. See Ref. [4].

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[2]Whether the condition $Y_b(M_{GUT}) = Y_\tau(M_{GUT})$ is realized or not in GUT's depends on the representation of the Higgs field which gives mass to the fermions. For SU(5), SO(10), and E_6 unifications, the Higgs boson must be in the **5**, **10**, and **27** representations, respectively.

[3]Note that since $m_t/m_b = (Y_t/Y_b)\tan\beta$, the region $Y_t \gg Y_b$ corresponds to small $\tan\beta$ while $Y_t \approx Y_b$ corresponds to large $\tan\beta$. The lower and upper limits of $1 \lesssim \tan\beta$ and $\tan\beta \lesssim 60$ are required to keep Y_t and Y_b in the perturbative region throughout evolution between M_Z and M_{GUT}.

[4]We assume the reader has some familiarity with the renormalization group equations (RGE's) for the Yukawa couplings.

alone, or when both Y_t and Y_b are large so that the two of them combined can have the desired effect. In the intermediate $\tan\beta$ region ($3 \lesssim \tan\beta \lesssim 40$) Y_t is not large enough to sufficiently suppress the increase of $R_{b\tau}$ by itself while Y_b is not large enough to compensate for it.

Of these two solutions, the small $\tan\beta$ case is often considered particularly attractive since the large size of $Y_t(M_{\rm GUT})$ will drive $Y_t(\mu)$ rapidly towards an infrared *quasi*fixed point [6] as it is evolved down in scale. As a result, the value of $Y_t(M_Z)$ is highly insensitive to its initial value $Y_t(M_{\rm GUT})$ at the GUT scale. On the other hand, the large $\tan\beta$ case opens the possibility of unifying the top Yukawa coupling with the other two:

$$Y_t(M_{\rm GUT}) = Y_b(M_{\rm GUT}) = Y_\tau(M_{\rm GUT}), \qquad (1.2)$$

as required in SO(10) unification with a **10** Higgs representation. However, the insensitivity to the initial condition at $M_{\rm GUT}$ is lost.[5]

In this paper, we wish to outline how these conclusions will be modified when the MSSM is extended with an addition of a generation–mirror-generation pair of extra matter superfields (the fourth and antifourth generations).[6] Each generation is assumed to consist of the usual 15 chiral fermion fields plus their superpartners. We will ignore the right-handed neutrino necessary to form the **16** representation of SO(10) since we will always assume it to have a superheavy Majorana mass and make it decouple from the RGE's.[7] Because of the mirror quantum number assignments between the fourth and antifourth generation fermions, they can develop $SU(2)_L \times U(1)_Y$-invariant masses enabling the left-handed neutrino to have a heavy Dirac mass thus circumventing the LEP limit for the number of massless neutrinos. Also, radiative corrections to LEP observables from the extra fermions can be made to decouple by making this gauge invariant mass large [11].

One immediate consequence of the presence of the two extra generations is that all three gauge couplings of $SU(3)_C \times SU(2)_L \times U(1)_Y$ will be asymptotically nonfree: they will become larger as they are evolved up to coincide at the unification scale [12]. This property is actually unique to the five generation model. In models with four generations[8] or less, the QCD coupling will stay asymptotically free, and in models with six generations or more the couplings will diverge before unification.

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[5]Another problem with the large $\tan\beta$ solution is that fine-tuning of the Higgs potential is necessary to achieve radiative electroweak symmetry breaking. In the small $\tan\beta$ case, radiative electroweak symmetry breaking is naturally achieved due to the initial condition $Y_t(M_{\rm GUT}) \gg Y_b(M_{\rm GUT})$. However, fine-tuning is necessary in this case also to obtain the correct value of $\tan\beta$. See, for instance, Ref. [7].

[6]Such pairs are well known to exist in many GUT scenarios. See Ref. [8].

[7]We do not consider an intermediate scale for the right-handed neutrino mass for the sake of simplicity. See Refs. [9,10] for analyses of the MSSM case with an intermediate scale.

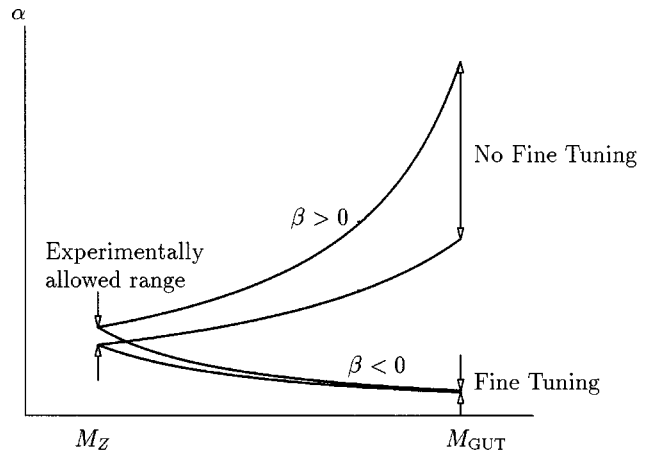[8]Four generation models have been discussed in Ref. [13].

FIG. 1. The difference between asymptotically free and nonfree theories.

As shown in the Appendix, the unification of gauge couplings is controlled solely by the *differences* of the beta function coefficients in the one-loop approximation. Since the differences of the coefficients are independent of the number of full generations, the gauge coupling unification in our five generation model works well just as in the MSSM.

However, an important difference between asymptotically free theories and asymptotically nonfree theories is that $\alpha = 0$ is an ultraviolet (UV) fixed point in the former but an infrared (IR) fixed point in the latter. This means that for asymptotically free (nonfree) theories, the RG flow will be such that a large region of α values in the IR (UV) will flow into a small region close to $\alpha = 0$ in the UV (IR), and the difference in the relative size of these regions will be more pronounced for larger separations in scale. Therefore, in order to get the desired value of $\alpha(M_Z)$ in asymptotically free theories, the value of $\alpha(M_{\rm GUT})$ must be tuned to extreme accuracy while for asymptotically nonfree theories, no fine-tuning is necessary. This is shown schematically in Fig. 1.

This absence of the necessity to fine-tune parameters at the UV cutoff is an extremely attractive feature of asymptotically nonfree theories. It means that the high energy theory effective above the UV cutoff can give the correct predictions at low energies as long as it predicts the values of the running couplings at the cutoff to be within an only mildly restricted range. This point has been emphasized previously by many authors [14] (though not necessarily from a modern point of view). In particular, Moroi, Murayama, and Yanagida [4] have studied the same five generation model as we are considering here and have shown that the values of the running couplings at $M_{\rm GUT}$ need not even be unified to predict the correct value of $\sin^2\theta_w$.

In this paper, we extend the analysis of Moroi *et al.* and study how the existence of the extra generations will affect the running of the Yukawa couplings of the third generation fermions. A similar problem for the nonsupersymmetric case has been considered in [15]. As in the MSSM case, we will impose a unification condition on the Yukawa couplings at $M_{\rm GUT}$ and determine the parameter range in which our model can predict the correct top, bottom, and τ-lepton masses. Babu and Pati [16] also studied the five generation model in the context of string scale unification. They showed that the extra two generation has several advantages for string unifi-

cation. Their model is different from ours in the Higgs sector where they assumed two extra Higgs singlets.

The attentive reader at this point may think that such a program is doomed to failure from the beginning. Since the QCD coupling is asymptotically nonfree, the QCD enhancement of $R_{b\tau}$ from M_{GUT} to M_Z will be even larger than the MSSM case making it impossible to bring $R_{b\tau}(M_Z)$ down to ~ 1.8 even with large Yukawa couplings. However, we would like to quickly point out that the unification condition need not be that of Eqs. (1.1) or (1.2). In fact, an SO(10)-GUT with a **126** Higgs representation predicts [17]

$$Y_t(M_{\text{GUT}}) = Y_b(M_{\text{GUT}}) = \frac{1}{3} Y_\tau(M_{\text{GUT}}), \qquad (1.3)$$

so that $R_{b\tau}(M_{\text{GUT}}) = 1/3$. This is the unification condition which we will adopt.[9] In this case, the extra enhancement from QCD is actually welcome since $R_{b\tau}$ must be enhanced by a factor of 5–6 to reproduce the experimental value of $R_{b\tau}(M_Z)$.

This paper is organized as follows. In Sec. II, we describe our model and specify the way we calculate the RG evolution of the gauge and Yukawa couplings. In Sec. III, we show how the gauge couplings can be unified in our model. Section IV discusses Yukawa coupling unification and the predictions for $R_{b\tau}(M_Z)$ and m_t. Section V concludes.

II. THE $4 + \bar{1}$ GENERATION MODEL

In extending the MSSM by introducing extra matter superfields, we must keep two things in mind: (1) the matter superfields must be introduced in such a way that gauge coupling unification (and anomaly cancellation) of the MSSM is preserved, and (2) the fermion content must be compatible with the constraints placed by LEP measurements at the CERN e^+e^- collider LEP, namely three massless neutrino species and the so-called Peskin-Takeuchi constraint [18].

The simplest way to satisfy these requirements is to introduce two extra generations which form a generation–mirror-generation pair. We will call them the fourth and antifourth generations. The fermion content of these extra generations will be ''vectorlike'' so that all of them, including the extra neutrinos, can develop $\text{SU}(2)_L \times \text{U}(1)_Y$-invariant Dirac masses. These masses will also suppress the size of radiative corrections to LEP observables from the extra fermions enabling them to circumvent the Peskin-Takeuchi constraint [11].

It should be noted that we can only introduce one such generation–mirror-generation pair. If we introduce two pairs or more, all three gauge couplings will reach their Landau poles and diverge well before the would-have-been unification scale [4].

We denote the extra fermion families (U,D,N,E) and $(\bar{U},\bar{D},\bar{N},\bar{E})$, respectively, and give them a common $\text{SU}(2)_L \times \text{U}(1)_Y$-invariant mass of M_{EVF}. Their superpartners, and all the other supersymmetric particles in the theory

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[9]A **126** Higgs representation is necessary to give a direct Majorana mass term to the right-handed neutrino.

will be given a common SUSY-breaking mass of M_{SUSY}. For the sake of simplicity, we take $M_{\text{EVF}} = M_{\text{SUSY}} = 1$ TeV.

In addition to the $\text{SU}(2)_L \times \text{U}(1)_Y$-invariant masses, we also couple the fourth and antifourth generation fermions to the two Higgs doublets in the same way as the other generations. Here we take the case where

$$Y_U = Y_t, \quad Y_{\bar{U}} = 0,$$

$$Y_D = Y_b, \quad Y_{\bar{D}} = 0,$$

$$Y_E = Y_\tau, \quad Y_{\bar{E}} = 0, \qquad (2.1)$$

and set all the first and second generation Yukawa couplings to zero. Furthermore, we impose the unification condition

$$Y_t(M_{\text{GUT}}) = Y_b(M_{\text{GUT}}) = \frac{1}{3} Y_\tau(M_{\text{GUT}}) \equiv Y_{\text{GUT}}, \quad (2.2)$$

as mentioned in the Introduction.

In view of the relatively large coupling strengths near the unification scale due to the asymptotically nonfreeness, we use the fully coupled two-loop renormalization group equations (RGE's) from Ref. [19] to evolve the gauge and Yukawa couplings. We ignore small differences in the masses of the fourth and antifourth generation particles or that of the supersymmetric particles which may be induced by the Yukawa couplings and simply set all their masses at $M_{\text{EVF}} = M_{\text{SUSY}} = 1$ TeV. We also ignore threshold corrections. Therefore, between M_{GUT} and $M_{\text{EVF}} = M_{\text{SUSY}}$, we evolve the couplings with the RGE's for the supersymmetric SM with five supergenerations and two super-Higgs doublets, while between $M_{\text{EVF}} = M_{\text{SUSY}}$ and M_Z, we use the RGE's for the SM with only three ordinary generations and one Higgs doublet. The gauge couplings are connected continuously at $M_{\text{EVF}} = M_{\text{SUSY}} = 1$ TeV while the up-type (down-type) Yukawa couplings are multiplied by $\sin\beta$ ($\cos\beta$) below $M_{\text{EVF}} = M_{\text{SUSY}}$ to take into account the decoupling of one of the Higgs bosons.

The number of adjustable parameters in our model is four: the unification scale M_{GUT}, the unified gauge coupling α_{GUT}, the unified Yukawa coupling Y_{GUT}, and the mixing angle of the low lying Higgs fields $\tan\beta = v_2/v_1$, where $\sqrt{v_1^2 + v_2^2} = v = 246$ GeV. We restrict α_{GUT} and Y_{GUT} to the region

$$\alpha_{\text{GUT}} < 1.0, \quad Y_{\text{GUT}} < 0.7. \qquad (2.3)$$

[Note that $Y_\tau(M_{\text{GUT}}) = 3Y_{\text{GUT}}$. Note also that the natural expansion coefficient corresponding to the $\alpha_i(\mu)$'s is $Y^2/(4\pi)$ for the Yukawa couplings.] As we will see later, this will keep the gauge and Yukawa couplings within their perturbative regions throughout the evolution from M_{GUT} to M_Z.

Since we do not consider the evolution of the soft supersymmetry- (SUSY-) breaking parameters of the Higgs potential in this paper, $\tan\beta$ will remain a phenomenological parameter to be fixed by hand. We will use the τ-lepton mass to fix $\tan\beta$ from
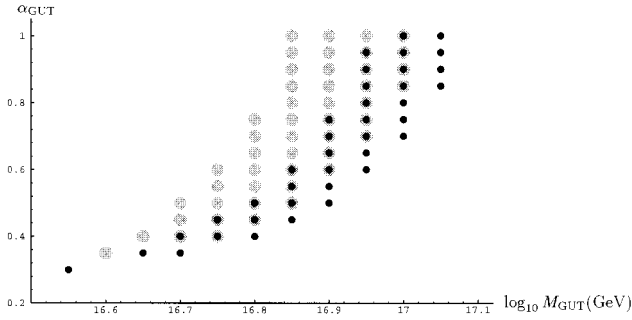
FIG. 2. The allowed region in the plane (α_{GUT}, M_{GUT}). The small black and large gray circles indicate the ranges $0.4 \leq Y_{GUT} < 0.7$ and $0.1 < Y_{GUT} < 0.4$, respectively.

$$m_\tau(M_{SUSY}) = \frac{v}{\sqrt{2}} Y_\tau(M_{SUSY}) \cos\beta. \qquad (2.4)$$

III. GAUGE COUPLING UNIFICATION

The values of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ coupling constants at $\mu = M_Z$ are given by [20]

$$\alpha_1(M_Z) = 0.01689 \pm 0.00005, \qquad (3.1)$$

$$\alpha_2(M_Z) = 0.03322 \pm 0.00025, \qquad (3.2)$$

$$\alpha_3(M_Z) = 0.12 \pm 0.01. \qquad (3.3)$$

Note that these are the $\overline{MS}$ coupling constants[10] and that the U(1) coupling is normalized to $\alpha_1 = (5/3)g'^2/4\pi$. Throughout the paper, we take rather larger error for $\alpha_3(M_Z)$ than that in [20].

For fixed values of Y_{GUT} in the range given in Eq. (2.3), we searched for values of α_{GUT} and M_{GUT} which reproduced the experimental data given above. The results are shown in Fig. 2.

We see that the allowed range of α_{GUT} is narrow for smaller M_{GUT} but still exist down to $M_{GUT} \approx 10^{16.55}$ GeV and becomes wider as M_{GUT} is increased. This result is as expected from our discussion on asymptotically nonfree theories: a wider range of α_{GUT} corresponds to a much smaller range of couplings at M_Z, and the allowed range will become wider as M_{GUT} is increased. However, if we increase M_{GUT} beyond $\sim 10^{17.1}$ GeV, then α_{GUT} and/or Y_{GUT} will have to be taken beyond the limits specified in Eq. (2.3) and they will be too large for the perturbative treatment of the RGE's to be reliable.

As an example, we show the running of the three gauge couplings in Fig. 3 for typical values of α_{GUT}, M_{GUT}, and Y_{GUT}. We see a small deviation from linear dependence on $\ln\mu$ near M_{GUT} where the couplings become large and the two-loop corrections start contributing to the running appreciably. However, it is clear that two-loop contributions are
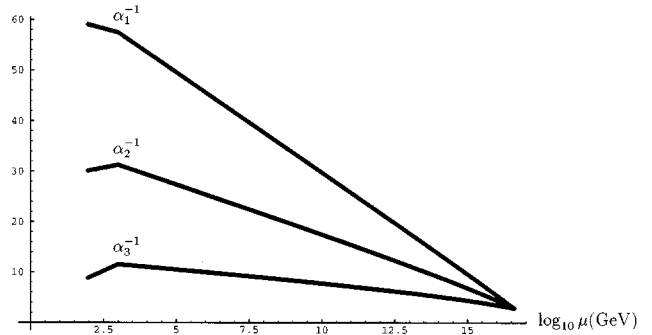

FIG. 3. Typical μ dependence of $\alpha_1(\mu)$, $\alpha_2(\mu)$, and $\alpha_3(\mu)$. The parameter values for this plot were $(\alpha_{GUT}, Y_{GUT}, M_{GUT}, \tan\beta) = (0.35, 0.3, 10^{16.6}$ GeV, 57.5$)$.

still not very serious within the range of α_{GUT} which we have chosen here and we may regard our perturbative treatment to be sufficient.

IV. YUKAWA COUPLING UNIFICATION

Next, we fix the values of α_{GUT} and M_{GUT} in the range allowed by gauge coupling unification and calculate the evolution of the Yukawa couplings for different values of Y_{GUT}. Typical evolutions of the τ, b, and t Yukawa couplings are shown in Figs. 4, 5, and 6. As is evident from these figures, the asymptotic nonfreeness of the gauge couplings has a strong focusing effect on the top and bottom Yukawa couplings as they evolve down in scale and as a result, the values of the two Yukawa's converge to IR fixed points by the time they reach the SUSY-breaking scale $M_{SUSY} = 1$ TeV. In the case of the τ Yukawa coupling, the situation is rather different. Near the GUT scale it tends to focus itself due to its larger size at M_{GUT} [recall $Y_\tau(M_{GUT}) = 3 Y_{GUT}$]. However, unlike the top and bottom Yukawa couplings, once Y_τ becomes small at lower scales, it runs slowly and does not quite converge to its IR fixed point $(y_\tau = 0)$.

Due to this IR fixed point behavior, the values of $Y_t(M_Z)$, $Y_b(M_Z)$ have almost no dependence on Y_{GUT}. They do depend on the value of α_{GUT} but even then only very mildly.
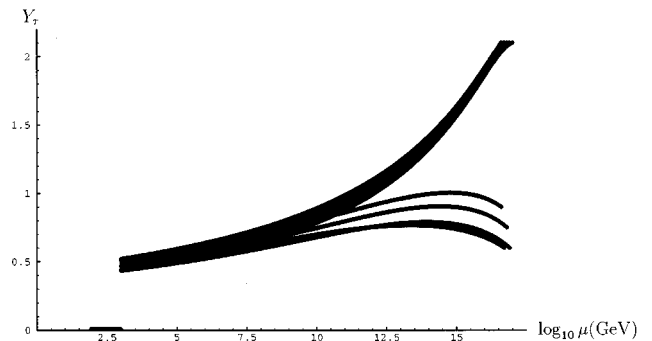

FIG. 4. The running of the τ-lepton Yukawa coupling for typical values of (α_{GUT}, M_{GUT}) shown in Fig. 2. The value of α_{GUT} is varied between 0.3 and 0.8. As an illustration we pick several points around $Y_\tau(M_{GUT}) = 3 Y_{GUT} \sim 0.6$ and several points around $Y_\tau(M_{GUT}) = 3 Y_{GUT} \sim 2.1$ for the plot. The Yukawa coupling is multiplied by $\cos\beta$ below M_{SUSY}.

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[10]This may be a confusing point since the ''effective'' QED coupling constant $\alpha(M_Z)$ and the ''effective'' weak angle $\sin^2\theta_W$ that are usually quoted by LEP are not modified minimal subtraction scheme ($\overline{MS}$) values.
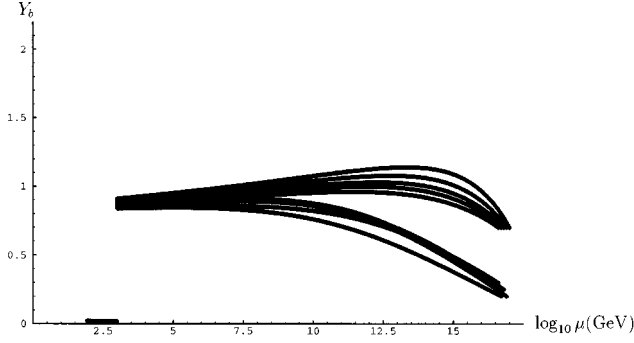
FIG. 5. The running of the bottom Yukawa coupling for typical values of (α_{GUT}, M_{GUT}) shown in Fig. 2. The value of α_{GUT} is varied between 0.3 and 0.8. As an illustration we pick several points around $Y_b(M_{GUT}) = Y_{GUT} \sim 0.2$ and several points around $Y_b(M_{GUT}) = Y_{GUT} \sim 0.7$ for the plot. The Yukawa coupling is multiplied by $\cos\beta$ below M_{SUSY}.

A. Bottom to tau mass ratio

The b-τ mass ratio has been the most intensively studied quantity in both supersymmetric and nonsupersymmetric GUT scenarios. Many authors have investigated the possibility of unifying the two couplings with various degrees of success [2,5,9,10,21–25].

Currently, the experimentally determined $\overline{\text{MS}}$ running masses of the τ-lepton and the b quark at $\mu = M_Z$ are given by [20]

$$m_\tau(M_Z) = 1.75 \pm 0.01 \ \text{GeV},$$

$$m_b(M_Z) = 3.1 \pm 0.4 \ \text{GeV}, \tag{4.1}$$

from which we conclude

$$R_{b\tau}(M_Z) = 1.6 \sim 2.0. \tag{4.2}$$

The dependence of $R_{b\tau}(M_Z)$ on α_{GUT} and M_{GUT} in our model is shown in Fig. 7. Obviously, whether our model can reproduce the experimental value of $R_{b\tau}(M_Z)$ or not depends almost solely on the value of α_{GUT}. If $\alpha_{GUT} > 0.6$, then the QCD interactions near M_{GUT} will be so strong that Y_b will
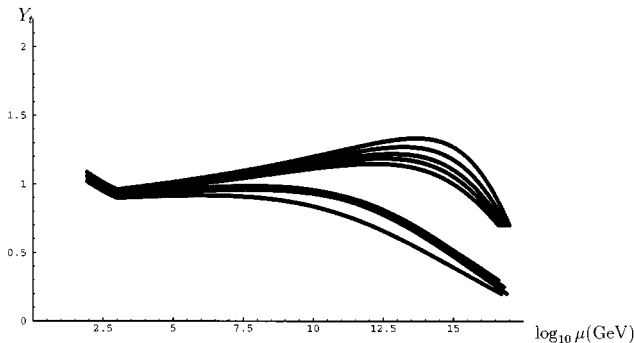

FIG. 6. The running of the top Yukawa coupling for typical values of (α_{GUT}, M_{GUT}) shown in Fig. 2. The value of α_{GUT} is varied between 0.3 and 0.8. As an illustration we pick several points around $Y_t(M_{GUT}) = Y_{GUT} \sim 0.2$ and several points around $Y_t(M_{GUT}) = Y_{GUT} \sim 0.7$ for the plot. The Yukawa coupling is multiplied by $\sin\beta$ below M_{SUSY}.
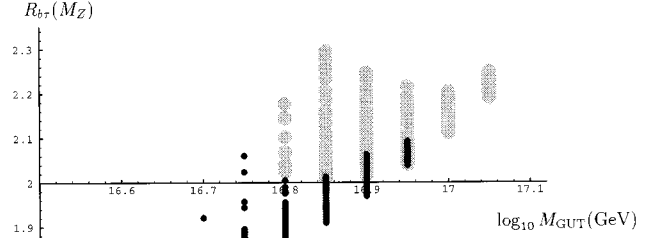

FIG. 7. The dependence of $R_{b\tau}(M_Z)$ on M_{GUT} and α_{GUT}. The small black and large gray circles indicate the ranges $0.3 < \alpha_{GUT} \lesssim 0.6$ and $0.6 < \alpha_{GUT} < 1$, respectively. The dependence of $R_{b\tau}(M_Z)$ on Y_{GUT} is negligible.

be enhanced too much relative to Y_τ. However, there is still a large set of (α_{GUT}, M_{GUT}) values which keeps $R_{b\tau}(M_Z)$ below 2.

Of course, since $\tan\beta$ is large in our model there is potentially a very large radiative correction to m_b from the loop induced coupling of the b quark to v_2 [2,23,26,27]. This correction can throw our prediction off the mark by a considerable amount.

However, such corrections can easily be compensated for. If the correction makes $R_{b\tau}(M_Z)$ smaller, we only need to make α_{GUT} larger. If it makes $R_{b\tau}(M_Z)$ larger, we only need to make α_{GUT} smaller, changing M_{EVF} and/or M_{SUSY} if necessary.

We therefore conclude that our model can accommodate the b-τ mass ratio quite easily without any fine tuning.

B. The top quark mass

Due to the IR fixed point behavior of Y_τ, Y_b, and Y_t we have seen above, for the range of (α, M_{GUT}, Y_{GUT}) values that yield the correct value of $R_{b\tau}(M_Z)$ we find

$$Y_t(M_Z) = 1.01 - 1.07. \tag{4.3}$$

Using the τ mass to fix $\tan\beta$, we find

$$m_t(M_Z) = 176 - 187 \ \text{GeV}, \tag{4.4}$$

which is in perfect agreement with the experimental value [20]:

$$m_t(M_Z) = 180 \pm 10 \ \text{GeV}. \tag{4.5}$$

This result is actually highly dependent on our choice Eq. (2.1) for the fourth and antifourth generation Yukawa couplings. Had we chosen a different condition such as

$$Y_U = Y_{\bar{U}} = Y_t,$$

$$Y_D = Y_{\bar{D}} = Y_b,$$

$$Y_E = Y_{\bar{E}} = Y_\tau, \tag{4.6}$$

then the IR fixed value for Y_t would have been

$$Y_t(M_Z) \sim 0.705 \tag{4.7}$$

leading to a prediction of

$$m_t(M_Z) \sim 154 \text{ GeV}. \tag{4.8}$$

V. SUMMARY AND CONCLUSIONS

We have presented an extension of the MSSM with a generation-antigeneration pair of extra matter superfields. The $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings are asymptotically nonfree in this model but still converge to a common value before any of them diverge. Consequences of this asymptotically nonfree theory are the following.

(1) The unified coupling α_{GUT} and the unification scale M_{GUT} need not be fine-tuned to reproduce the values of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings at $\mu = M_Z$. Larger values of M_{GUT} allow for a larger range of α_{GUT} values. However, M_{GUT} must be taken below $\sim 10^{17.1}$ GeV to keep α_{GUT} and Y_{GUT} within perturbative range.

(2) The top and bottom Yukawa couplings are strongly focused onto IR fixed points. This makes the IR values of the two Yukawa couplings insensitive to their initial value Y_{GUT} at the GUT scale.

(3) The model can reproduce the ratio of bottom and tau masses $R_{b\tau}(M_Z) = Y_b(M_Z)/Y_\tau(M_Z) = 1.6$–2, provided we assume the unification condition $R_{b\tau}(M_{GUT}) = 1/3$ (**126** Higgs representation) instead of the usual $R_{b\tau}(M_{GUT}) = 1$ (**10** Higgs representation).

(4) The model can also give the correct top mass provided the Yukawa couplings of the fourth and antifourth generations are taken as in Eq. (2.1).

It would be most interesting if this insensitivity of the low energy predictions to the initial conditions at the GUT scale could be taken further to include the soft SUSY-breaking parameters of the Higgs sector. In particular, if a large $\tan\beta$ could be generated without fine-tuning, it could provide an answer to the question why the top is so much heavier than the bottom.

Above M_{GUT}, our model is a supersymmetric $SO(10)$ theory which includes four **16**-plets and a $\overline{\textbf{16}}$-plet, and a Higgs sector which consist of at least a **126** and a $\overline{\textbf{126}}$ (in order for **126** to have a mass term). This makes the $SO(10)$ gauge coupling asymptotically nonfree also.

It has recently been argued that such asymptotic nonfreeness of the gauge couplings can be consistently interpreted as a sign of compositeness [28]. The basic reasoning is as follows: The general compositeness condition of gauge bosons is given by $Z(M_{comp}) = 0$ where $Z(\mu)$ is the wave-function renormalization constant and M_{comp} is the compositeness scale. If one enforces conventional normalization $Z(\mu) = 1$ at all scales μ, then superficially the running gauge coupling $\alpha(\mu)$ will diverge at $\mu = M_{comp}$ making it look like an asymptotically nonfree theory. This is analogous to theories with dynamically generated Higgs bosons where compositeness manifests itself as the divergence of the Yukawa coupling at the compositeness scale in the effective Higgs-Yukawa theory [29].

In the SM, the large number of arbitrary parameters have lead most people to believe that going beyond the SM will somehow reduce the number of parameters and make theories more predictive. However, most extensions of the SM such as the MSSM or Technicolor actually *increases* the number of parameters by a huge amount. Reduction of the number of parameters is usually achieved by imposing *ad hoc* symmetries such as R-parity, universality of the scalar masses at the GUT scale, etc. What our analysis has shown is that for certain types of theories with IR fixed points, it may happen that most of the parameters simply *do not matter* or only needs to be specified to an order of magnitude to make precise low energy predictions. Clearly, the IR fixed point phenomena is an alternative to symmetries for making theories more predictive and deserves thorough and systematic investigation.

APPENDIX: SOLUTION TO THE ONE-LOOP RG EQUATIONS

In this appendix, we give a qualitative interpretation of our results in the one-loop approximation.

The one-loop renormalization group equations above the SUSY scale in our model are

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi}\alpha_i^2 \quad (i=1,2,3), \tag{A1}$$

$$\frac{d\alpha_t}{dt} = \frac{\alpha_t}{2\pi}\left[-\left(\frac{13}{15}\alpha_1 + 3\alpha_2 + \frac{16}{3}\alpha_3\right) + (9\alpha_t + \alpha_b)\right], \tag{A2}$$

$$\frac{d\alpha_b}{dt} = \frac{\alpha_b}{2\pi}\left[-\left(\frac{7}{15}\alpha_1 + 3\alpha_2 + \frac{16}{3}\alpha_3\right) + (\alpha_t + 9\alpha_b + 2\alpha_\tau)\right], \tag{A3}$$

$$\frac{d\alpha_\tau}{dt} = \frac{\alpha_\tau}{2\pi}\left[-\left(\frac{9}{5}\alpha_1 + 3\alpha_2\right) + (6\alpha_b + 5\alpha_\tau)\right]. \tag{A4}$$

Here, $t = \ln\mu$, $\alpha_{t,b,\tau} \equiv Y_{t,b,\tau}^2/4\pi$, and the one-loop beta function coefficients are given by the formula

$$(b_1,b_2,b_3)=(0,-6,-9)+(3+2n_\text{vector})(2,2,2)+2\left(\frac{3}{10},\frac{1}{2},0\right) \quad \text{(for the MSSM plus } n_\text{vector} \text{ full generation pairs)}$$

$$=\left(\frac{53}{5},5,1\right) \quad \text{(for } n_\text{vector}=1\text{)}.$$

Since the model is nothing but the standard model below the SUSY scale, as can be seen from Fig. 3, the experimental inputs are essentially equivalent to

$$\alpha_1^{-1}(t_\text{SUSY})\approx58, \quad \alpha_2^{-1}(t_\text{SUSY})\approx31, \quad \alpha_3^{-1}(t_\text{SUSY})\approx12.$$

The solution to Eq. (A1) is

$$\frac{1}{\alpha_i(t)}=\frac{b_i}{2\pi}(t_\text{GUT}-t)+\frac{1}{\alpha_i(t_\text{GUT})} \quad (i=1,2,3).$$

From the above equation, and using $\alpha_1^{-1}(t_\text{SUSY})$ and $\alpha_2^{-1}(t_\text{SUSY})$ as inputs, we can predict $\alpha_3^{-1}(t_\text{SUSY})$ as

$$\alpha_3^{-1}(t_\text{SUSY})=\frac{b_3-b_2}{b_1-b_2}[\alpha_1^{-1}(t_\text{SUSY})-\alpha_2^{-1}(t_\text{SUSY})]$$
$$+\alpha_2^{-1}(t_\text{SUSY})\approx12.$$

Note that only the differences of beta function coefficients appear in this expression. Therefore, at the one-loop level, the prediction of $\alpha_3^{-1}(t_\text{SUSY})$ would be exactly the same for any n_vector. However, the two-loop correction which is $O(\max[\alpha^2(t)])$ would be different depending on whether the theory is asymptotically free or nonfree. In the MSSM, the expected correction would be as large as $O(\alpha^2(t_\text{SUSY}))\sim1\%$, whereas in the $n_\text{vector}=1$ case, the correction would be as large as $O(\alpha^2(t_\text{GUT}))\sim10\%$. Thus if the one-loop prediction differs from the experiment by more than a few percent, one has to consider rather large threshold correction at the GUT scale to explain the discrepancy in the MSSM whereas in the latter model there is still room for two-loop corrections to account for it.

Next, let us solve Eqs. (A2)–(A4) to obtain the low energy behavior of the Yukawa couplings. The first term in each equation is the contribution from the gauge interactions and the second term in each equation is that from the Yukawa interactions. The gauge interactions try to enhance the Yukawa couplings as the scale is lowered whereas the Yukawa interactions tend to reduce it. Therefore, one can naively expect that the Yukawa couplings fall into infrared fixed points where the gauge contribution and the Yukawa contribution balance. Whether this is true or not depends on initial values and the details of the beta function coefficients.

In order to see this more explicitly, let us make the following three assumptions and reduce Eqs. (A2)–(A4) into a much simpler form.

(1) Since at low energy, $\alpha_1,\alpha_2,\ll\alpha_3$, we can neglect α_1,α_2.

(2) At low energy, the contribution from α_τ is not so dominant compared to those from $\alpha_{t,b}$. This is partly because the coefficient of α_τ in Eq. (A3) is not so large and partly because in Eq. (A4) there is no contribution from α_3

which can prevent α_yukawa getting small thus α_τ gets small at lower energy much faster than $\alpha_{t,b}$.

(3) Assuming 1 and 2, the difference between α_t and α_b is almost negligible. This is because we impose $\alpha_t(t_\text{GUT})=\alpha_b(t_\text{GUT})$ as the GUT scale initial condition, and because the approximate RG equation is symmetric under the interchange $\alpha_t\leftrightarrow\alpha_b$.

In the following, we will obtain the solutions to the resulting approximate equations. Of course, the behavior of those solutions will be correct only qualitatively since the corrections from the neglected terms are not completely negligible. (In principle, it is possible to check the validity of this approximation by solving the full equations.) However, since we are only interested in the qualitative features, we will not discuss the corrections from the neglected terms here.

With these assumptions and by setting $\alpha_t=\alpha_b\equiv\alpha_Q$, Eqs. (A2), (A3) become

$$\frac{d\alpha_Q}{dt}=\frac{\alpha_Q}{2\pi}\left(-\frac{16}{3}\alpha_3+10\alpha_Q\right). \quad \text{(A5)}$$

Let us define the ratio $z_Q=\alpha_Q/\alpha_3$. Equation (A5) is then

$$\frac{dz_Q}{dt}=\frac{z_Q}{2\pi}\alpha_3\left(-\frac{19}{3}+10z_Q\right). \quad \text{(A6)}$$

It is easy to see that the solution to Eq. (A6) is

$$\frac{z_Q(t)-\frac{19}{30}}{z_Q(t)}=\frac{z_Q(t_\text{GUT})-\frac{19}{30}}{z_Q(t_\text{GUT})}\exp\left[-\frac{19}{3}\ln\left(\frac{\alpha_3(t_\text{GUT})}{\alpha_3(t)}\right)\right]. \quad \text{(A7)}$$

The exponent $\frac{19}{3}\ln(\alpha_3(t_\text{GUT})/\alpha_3(t))$ at $t=t_\text{SUSY}$ ranges roughly from 8 to 12, thus at t_SUSY the quark Yukawa coupling gets very close to the infrared fixed point

$$\alpha_Q(t)\overset{t\rightarrow t_\text{SUSY}}{\Rightarrow}\frac{19}{30}\alpha_3(t_\text{SUSY}). \quad \text{(A8)}$$

This behavior of the quark Yukawa coupling is consistent with what we see from the numerical solution to the two-loop RG equation.

On the other hand, the one-loop RG equation for the bottom-tau ratio $R_{b\tau}$ can be obtained from Eqs. (A3), (A4). Using the assumptions, the equation at low energy can be simplified to

$$\frac{dR_{b\tau}}{dt}=\frac{R_{b\tau}}{2\pi}\left(-\frac{8}{3}\alpha_3+2\alpha_Q\right)\approx\frac{R_{b\tau}}{2\pi}\left(-\frac{7}{5}\alpha_3\right). \quad \text{(A9)}$$

It is easy to see that the solution to Eq. (A9) is

$$R_{b\tau}(t) = R_{b\tau}(t_{\mathrm{GUT}}) \exp\left[\frac{7}{5}\ln\left(\frac{\alpha_3(t_{\mathrm{GUT}})}{\alpha_3(t)}\right)\right]. \quad (A10)$$

The factor $\exp[\frac{7}{5}\ln(\alpha_3(t_{\mathrm{GUT}})/\alpha_3(t))]$ at $t = t_{\mathrm{SUSY}}$ is about 4.1 to 7.5 for $\alpha_{\mathrm{GUT}} = 0.3\text{–}0.55$. This gives roughly the right enhancement for $R_{b\tau}$.

On the other hand, the RG equations in the MSSM are given by

$$\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi}\alpha_i^2 \quad (i = 1,2,3), \quad (A11)$$

$$\frac{d\alpha_t}{dt} = \frac{\alpha_t}{2\pi}\left[-\left(\frac{13}{15}\alpha_1 + 3\alpha_2 + \frac{16}{3}\alpha_3\right) + (6\alpha_t + \alpha_b)\right], \quad (A12)$$

$$\frac{d\alpha_b}{dt} = \frac{\alpha_b}{2\pi}\left[-\left(\frac{7}{15}\alpha_1 + 3\alpha_2 + \frac{16}{3}\alpha_3\right) + (\alpha_t + 6\alpha_b + \alpha_\tau)\right], \quad (A13)$$

$$\frac{d\alpha_\tau}{dt} = \frac{\alpha_\tau}{2\pi}\left[-\left(\frac{9}{5}\alpha_1 + 3\alpha_2\right) + (3\alpha_b + 4\alpha_\tau)\right]. \quad (A14)$$

The one-loop beta function coefficients are given by the formula

$$(b_1, b_2, b_3) = \left(\frac{33}{5}, 1, -3\right).$$

The equation for z_Q is

$$\frac{dz_Q}{dt} = \frac{z_Q}{2\pi}\alpha_3\left(-\frac{19}{3} + 7z_Q\right). \quad (A15)$$

It is easy to see that the solution to Eq. (A15) is

$$\frac{z_Q(t) - \frac{19}{21}}{z_Q(t)} = \frac{z_Q(t_{\mathrm{GUT}}) - \frac{19}{21}}{z_Q(t_{\mathrm{GUT}})}\exp\left[-\frac{19}{9}\ln\left(\frac{\alpha_3(t)}{\alpha_3(t_{\mathrm{GUT}})}\right)\right]. \quad (A16)$$

The exponent $\frac{19}{9}\ln(\alpha_3(t)/\alpha_3(t_{\mathrm{GUT}}))$ at $t = t_{\mathrm{SUSY}}$ is roughly 2, thus at t_{SUSY} the focusing effect of quark Yukawa coupling is not as strong as in our five generation model.

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