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The Auxiliary Mass Method beyond the Local Potential Approximation

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We show that the evolution equation of the effective potential in the auxiliary mass method corresponds to a leading approximation of a certain series. This series is derived from an evolution equation of an effective action using a derivative expansion. We derive an expression of the next-to-leading approximation of the evolution equation, which is a simultaneous partial differential equation.

§1. Introduction

Finite temperature field theory, which is based only on a statistical mechanism, adequately describes many physical phenomena, such as phase transitions and mass spectra in a thermal bath.[1)-3)] The corresponding perturbation theory, however, breaks down at a high finite temperature. This happens because the mass squared becomes negative even in a vacuum at finite temperature. This problem is solved using the ring resummation, which adds thermal mass to the zero temperature mass beforehand.[4),5)] This procedure, however, is still insufficient to make the perturbative expansion reliable, especially around the critical temperature. For example, the perturbation theory incorrectly indicates that the phase transition of the Z_2-invariant scalar theory is first order.[6)] As another example, different properties are suggested by the perturbation theory and lattice simulations in investigations of the Abelian Higgs model[4),7)-15)] and the Standard Model[16)-24)] in the large Higgs boson mass range.

These failures of the perturbation theory are caused by bad infrared behavior around the critical temperature.[25),26)] The loop expansion parameter becomes $\lambda T/m$ even after the ring resummation, due to this infrared effect. Here, λ is a small coupling constant and m is the mass at the temperature T. The perturbation theory is, therefore, unreliable at high temperature and small mass. This situation arises around the critical temperature of second order phase transitions or weakly first order phase transitions. The auxiliary mass method controls this infrared behavior by introducing an "auxiliary mass".[27)-32)] We first calculate an effective potential with large auxiliary mass, $M \sim T$. This effective potential is reliable because the loop expansion parameter is small, thanks to the auxiliary mass. We next calculate the effective potential at the true mass from this effective potential through an evolution

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equation. We solve the evolution equation of the effective potential with respect to the variation of the mass squared.

We used a certain approximation to derive the evolution equation in Refs. 28) and 29). Though we obtained quite good results using this approximation, we did not have a method to improve the approximation. In the present paper, we show that the evolution equation obtained previously is a leading approximation of a certain series. We then derive the next-to-leading evolution equation, which is a simultaneous partial differential equation. Though it is difficult to solve this evolution equation at arbitrary temperature due to numerical problems, we can, in principle, improve the approximation systematically.

§2. Evolution equation

In this section, we explain the idea pursued in this paper and derive the evolution equation for the Z_2-invariant scalar theory using the auxiliary mass method.*) Let us consider the following Euclidean Lagrangian density with mass squared m^2:

$$\mathcal{L}_E(\phi; m^2) = -\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 - \frac{1}{2}(\boldsymbol{\nabla}\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}(\phi^2)^2 + J_m\phi + \text{c.t.} \quad (2\cdot1)$$

Here, $J_m(x)$ is an external source function which gives

$$\langle\phi(x)\rangle = \phi_c(x), \quad (2\cdot2)$$

and hence depends on m^2. We set the true mass squared to a negative value, $m^2 = -\mu^2$, since we investigate the phase transition of this theory. We assume that the coupling constant λ is small, so that the perturbation theory be reliable at a low temperature ($T \ll m$).

Our idea is the following. The effective action for the theory, $\mathit{\Gamma}[\phi_c; m^2)$, satisfies the following identity,

$$\mathit{\Gamma}[\phi_c; -\mu^2) = \int_{M^2}^{-\mu^2}\left(\frac{\partial\mathit{\Gamma}[\phi_c; m^2)}{\partial m^2}\right)dm^2 + \mathit{\Gamma}[\phi_c; M^2). \quad (2\cdot3)$$

We choose M^2 so large ($\sim T^2$) that we can calculate the initial condition, $\mathit{\Gamma}[\phi_c; M^2)$, reliably using the perturbation theory. If we can evaluate the derivative $\partial\mathit{\Gamma}(\phi_c)/\partial m^2$ correctly, we can calculate the effective action accurately.**)

2.1. *Derivative of the effective action with respect to the mass squared*

In this subsection we calculate the derivative $\frac{\partial\mathit{\Gamma}[\phi_c]}{\partial m^2}$.***) This is formally given by [28), 29)]

$$\frac{\partial\mathit{\Gamma}[\phi_c]}{\partial m^2} = -\frac{1}{2}\int d^4x\langle\phi(x)^2\rangle$$

*) Those for the other theories can be derived similarly.

**) Hereafter, we omit the argument m^2.

***) See the Appendix for details.

$$= -\frac{1}{2}\int d^4x d^4y \langle\phi(x)\phi(y)\rangle\delta(y-x)$$
$$= -\frac{1}{2}\int d^4x \phi_c(x)^2 - \frac{1}{2}\int d^4x d^4y \left(\frac{\delta^2(-\Gamma[\phi_c])}{\delta\phi_c(x)\delta\phi_c(y)}\right)^{-1}\delta(y-x). \qquad (2\cdot4)$$

Here $\int d^4x$ is as an abbreviated expression $\int_0^\beta d\tau \int_{-\infty}^{\infty} d^3x$. We use this notation hereafter.

Since Eq. (2·4) is a functional equation, it cannot be solved directly. We, therefore, limit the functional space and expand $\Gamma[\phi_c]$ in powers of derivatives [33), 34), *)] as

$$\Gamma[\phi_c] = \int d^4x \left[-V(\phi_c^2) - \frac{1}{2}K_0(\phi_c^2)(\partial_0\phi_c)^2 - \frac{1}{2}K_s(\phi_c^2)(\nabla\phi_c)^2 + \cdots\right], \qquad (2\cdot5)$$

where the dots represent terms with higher derivatives. Note that the coefficient functional of $(\partial_0\phi_c)^2$ differs from that of $(\nabla\phi_c)^2$, due to the absence of 4-dimensional Euclidean symmetry. We then expand both sides of Eq. (2·4) with respect to derivatives, as Eq. (2·5), and match the coefficient functionals of all terms. In practice, we have to truncate the series in Eq. (2·5). In the present paper, we leave the three terms in Eq. (2·5). This is the next-to-leading order approximation of the derivative expansion. We obtain the leading order approximation, which corresponds to the evolution equation of Refs. 28) and 29), by setting $K_0 = 1$ and $K_s = 1$.

From Eq. (2·5), the l.h.s of Eq. (2·4) becomes

$$-\frac{\partial\Gamma}{\partial m^2} = \int d^4x \left[\frac{\partial V}{\partial m^2} + \frac{1}{2}\frac{\partial K_0}{\partial m^2}(\partial_0\phi_c)^2 + \frac{1}{2}\frac{\partial K_s}{\partial m^2}(\nabla\phi_c)^2\right]. \qquad (2\cdot6)$$

From Eq. (2·5), up to second derivative terms, [**)] we have

$$\begin{aligned}
M_{yx} &\equiv -\frac{\delta^2\Gamma[\phi_c]}{\delta\phi_c(x)\delta\phi_c(y)} \\
&= \delta(y-x)\Big[V''(\phi_c(y)) \\
&\quad -\Big\{\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2 + K_0'(\phi_c(y))\partial_0\phi_c(y)\partial_{y0} \\
&\quad +K_0'(\phi_c(y))\partial_0^2\phi_c(y) + K_0(\phi_c(y))\partial_{y0}^2\Big\} \\
&\quad -\{K_0 \leftrightarrow K_s, \partial_{y0} \leftrightarrow \nabla_y\}\Big], \qquad (2\cdot7) \\
&= \delta(y-x)(-\overline{K}_0\partial_{y0}^2 - \overline{K}_s\nabla_y^2 + \overline{V}'') \\
&\quad +\delta(y-x)\Big[\widetilde{V}''(\phi_c(y))
\end{aligned}$$

*) There is a problem with a similar derivation in Refs. 33) and 34): There, $\exp iqx$ is treated not as a distribution but as an ordinary function.

**) Hereafter, we also use notation like $\overline{V} = V(\bar{\phi})$ for values of functions in a constant configuration, $\phi_c(x) = \bar{\phi}$.

$$
\begin{aligned}
&-\Big\{\frac{1}{2}K_0^{''}(\phi_c(y))(\partial_0\phi_c(y))^2+K_0^{'}(\phi_c(y))\partial_0\phi_c(y)\partial_{y_0}\\
&+K_0^{'}(\phi_c(y))\partial_0^2\phi_c(y)+\widetilde{K}_0(\phi_c(y))\partial_{y_0}^2\Big\}\\
&-\{K_0\leftrightarrow K_s,\partial_{y_0}\leftrightarrow\nabla_y\}\Big], \qquad (2\cdot 8)
\end{aligned}
$$

$$
\begin{aligned}
&= A_{yx}-B_{yx}, \qquad (2\cdot 9)\\
A_{yx}&\equiv\delta(y-x)(-\overline{K}_0\partial_{y_0}^2-\overline{K}_s\nabla_y^2+\overline{V}''),\\
B_{yx}&\equiv A_{yx}-M_{yx},\\
\widetilde{V}''(\phi_c(y))&\equiv V''(\phi_c(y))-\overline{V}'',\\
\widetilde{K_0}(\phi_c(y))&\equiv K_0(\phi_c(y))-\overline{K}_0.
\end{aligned}
$$

Here, we divide M_{yx} into two parts, A_{yx}, which remains finite, and B_{yx}, which vanishes at $\phi_c(x)=\bar{\phi}$. Details of the following calculation are explained in the Appendix. The right-hand side of Eq. (2·4) can now be expanded around $\phi_c(x)=\bar{\phi}$ up to second derivative terms as

$$
\begin{aligned}
&\int d^4x d^4y\left(\frac{\delta^2(-\Gamma[\phi_c])}{\delta\phi_c(x)\delta\phi_c(y)}\right)^{-1}\delta(y-x)\\
&=\int d^4x(A^{-1})_{xx}\\
&\quad+\int d^4xd^4yd^4z(A^{-1})_{xy}B_{yz}(A^{-1})_{zx}\\
&\quad+\int d^4xd^4yd^4zd^4ud^4v(A^{-1})_{xy}B_{yz}(A^{-1})_{zu}B_{uv}(A^{-1})_{vx}\\
&\quad+\cdots\\
&=\int_{px}\frac{1}{\nu}\\
&\quad+\int_{px}\frac{1}{\nu^2}\left[-\frac{1}{2}\left\{\overline{K}_0''(\partial_0\phi_c)^2+\overline{K}_s''(\nabla\phi_c)^2\right\}\right]\\
&\quad+\int_{px}\frac{1}{\nu^3}\Big[\left\{\overline{K}_0'p_0(\partial_0\phi_c)+\overline{K}_s'p_i(\partial_i\phi_c)\right\}^2\\
&\qquad\qquad+2\left\{\overline{K}_0'(\partial_0\phi_c)^2+\overline{K}_s'(\nabla\phi_c)^2\right\}\nu'\Big]\\
&\quad+\int_{px}\frac{1}{\nu^4}\Big[-2\left\{\overline{K}_0'p_0\partial_0\phi_c+\overline{K}_s'p_i\partial_i\phi_c\right\}\left\{\overline{K}_0p_0\partial_0\phi_c+\overline{K}_sp_i\partial_i\phi_c\right\}\nu'\\
&\qquad-\frac{1}{2}\{\nu'\}^2\left\{\overline{K}_0(\partial_0\phi_c)^2+\overline{K}_s(\nabla\phi_c)^2\right\}\Big]\\
&\quad+\cdots+(\text{terms which vanish at } \phi_c(x)=\bar{\phi} \text{ like } \widetilde{V}\times(\text{something})), \qquad (2\cdot 10)
\end{aligned}
$$

where $\nu=\overline{K}_0p_0^2+\overline{K}_s\boldsymbol{p}^2+\overline{V}''$, $\nu'=\overline{K}_0'p_0^2+\overline{K}_s'\boldsymbol{p}^2+\overline{V}'''$ [with $p_0=2\pi nT(n=0,\pm1,\cdots)$ and $\boldsymbol{p}^2\equiv\sum_{i=1}^3p_i^2$], and $\int_{px}$ is used as an abbreviation of $T\sum_{n=-\infty}^{\infty}\int d^3p/(2\pi)^3\int d^4x$.

We match both sides of Eq. (2·4) using Eqs. (2·6) and (2·10) and equate the coefficient functionals of $(\partial\phi_c)$. After the matching, we set $\phi_c(x) = \bar{\phi}$ and obtain the following simultaneous partial differential equation:

$$\frac{\partial \overline{V}}{\partial m^2} = \frac{1}{2}\bar{\phi}^2 + \frac{1}{2}\int_p \frac{1}{\nu}, \tag{2·11}$$

$$\begin{aligned}\frac{\partial \overline{K}_0}{\partial m^2} &= -\frac{1}{2}\overline{K}_0'' \int_p \frac{1}{\nu^2} \\ &\quad + \overline{K}_0^2 \int_p \frac{p_0^2}{\nu^3} + 2\overline{K}_0' \int_p \frac{\nu'}{\nu^3} \\ &\quad - 2\overline{K}_0'\overline{K}_0 \int_p p_0^2 \frac{\nu'}{\nu^4} - \frac{1}{2}\overline{K}_0 \int_p \frac{\{\nu'\}^2}{\nu^4},\end{aligned} \tag{2·12}$$

$$\begin{aligned}\frac{\partial \overline{K}_s}{\partial m^2} &= -\frac{1}{2}\overline{K}_s'' \int_p \frac{1}{\nu^2} \\ &\quad + \frac{\overline{K}_s^2}{3} \int_p \frac{\boldsymbol{p}^2}{\nu^3} + 2\overline{K}_s' \int_p \frac{\nu'}{\nu^3} \\ &\quad - \frac{2}{3}\overline{K}_s'\overline{K}_s \int_p \boldsymbol{p}^2 \frac{\nu'}{\nu^4} - \frac{1}{2}\overline{K}_s \int_p \frac{\{\nu'\}^2}{\nu^4}.\end{aligned} \tag{2·13}$$

Here, $\int_p$ is used as an abbreviated expression of $T\sum_{n=-\infty}^{\infty}\int d^3p/(2\pi)^3$. Since we set $\phi_c(x) = \bar{\phi}$ finally, there is no contribution from the terms in Eq. (2·10) that vanish at $\phi_c(x) = \bar{\phi}$. This is the evolution equation of the next-to-leading order approximation of the derivative expansion.

2.2. *Initial condition*

We can calculate the initial condition $\Gamma[\bar{\phi}; M^2)$ using the perturbation theory to the one-loop level, thanks to the large auxiliary mass, $M \sim T$. The effective potential, $V(\bar{\phi}; M^2)$, is calculated within the one-loop approximation as

$$V(\bar{\phi}; M^2) = \frac{1}{2}M^2\bar{\phi} + \frac{\lambda}{24}\bar{\phi}^4 + \frac{1}{2}\int_p \log\left(p_0^2 + \boldsymbol{p}^2 + M^2 + \frac{\lambda}{2}\bar{\phi}^2\right). \tag{2·14}$$

The loop correction to $K_0(\bar{\phi}; M^2)$ and $K_s(\bar{\phi}; M^2)$ comes from the self-energy graph depicted in Fig. 1 at one-loop level,

$$\Pi(q_0^2, \boldsymbol{q}^2) = \frac{\lambda^2\bar{\phi}^2}{2}\int_p \frac{1}{p_0^2 + \boldsymbol{p}^2 + M^2 + \frac{\lambda\bar{\phi}^2}{2}} \frac{1}{(q_0 - p_0)^2 + (\boldsymbol{q} - p)^2 + M^2 + \frac{\lambda\bar{\phi}^2}{2}}. \tag{2·15}$$

The initial conditions of $K_0(\bar{\phi}; M^2)$ and $K_s(\bar{\phi}; M^2)$ are given by the coefficients of $-q_0^2$ and $-\boldsymbol{q}^2$, respectively. We, therefore, obtain[*)]

$$K_0(\bar{\phi}; M^2) = 1 - \left.\frac{d\Pi(q_0^2, \boldsymbol{q}^2)}{dq_0^2}\right|_{q_i=0, q_0=0}$$

[*)] Strictly speaking, we first set $q_i = 0$ and then set $q_0 = 0$ in taking the limits $q_i \to 0$ and $q_0 \to 0$. For the function K_0 to be defined, this order is essential, while for K_s the order of the limits is irrelevant (see Ref. 35)).

$$= 1 - \frac{\lambda^2\bar{\phi}^2}{2}\int_p \left[\frac{1}{(p_0^2+\boldsymbol{p}^2+M^2+\frac{\lambda\bar{\phi}^2}{2})^3} - \frac{M^2+\frac{\lambda\bar{\phi}^2}{2}}{(p_0^2+\boldsymbol{p}^2+M^2+\frac{\lambda\bar{\phi}^2}{2})^4}\right], \tag{2·16}$$

$$K_s(\bar{\phi};M^2) = 1 - \left.\frac{d\Pi(q_0^2,\boldsymbol{q}^2)}{d\boldsymbol{q}^2}\right|_{q_0=0,q_i=0}$$
$$= 1 + \frac{\lambda\bar{\phi}^2}{6}\int_p \frac{1}{(p_0^2+\boldsymbol{p}^2+M^2+\frac{\lambda^2\bar{\phi}^2}{2})^3}. \tag{2·17}$$

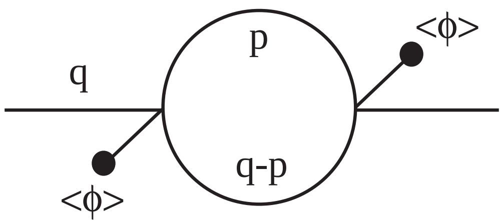

Fig. 1. Diagram of the external-momentum dependent self-energy at one loop.

We have calculated the evolution equation in §2.1 and the initial condition in §2.2. Since some of them, e.g. the one-loop effective potential, have an ultraviolet divergence, we have to renormalize it using the counter terms. Instead of considering this contribution, we simply assume that the renormalization effect is small and discard it. This contribution is, actually, small compared with the finite temperature contribution around a critical temperature in most cases. We thus need only to deal with the temperature dependent pieces in the integrals of $\frac{\partial \Gamma}{\partial m^2}$ and the initial condition. To do so, we carry out the following replacement in Eqs. (2·11) – (2·14), (2·16) and (2·17):

$$\int_p \quad \longrightarrow \quad 2\int \frac{d^4p}{(2\pi)^4 i}\frac{1}{\exp(p_0/T)-1}.$$

We note that under the local potential approximation, in which $K_0 = K_s = 1$, the evolution equation of the effective potential reproduces to our previous equation.[28), 29)]

§3. Results and summary

We solved the simultaneous evolution equations (2·11) – (2·13) with the initial conditions (2·14), (2·16) and (2·17) numerically. In this computation, we used an extended Crank-Nicholson method, which is explained in the Appendix of Ref. 29), to solve the partial differential equation. We were able to solve this equation at most temperatures above the critical temperature.[*)] Unfortunately, however, we cannot solve the equation at temperatures very close to the critical temperature due to numerical problems. We tried several improvements of the numerical method, but these all failed. In the present paper, we give the results we were able to obtain. This

*) We define the critical temperature as the temperature at which the curvature of the effective potential at origin is zero for a second order phase transition. For a first order phase transition, we define it as the temperature at which the effective potential has two degenerate minima.

equation could be solved at arbitrary temperature if sufficiently effective numerical methods for solving partial differential equations are developed or if the computing power of computers progresses to the point that we can calculate with sufficiently high precision using present numerical method. In our computation, we use mass units such that $\mu = 1$.

We display the effective potential above the critical temperature in Fig. 2 for $\lambda = 1$. We observe behavior of a second order phase transition up to this temperature. This effective potential seems to be cone shaped. This shape indicates that

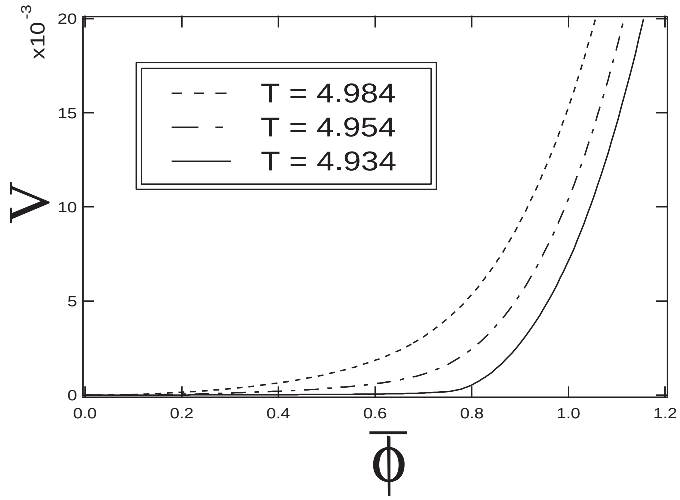

Fig. 2. Effective potential near the critical temperature ($\lambda = 1$).

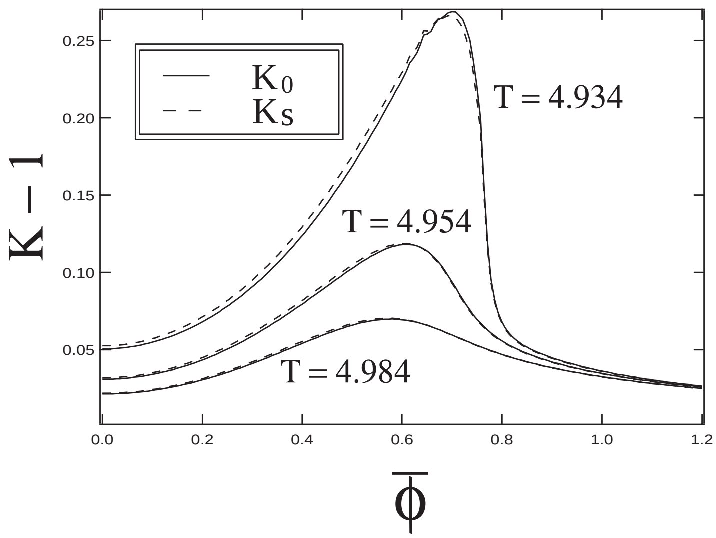

Fig. 3. The coefficient functions of the second derivative terms in the effective action, K_0 and K_s ($\lambda = 1$).

the effective potential becomes convex due to non-perturbative effects for any temperature. We, however, believe that the effective potential becomes non-convex at slightly smaller temperature. Actually, the effective potential is non-convex at zero temperature, because it is merely the tree level effective potential in our approximation. The critical temperature is estimated to be lower than that of the local potential approximation by 2%.*)

We show K_0 and K_s in Fig. 3 for $\lambda = 1$. We see that they are very similar, in spite of the violation of the Lorenz invariance in this case, while the initial condition is quite different.

In summary, we derived an evolution equation of the effective action with respect to the mass squared in the Z_2-invariant scalar theory. We then approximated the effective action as a derivative expansion. We showed that the previous evolution equation of the effective potential can be derived as the leading order approximation in this expansion, the local potential approximation. We next derived the evolution equation of the next-to-leading order approximation, which is a simultaneous partial differential equation. We finally solved this equation numerically. Though we could solve it at most temperatures above the critical temperature, we were not able to do so below a certain temperature very close to the critical temperature. In any case, we constructed a systematic method to improve the auxiliary mass method in principle.

Acknowledgements

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Appendix

In this appendix, we give the derivation of $\partial\Gamma/\partial m^2$ in detail. Part of the notation used here is given in §2. The effective action $\Gamma[\phi_c]$ for the Lagrangian Eq. (2·1) is defined as usual,

$$\begin{aligned}\Gamma[\phi_c] &\equiv W[J_m] - \int d^4x J_m(x)\phi_c(x),\\ W[J_m] &\equiv \log(Z[J_m]),\\ Z[J_m] &\equiv \int \mathcal{D}[\phi]\exp\left(\int d^4x\mathcal{L}_E\right).\end{aligned} \tag{A·1}$$

Equation (2·4) is, then, derived as

$$\begin{aligned}\frac{\partial\Gamma[\phi_c]}{\partial m^2} &= \frac{1}{Z[J_m]}\int\mathcal{D}[\phi]\left[\int\left\{d^4x\frac{1}{2}(-\phi(x)^2) + \frac{\partial J_m(x)}{\partial m^2}\phi(x)\right\}\exp\left\{\int d^4x\mathcal{L}_E\right\}\right]\\ &\quad - \int d^4x\frac{\partial J_m(x)}{\partial m^2}\phi_c(x)\end{aligned}$$

*) We estimated the critical temperature by extrapolating the curvature of the origin as a function of the temperature and determining the temperature at which the curvature becomes zero.

$$
\begin{aligned}
&= -\frac{1}{2}\int d^4x \langle \phi(x)^2\rangle \\
&= -\frac{1}{2}\int d^4x\ \phi_c(x)^2 - \frac{1}{2}\int d^4x (M)^{-1}_{xx}. \qquad (\mathrm{A{\cdot}2})
\end{aligned}
$$

Since Eq. (2·4) is a functional equation, it cannot be solved directly. We, therefore, limit functional space and expand $\Gamma[\phi_c]$ in powers of derivatives[33),34)] (see Eq. (2·5)) as

$$
\Gamma[\phi_c] = \int d^4x \left[-V(\phi_c^2) - \frac{1}{2}K_0(\phi_c^2)(\partial_0\phi_c)^2 - \frac{1}{2}K_s(\phi_c^2)(\nabla\phi_c)^2 + \cdots\right],
$$

where the dots represent terms of higher order derivatives, which are omitted here, and equating the coefficient functionals of $(\partial\phi_c)$ on both sides of Eq. (2·4).

The left-hand side of Eq. (A·2) is found to be given by Eq. (2·6) by simply differentiating Γ with respect to m^2. We have

$$
-\frac{\partial \Gamma}{\partial m^2} = \int d^4x \left[\frac{\partial V}{\partial m^2} + \frac{1}{2}\frac{\partial K_0}{\partial m^2}(\partial_0\phi_c)^2 + \frac{1}{2}\frac{\partial K_s}{\partial m^2}(\nabla\phi_c)^2\right].
$$

It is very complicated to calculate the rihgt-hand side of Eq. (A·2). First, we calculate M_{yx}. The derivative of the effective action $\Gamma[\phi_c]$ with respect to $\phi_c(x)$ is,

$$
\begin{aligned}
\frac{\delta\Gamma[\phi_c]}{\delta\phi_c(x)} = &- V^{'}(\phi_c(x)) \\
&+ \frac{1}{2}K_0^{'}(\phi_c(x))(\partial_0\phi_c(x))^2 + K_0(\phi_c(x))(\partial_0^2\phi_c(x)) \\
&+ \{K_0 \leftrightarrow K_s, \partial_0 \leftrightarrow \nabla\}. \qquad (\mathrm{A{\cdot}3})
\end{aligned}
$$

Here, we have assumed that we can carry out a partial integral freely, without a surface term.

We define the operator M_{xy} in the following manner. An operator O_{xy} defined through functional derivative with respect to $\phi(x)$, say $\delta F(\phi(y))/\delta\phi(x)$, acts on any appropriate test function, say $T(y)$, as

$$
OT(x) \equiv \int d^4y \left\{\frac{\delta}{\delta\phi(x)}F(\phi(y))\right\} T(y).
$$

In particular, if $F(\phi(y))$ contains the derivative of $\phi(y)$, say $F(\phi(y)) = G(\phi(y))\partial\phi(y)$, we have

$$
\begin{aligned}
OT(x) &\equiv \int d^4y \left\{\frac{\delta}{\delta\phi(x)}G(\phi(y))\partial\phi(y)\right\} T(y). \\
&\equiv \int d^4y\delta(y-x)\left[G^{'}(\phi(y))\partial\phi(y)T(y) - \partial\left\{G(\phi(y))T(y)\right\}\right]. \qquad (\mathrm{A{\cdot}4})
\end{aligned}
$$

In order to obtain Eq. (2·7) and to determine the inverse of M_{yx} near $\bar{\phi}$, we divide Eq. (2·7) into two pieces (Eq. (2·9)) as

$$
M_{yx} = A_{yx} - B_{yx},
$$

$$A_{yx} = \delta(y-x)(-\overline{K}_0\partial_0^2 - \overline{K}_s\nabla^2 + \overline{V}''),$$
$$\begin{aligned} B_{yx} = -\delta(y-x)\Big[&\widetilde{V}''(\phi_c(y)) \\ &-\Big\{\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2 + K_0'(\phi_c(y))\partial_0\phi_c(y)\partial_{y0} \\ &+K_0'(\phi_c(y))\partial_0^2\phi_c(y) + \widetilde{K}_0(\phi_c(y))\partial_{y0}^2\Big\} \\ &-\{K_0 \leftrightarrow K_s, \partial_{y0} \leftrightarrow \nabla_y\}\Big]. \end{aligned}$$

The inverse of M_{yx} is, then, expanded as

$$(M^{-1})_{xy} = \left\{(A^{-1})\left(\sum_{n=0}^{n=\infty}(BA^{-1})^n\right)\right\}_{xy}. \tag{A·5}$$

Here, the multiplication of the "matrices" A and B is defined as

$$(AB)_{xy} = \int d^4z A_{xz}B_{zy}.$$

The inverse of A_{xy} is easily calculated to be

$$(A^{-1})_{xy} = \int_p \frac{1}{\nu_p}\exp\{ip(x-y)\}, \tag{A·6}$$

where $\nu_p = \overline{K}_0 p_0^2 + \overline{K}_s\boldsymbol{p}^2 + \overline{V}''$.

We need terms with $(\partial\phi_c)^n$ $(n = 0, 2)$ to evaluate right-hand side of Eq. (A·2). Such terms are contained only in the first three terms of Eq. (A·5),

$$\begin{aligned} (M^{-1})_{xy} = &(A^{-1})_{xy} \\ &+(A^{-1}BA^{-1})_{xy} \\ &+(A^{-1}BA^{-1}BA^{-1})_{xy}. \end{aligned} \tag{A·7}$$

The relevant terms in Eq. (A·2) are, therefore, the following:

$$\int d^4x(M^{-1})_{xx} = \int d^4x(A^{-1})_{xx} \tag{A·8}$$
$$+\int d^4x(A^{-1}BA^{-1})_{xx} \tag{A·9}$$
$$+\int d^4x(A^{-1}BA^{-1}BA^{-1})_{xx}. \tag{A·10}$$

We next calculate the quantities in Eqs. (A·8) – (A·10) up to terms containing second derivatives of $\phi_c(x)$. From Eq. (A·6) the first term, (A·8), is easy to calculate:

$$\int d^4x(A^{-1})_{xx} = \int d^4x \int_p \frac{1}{\nu_p}. \tag{A·11}$$

The second term, Eq. (A·9), is

$$
\begin{aligned}
&\int d^4x (A^{-1}BA^{-1})_{xx} \\
&= \int d^4x \int d^4y \int d^4z \int_p \frac{1}{\nu_p} \exp\{ip(x-y)\} \\
&\quad \times \delta(y-z)\Big[-\widetilde{V}''(\phi_c(y)) + \Big\{ \frac{1}{2} K_0''(\phi_c(y))(\partial_0\phi_c(y))^2 + K_0'(\phi_c(y))\partial_0\phi_c(y)\partial_{y0} \\
&\quad + K_0'(\phi_c(y))\partial_0^2\phi_c(y) + \tilde{K}_0(\phi_c(y))\partial_{y0}^2 \Big\} + \{K_0 \leftrightarrow K_s, \partial_{y0} \leftrightarrow \nabla_y\} \Big] \\
&\quad \times \int_q \frac{1}{\nu_q} \exp\{iq(z-x)\} \\
&\quad \text{(integrating over } z\text{)} \\
&= \int d^4x \int d^4y \int_p \int_q \frac{1}{\nu_p}\frac{1}{\nu_q} \exp\{ip(x-y)\}\exp\{iq(y-x)\} \\
&\quad \times \Big[-\widetilde{V}''(\phi_c(y)) + \Big\{ \frac{1}{2} K_0''(\phi_c(y))(\partial_0\phi_c(y))^2 + K_0'(\phi_c(y))\partial_0\phi_c(y) i q_0 \\
&\quad + K_0'(\phi_c(y))\partial_0^2\phi_c(y) - \tilde{K}_0(\phi_c(y)) q_0^2 \Big\} + \{K_0 \leftrightarrow K_s, \partial_0 \leftrightarrow \nabla, q_0 \leftrightarrow \boldsymbol{q}\} \Big] \\
&\quad \text{(integrating over } x \text{ and } q \text{ and then replacing } y \text{ by } x\text{)} \\
&= \int d^4x \int_p \frac{1}{\nu_p^2} \\
&\quad \times \Big[-\widetilde{V}''(\phi_c(x)) + \Big\{ \frac{1}{2} K_0''(\phi_c(x))(\partial_0\phi(x))^2 + K_0'(\phi_c(x))\partial_0\phi(x) i p_0 \\
&\quad + K_0'(\phi_c(x))\partial_0^2\phi(x) - \tilde{K}_0(\phi_c(x)) p_0^2 \Big\} + \{K_0 \leftrightarrow K_s, \partial_0 \leftrightarrow \nabla, p_0 \leftrightarrow \boldsymbol{p}\} \Big] \\
&\quad \text{(partially integrating the fourth term)} \\
&= \int d^4x \int_p \frac{1}{\nu_p^2} \Big[-\widetilde{V}''(\phi_c(x)) - \Big\{ \frac{1}{2} K_0''(\phi_c(x))(\partial_0\phi_c(x))^2 + \tilde{K}_0(\phi_c(x)) p_0^2 \Big\} \\
&\quad - \{K_0 \leftrightarrow K_s, \partial_0 \leftrightarrow \nabla, p_0 \leftrightarrow \boldsymbol{p}\} \Big]. \\
&= \int d^4x \int_p \frac{1}{\nu_p^2} \Big[-\Big\{ \frac{1}{2} \overline{K}_0''(\partial_0\phi_c)^2 + \frac{1}{2} \overline{K}_s''(\nabla\phi_c)^2 \Big\} \Big] \\
&\quad + \text{(terms which vanish at } \phi_c(x) = \bar{\phi}\text{, like } \widetilde{V}'' \times \text{(something))}. \qquad (\text{A·12})
\end{aligned}
$$

Since we set $\phi_c(x) = \bar{\phi}$, after the matching of Eq. (2·4), we have no contribution to the evolution equation from the terms which vanish at $\phi_c(x) = \bar{\phi}$. We thus keep only terms which remain finite at $\phi_c(x) = \bar{\phi}$ from this point. The third term, (A·10), is very complicated to evaluate. We find

$$
\begin{aligned}
&\int d^4x (A^{-1}BA^{-1}BA^{-1})_{xx} \\
&= \int d^4x \int d^4y \int d^4z \int d^4u \int d^4v \int_p \frac{1}{\nu_p} \exp\{ip(x-y)\}
\end{aligned}
$$

$$\times\delta(y-z)\Big[-\widetilde{V}''(\phi_c(y))+\Big\{\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2+K_0'(\phi_c(y))\partial_0\phi_c(y)\partial_{y_0}$$
$$+K_0'(\phi_c(y))\partial_0^2\phi_c(y)+\tilde{K}_0(\phi_c(y))\partial_{y_0}^2\Big\}+\{K_0\leftrightarrow K_s,\partial_{y_0}\leftrightarrow\nabla_y\}\Big]$$
$$\times\int_q\frac{1}{\nu_q}\exp\{iq(z-u)\}$$
$$\times\delta(u-v)\Big[-\widetilde{V}''(\phi_c(u))+\Big\{\frac{1}{2}K_0''(\phi_c(u))(\partial_0\phi_c(u))^2+K_0'(\phi_c(u))\partial_0\phi_c(u)\partial_{u_0}$$
$$+K_0'(\phi_c(u))\partial_0^2\phi_c(u)+\tilde{K}_0(\phi_c(u))\partial_{u_0}^2\Big\}+\{K_0\leftrightarrow K_s,\partial_{u_0}\leftrightarrow\nabla_u\}\Big]$$
$$\times\int_r\frac{1}{\nu_r}\exp\{ir(v-x)\}$$

(integrating over x,z,v,r, and replacing u with x)

$$=\int d^4x\int d^4y\int_p\int_q\frac{1}{\nu_p^2}\frac{1}{\nu_q}\exp\{i(p-q)(x-y)\}$$
$$\times\Big[-\widetilde{V}''(\phi_c(x))+\Big\{\frac{1}{2}K_0''(\phi_c(x))(\partial_0\phi_c(x))^2+K_0'(\phi_c(x))\partial_0\phi_c(x)ip_0$$
$$+K_0'(\phi_c(x))\partial_0^2\phi_c(x)-\tilde{K}_0(\phi_c(x))p_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p}\}\Big]$$
$$\times\Big[-\widetilde{V}''(\phi_c(y))+\Big\{\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2+K_0'(\phi_c(y))\partial_0\phi_c(y)iq_0$$
$$+K_0'(\phi_c(y))\partial_0^2\phi_c(y)-\tilde{K}_0(\phi_c(y))q_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,q_0\leftrightarrow\boldsymbol{q}\}\Big].$$

(partially integrating the term with $\partial_0^2\phi$)

$$=\int d^4x\int d^4y\int_p\int_q\frac{1}{\nu_p^2}\frac{1}{\nu_q}\exp\{i(p-q)(x-y)\}$$
$$\times\Big[-\widetilde{V}''(\phi_c(x))+\Big\{-\frac{1}{2}K_0''(\phi_c(x))(\partial_0\phi_c(x))^2+K_0'(\phi_c(x))\partial_0\phi_c(x)iq_0$$
$$-\tilde{K}_0(\phi_c(x))p_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]$$
$$\times\Big[-\widetilde{V}''(\phi_c(y))+\Big\{-\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2+K_0'(\phi_c(y))\partial_0\phi_c(y)ip_0$$
$$-\tilde{K}_0(\phi_c(y))q_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big].$$

By the variable exchange

$$\begin{cases} p & \longrightarrow & p+q, \\ q & \longrightarrow & p, \end{cases}$$

we obtain

$$=\int d^4x\int d^4y\int_p\int_q\frac{1}{\nu_{p+q}^2}\frac{1}{\nu_p}\exp\{iq(x-y)\}$$

$$
\begin{aligned}
&\times\Big[-\widetilde{V}''(\phi_c(x))+\Big\{-\frac{1}{2}K_0''(\phi_c(x))(\partial_0\phi_c(x))^2+K_0'(\phi_c(x))\partial_0\phi_c(x)ip_0\\
&-\tilde{K}_0(\phi_c(x))(p+q)_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]\\
&\times\Big[-\widetilde{V}''(\phi_c(y))+\Big\{-\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2+K_0'(\phi_c(y))\partial_0\phi_c(y)i(p+q)_0\\
&-\tilde{K}_0(\phi_c(y))p_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]\\
=&\int d^4x\int d^4y\int_p\int_q\frac{\exp\{iq(x-y)\}}{\nu_p^3}\\
&\times\left\{1-\frac{\left\{\overline{K}_0(4p_0q_0+2q_0^2)+\overline{K}_s(4\boldsymbol{p}\boldsymbol{q}+2\boldsymbol{q}^2\right\}}{\nu_p}+\frac{12\left\{\overline{K}_0p_0q_0+\overline{K}_s\boldsymbol{p}\boldsymbol{q}\right\}^2}{\nu_p^2}+\cdots\right\}\\
&\times\Big[-\widetilde{V}''(\phi_c(x))+\Big\{-\frac{1}{2}K_0''(\phi_c(x))(\partial_0\phi_c(x))^2+K_0'(\phi_c(x))\partial_0\phi_c(x)ip_0\\
&-\tilde{K}_0(\phi_c(x))(p+q)_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]\\
&\times\Big[-\widetilde{V}''(\phi_c(y))+\Big\{-\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2+K_0'(\phi_c(y))\partial_0\phi_c(y)i(p+q)_0\\
&-\tilde{K}_0(\phi_c(y))p_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big],
\end{aligned}
\tag{A·13}
$$

where we keep the terms up to those proportional to q^2, since we will replace q with a derivative, and we need terms up to those with second derivatives. Similarly, by the exchange of the variables

$$
\left\{\begin{array}{ccc} p & \longrightarrow & p, \\ q & \longrightarrow & p+q, \end{array}\right.
$$

we have

$$
\begin{aligned}
=&\int d^4x\int d^4y\int_p\int_q\frac{1}{\nu_{p+q}}\frac{1}{\nu_p^2}\exp\{iq(x-y)\}\\
&\times\Big[-\widetilde{V}''(\phi_c(x))+\Big\{-\frac{1}{2}K_0''(\phi_c(x))(\partial_0\phi_c(x))^2+K_0'(\phi_c(x))\partial_0\phi_c(x)ip_0\\
&-\tilde{K}_0(\phi_c(x))(p+q)_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]\\
&\times\Big[-\widetilde{V}''(\phi_c(y))+\Big\{-\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2+K_0'(\phi_c(y))\partial_0\phi_c(y)i(p+q)_0\\
&-\tilde{K}_0(\phi_c(y))p_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]\\
=&\int d^4x\int d^4y\int_p\int_q\frac{\exp\{iq(x-y)\}}{\nu_p^3}
\end{aligned}
$$

$$\times\left\{1-\frac{\left\{\overline{K}_0(2p_0q_0+q_0^2)+\overline{K}_s(2\boldsymbol{p}q+\boldsymbol{q}^2)\right\}}{\nu_p}+\frac{4\left\{\overline{K}_0p_0q_0+\overline{K}_s\boldsymbol{p}q\right\}^2}{\nu_p^2}+\cdots\right\}$$
$$\times\Big[-\widetilde{V}''(\phi_c(x))+\Big\{-\frac{1}{2}K_0''(\phi_c(x))(\partial_0\phi_c(x))^2+K_0'(\phi_c(x))\partial_0\phi_c(x)ip_0$$
$$-\tilde{K}_0(\phi_c(x))(p+q)_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]$$
$$\times\Big[-\widetilde{V}''(\phi_c(y))+\Big\{-\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2+K_0'(\phi_c(y))\partial_0\phi_c(y)i(p+q)_0$$
$$-\tilde{K}_0(\phi_c(y))p_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]. \qquad (\text{A·14})$$

By comparing Eq. (A·13) with Eq. (A·14), we have the following "identity":

$$0=\int d^4x\int d^4y\int_p\int_q\frac{\exp\{iq(x-y)\}}{\nu_p^3}$$
$$\times\left\{-\frac{\left\{\overline{K}_0(2p_0q_0+q_0^2)+\overline{K}_s(2\boldsymbol{p}q+\boldsymbol{q}^2)\right\}}{\nu_p}+\frac{8\left\{\overline{K}_0p_0q_0+\overline{K}_s\boldsymbol{p}q\right\}^2}{\nu_p^2}+\cdots\right\}$$
$$\times\Big[-\widetilde{V}''(\phi_c(x))+\Big\{-\frac{1}{2}K_0''(\phi_c(x))(\partial_0\phi_c(x))^2+K_0'(\phi_c(x))\partial_0\phi_c(x)ip_0$$
$$-\tilde{K}_0(\phi_c(x))(p+q)_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]$$
$$\times\Big[-\widetilde{V}''(\phi_c(y))+\Big\{-\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2+K_0'(\phi_c(y))\partial_0\phi_c(y)i(p+q)_0$$
$$\tilde{K}_0(\phi_c(y))p_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]. \qquad (\text{A·15})$$

Indeed, this identity holds exactly for the coefficient functions of $(\nabla\phi_c)^2$, although it does not for those of $(\partial_0\phi_c)^2$. In any case, we use this identity and rewrite (or define) the third term, (A·10), as follows:

$$\int d^4x(A^{-1}BA^{-1}BA^{-1})_{xx}$$
$$=\int d^4x\int d^4y\int_p\int_q\frac{\exp\{iq(x-y)\}}{\nu_p^3}$$
$$\times\left\{1-\frac{1}{2}\frac{\left\{\overline{K}_0(2p_0q_0+q_0^2)+\overline{K}_s(2\boldsymbol{p}q+\boldsymbol{q}^2)\right\}}{\nu_p}+\cdots\right\}$$
$$\times\Big[-\widetilde{V}''(\phi_c(x))+\Big\{-\frac{1}{2}K_0''(\phi_c(x))(\partial_0\phi_c(x))^2+K_0'(\phi_c(x))\partial_0\phi_c(x)ip_0$$
$$-\tilde{K}_0(\phi_c(x))(p+q)_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]$$

$$\times\Big[-\tilde{V}''(\phi_c(y))+\Big\{-\frac{1}{2}K_0''(\phi_c(y))(\partial_0\phi_c(y))^2+K_0'(\phi_c(y))\partial_0\phi_c(y)i(p+q)_0$$
$$-\tilde{K}_0(\phi_c(y))p_0^2\Big\}+\{K_0\leftrightarrow K_s,\partial_0\leftrightarrow\nabla,p_0\leftrightarrow\boldsymbol{p},q_0\leftrightarrow\boldsymbol{q}\}\Big]. \qquad (\text{A}\cdot 16)$$

Note that there is no contribution from Eq. (A·16) to the evolution equation for the effective potential V, since the constant terms always include $\tilde{K}$ and $\tilde{V}''$, which are zero at $\phi_c(x)=\bar{\phi}$ by definition.

We now evaluate the coefficient functions of $(\partial\phi_c(x))^2$. First we calculate the first term of Eq. (A·16). There are three contributions from this term,

$$\int d^4x\int d^4y\int_p\int_q\frac{\exp\{iq(x-y)\}}{\nu_p^3}$$
$$\times\Big\{K_0'(\phi_c(x))\partial_0\phi_c(x)ip_0+K_s'(\phi_c(x))\nabla\phi_c(x)i\boldsymbol{p}\Big\}$$
$$\times\Big\{K_0'(\phi_c(y))\partial_0\phi_c(y)ip_0+K_s'(\phi_c(y))\nabla\phi_c(y)i\boldsymbol{p}\Big\}$$
$$\text{(integrating over } q \text{ and } y)$$
$$=\int d^4x\int_p\frac{-1}{\nu_p^3}\Big\{\overline{K}_0'(\partial_0\phi_c)p_0+\overline{K}_s'(\nabla\phi_c)\boldsymbol{p}\Big\}^2, \qquad (\text{A}\cdot 17)$$

$$\int d^4x\int d^4y\int_p\int_q\frac{\exp\{iq(x-y)\}}{\nu_p^3}$$
$$\times\Big[\Big\{K_0'(\phi_c(x))\partial_0\phi_c(x)iq_0+K_s'(\phi_c(x))\nabla\phi_c(x)i\boldsymbol{q}\Big\}$$
$$\times\Big\{-\tilde{K}_0(\phi_c(y))p_0^2-\tilde{K}_s(\phi_c(y))\boldsymbol{p}^2-V''(\phi_c(y))\Big\}$$
$$+2\Big\{K_0'(\phi_c(x))\partial_0\phi_c(x)ip_0+K_s'(\phi_c(x))\nabla\phi_c(x)i\boldsymbol{p}\Big\}$$
$$\times\Big\{-\tilde{K}_0(\phi_c(y))p_0q_0-\tilde{K}_s(\phi_c(y))\boldsymbol{p}\boldsymbol{q}\Big\}\Big]$$
$$=\int d^4x\int d^4y\int_p\int_q\frac{1}{\nu_p^3}$$
$$\times\Big[\Big\{K_0'(\phi_c(x))\partial_0\phi_c(x)\partial_{x_0}+K_s'(\phi_c(x))\nabla\phi_c(x)\nabla_x\Big\}$$
$$\times\Big\{-\tilde{K}_0(\phi_c(y))p_0^2-\tilde{K}_s(\phi_c(y))\boldsymbol{p}^2-V''(\phi_c(y)\Big\}$$
$$+2\Big\{K_0'(\phi_c(x))\partial_0\phi_c(x)ip_0+K_s'(\phi_c(x))\nabla\phi_c(x)i\boldsymbol{p}\Big\}$$
$$\times\Big\{\tilde{K}_0(\phi_c(y))ip_0\partial_{x_0}+\tilde{K}_s(\phi_c(y))i\boldsymbol{p}\nabla_x\Big\}\Big]$$
$$\times\exp\{iq(x-y)\}$$
$$\text{(partially integrating over } x\text{, and then integrating over } q \text{ and } y)$$
$$=\int d^4x\int_p\frac{1}{\nu_p^3}\Big[\Big\{\overline{K}_0'(\partial_0\phi_c)^2+\overline{K}_s'(\nabla\phi_c)^2\Big\}\Big\{\overline{K}_0'p_0^2+\overline{K}_s'\boldsymbol{p}^2+\overline{V}'''\Big\}$$
$$+2\Big\{\overline{K}_0'(\partial_0\phi_c)p_0+\overline{K}_s'(\nabla\phi_c)\boldsymbol{p}\Big\}^2\Big], \qquad (\text{A}\cdot 18)$$

and

$$\int d^4x \int d^4y \int_p \int_q \frac{\exp\{iq(x-y)\}}{\nu_p^3}$$
$$\times \left[\left\{-\tilde{K}_0(\phi_c(x))q_0^2 - \tilde{K}_s(\phi_c(x))\boldsymbol{q}^2\right\}\right.$$
$$\left.\times \left\{-\tilde{K}_0(\phi_c(y))p_0^2 - \tilde{K}_s(\phi_c(y))\boldsymbol{p}^2 - V''(\phi_c(y))\right\}\right]$$
$$= \int d^4x \int_p \frac{1}{\nu_p^3} \left\{\overline{K}_0'(\partial_0\phi_c)^2 + \overline{K}_s'(\nabla\phi_c)^2\right\} \left\{\overline{K}_0' p_0^2 + \overline{K}_s'\boldsymbol{p}^2 + \overline{V}'''\right\}. \quad (\text{A}\cdot 19)$$

In total, we have the following contribution from the first term of Eq. (A·16)

$$\int d^4x \int_p \frac{1}{\nu_p^3} \left[2\left\{\overline{K}_0'(\partial_0\phi_c)^2 + \overline{K}_s'(\nabla\phi_c)^2\right\}\nu_p'\right.$$
$$\left. + \left\{\overline{K}_0'\partial_0\phi_c p_0 + \overline{K}_s'\nabla\phi_c\boldsymbol{p}\right\}^2\right]. \quad (\text{A}\cdot 20)$$

Similarly, we have the following contribution from the second term of Eq. (A·16):

$$\int d^4x \int_p \frac{1}{\nu_p^4} \left[-\frac{1}{2}\left\{\overline{K}_0' p_0^2 + \overline{K}_s'\boldsymbol{p}^2 + \overline{V}'''\right\}^2 + \left\{\overline{K}_0(\partial_0\phi_c)^2 + \overline{K}_s(\nabla\phi_c)^2\right\}\right.$$
$$\left. -2\left\{\overline{K}_0'(\partial_0\phi_c)p_0 + \overline{K}_s'(\nabla\phi_c)\boldsymbol{p}\right\}\left\{\overline{K}_0(\partial_0\phi_c)p_0 + \overline{K}_s(\nabla\phi_c)\boldsymbol{p}\right\}\nu_p'\right]. \quad (\text{A}\cdot 21)$$

Finally, combining Eqs. (A·11), (A·12), (A·20) and (A·21), we obtain the evolution equation, Eq. (2·10).

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