

Critical exponents and critical amplitude ratio of the scalar model from finite-temperature field theory

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The critical exponents and the critical amplitude ratio of the scalar model are determined using finite-temperature field theory with an auxiliary mass. A new numerical method is developed to solve an evolution equation. The results are discussed in comparison with values obtained from the other methods. [S0556-2821(98)05012-7]

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I. INTRODUCTION

The phase transition is an important phenomenon in particle physics, cosmology, and condensed matter physics. The quark gluon plasma should be present in heavy ion collisions and will give us a lot of valuable information on particle physics [1]. Investigation into the chiral phase transition suggests that a number of flavors may be bounded from above [2,3]. In cosmology, the electroweak phase transition should be first order for electroweak baryogenesis [4,5] and has been investigated attentively [6–14]. Needless to say, a variety of phase transitions has been observed and investigated precisely in condensed matter physics.

The field theoretical approach is essential in order to investigate these phase transitions: finite-temperature–chemical-potential field theory [1,15], perturbative and non-perturbative renormalization group [16–22], field theory on lattices [23], and so on. Temperature can be naturally introduced by statistical principles using finite-temperature field theory. Not all the phase transitions, however, can be investigated by it; perturbation theory, which is the most powerful method at zero temperature, often breaks down around the critical temperature because of many interactions in the thermal bath [24,25]. Indeed, perturbation theory fails when it is applied to either a second-order or a weakly first-order phase transition.

Drummond *et al.* [26] proposed a new method using an auxiliary mass in order to avoid this difficulty. We utilized their idea and developed a new method to calculate the effective potential. We then investigated the phase transition of the scalar model using the *auxiliary-mass method* and showed it is second order correctly [27]. It is a great advance in finite-temperature field theory, because the phase transition in the scalar model is indicated to be first order incorrectly by perturbation theory with a daisy resummation [7,28]. We note that the method was able to reproduce this result with a superdaisy approximation [29].

Since the equation we must solve in the auxiliary-mass method is a nonlinear partial differential equation for the effective potential, it cannot be solved analytically and must be solved by a numerical method. It is, however, difficult to solve partial differential equations numerically because of numerical instability [30]. What is worse, the nonlinearity of the equation prevents us from using methods established in the case of a linear equation. We could not, therefore, make the mesh size arbitrary small; the investigation in Ref. [27] was not accurate quantitatively. In the present paper, we use an improved numerical method given in the Appendix, which does not suffer from instability and obtains accurate universal quantities. Unlike the rough values in [27], they are beyond the values obtained from the Landau approximation.

The present paper is organized as follows. In the next section we review the auxiliary-mass method developed in [27]. In Sec. III the effective potential is shown as the temperature varies. We then focus on the behavior of it around the critical temperature, and calculate the universal quantities. These values are compared with values obtained from other methods. The summary and discussion are presented in Sec. IV. In the Appendix we explain the numerical method we used.

II. REVIEW OF THE AUXILIARY-MASS METHOD

We review the method to calculate an effective potential at the temperature where perturbation theory is not reliable [27]. We consider $\lambda\phi^4$ theory which is defined by the Lagrangian density

$$\mathcal{L}_E = -\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + J\phi + \text{c.t.}, \quad (1)$$

where J is an external source function. If m^2 is negative, the scalar field ϕ develops the nonvanishing field expectation value at $T=0$. First, the effective potential is calculated with a positive mass squared M^2 which is as large as the temperature T^2 . This selection of the mass permits us to use perturbation theory without failure, because the loop expansion

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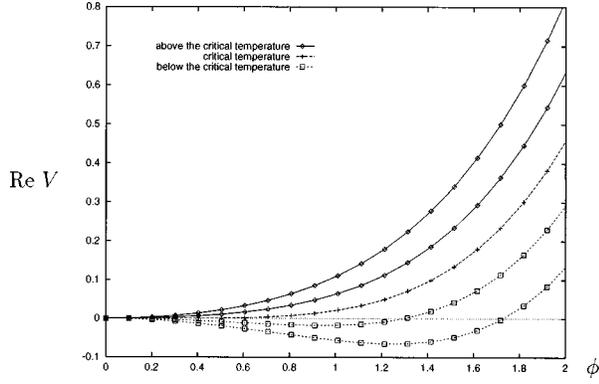


FIG. 1. Real part of the effective potential ($\lambda=1$). The values of the origin are set to zero. A stable point comes to be zero smoothly as the temperature increases.

parameter there is $\lambda T/M \sim \lambda$ [7,31,32], which is small when the coupling constant λ is small. Using perturbation theory, the effective potential is calculated as follows:

$$V = \frac{1}{2} M^2 \bar{\phi}^2 + \frac{\lambda}{4!} \bar{\phi}^4 + \frac{T}{2\pi^2} \times \int_0^\infty dr r^2 \log \left[1 - \exp \left(-\frac{1}{T} \sqrt{r^2 + M^2 + \frac{\lambda}{2} \bar{\phi}^2} \right) \right]. \quad (2)$$

Here, only the one-loop thermal correction is left and the quantum correction is neglected, because it should be negligible when the coupling constant λ is sufficiently small.

We then extrapolate the effective potential (2) to the negative mass squared¹ $m^2 = -\mu^2$ using the following evolution equation:

$$\frac{\partial V}{\partial m^2} = \frac{1}{2} \bar{\phi}^2 + \frac{1}{2\pi i} \int_{-i\infty+\epsilon}^{+i\infty+\epsilon} dp_0 \times \int \frac{d^3 p}{(2\pi)^3} \frac{1}{-p_0^2 + \mathbf{p}^2 + m^2 + (\lambda/2) \bar{\phi}^2 + \Pi} \frac{1}{e^{\beta p_0} - 1}, \quad (3)$$

where $\bar{\phi}$ is the expectation value of the field and $\Pi = \Pi(\mathbf{p}^2, -p_0^2, \bar{\phi}, m^2, \tau)$ is the full self-energy. The thermal correction is left and the quantum correction is neglected here, too. Of course, Π cannot be calculated exactly; we need an appropriate approximation in order to calculate the effective potential from Eq. (2). Because the effective potential is a generating function of n -point functions with zero external momentum, neglect of the momentum dependence in Π allows us to make the following replacement:

$$m^2 + \frac{\lambda}{2} \bar{\phi}^2 + \Pi(0,0, \bar{\phi}, m^2, \tau) \rightarrow \frac{\partial^2 V}{\partial \bar{\phi}^2}. \quad (4)$$

¹Hereafter we use the unit $\mu=1$. All dimensionful quantities are measured in this unit.

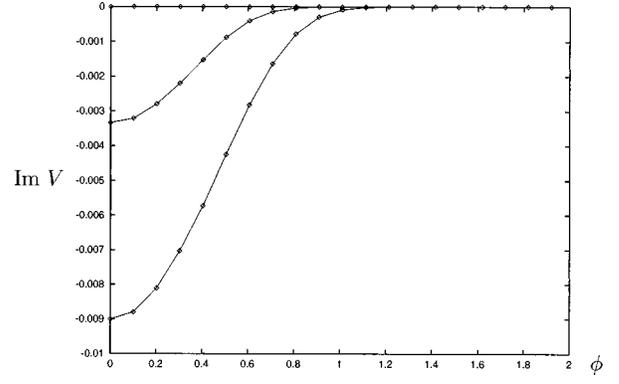


FIG. 2. Imaginary part of the effective potential ($\lambda=1$). The magnitude, which shows the instability of the state, increases as the field expectation value decreases.

The evolution equation (3) can be converted to a partial differential equation using this replacement as follows:

$$\frac{\partial V}{\partial m^2} = \frac{1}{2} \bar{\phi}^2 + \frac{1}{4\pi^2} \int_0^\infty dr r^2 \frac{1}{\sqrt{r^2 + (\partial^2 V / \partial \bar{\phi}^2)}} \times \frac{1}{\exp[(1/T) \sqrt{r^2 + (\partial^2 V / \partial \bar{\phi}^2)}] - 1}. \quad (5)$$

The effective potential can be calculated by solving the partial differential equation (5) with the initial condition (2). The effective potential has an imaginary part below the critical temperature and an analytic continuation is done so that this imaginary part is negative [27]. Since the evolution equation (5) is a complicated nonlinear partial differential equation, it can be solved only by numerical methods.

III. RESULTS

We calculate the effective potential numerically using the method in the Appendix. The real part of the effective potential as the temperature varies is shown in Fig. 1. A stable

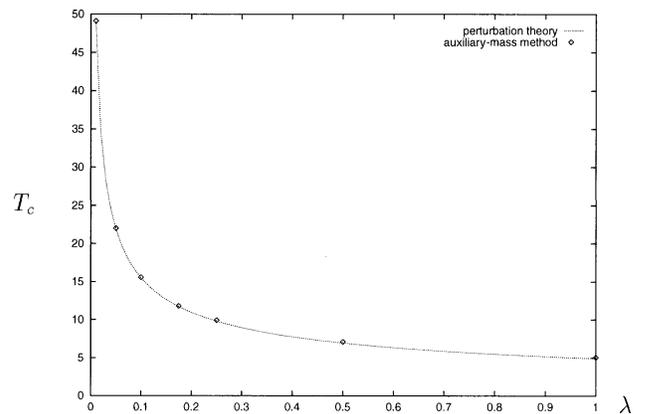


FIG. 3. Phase diagram of $\lambda \phi^4$ theory. A second-order phase transition is observed on the boundary. The dots represent values calculated using the auxiliary-mass method. The dotted line represents the leading result of perturbation theory [35].

TABLE I. Critical exponents and critical amplitude obtained from various methods. Since a first-order phase transition is indicated, the critical exponents cannot be determined using finite-temperature field theory (FT) within one-loop order. We note that there are many nonperturbative methods based on the renormalization group (RG) idea which we do not refer to here. The central values of them are shown. Values in the parentheses are determined using scaling relations.

			γ	ν	β	α	δ	η	χ_+/χ_-
Auxiliary-mass method			1.37		0.385	0.12	4.0		3.4
FT	Perturbation theory	One-loop	*	*	*	*	*	*	*
		Two-loop	1.0	0.5	0.5	0.0	3.0	0.0	2.0
	Perturbation theory [36]	Fixed dim.	1.24	0.630	0.325	0.11	4.82	0.317	4.82
RG	Nonperturbative	ϵ exp.	1.24	0.631	0.327	0.11	4.79	0.349	4.70
		Sharp cutoff [19]	(1.38)	0.690	(0.345)	(-0.07)	(5.0)	0.0	*
		Smooth cutoff [17]	∂^0 (1.32) ∂^2 (1.20)	0.660 (0.33) 0.618 (0.327)	(0.02) (0.146)	(5.0) (4.67)	0.00 0.054	*	
	Lattice Monte Carlo [37]		1.24	0.629	0.324	0.113	4.83	0.027	
	Binary fluids		1.236	0.625	0.325	0.112			4.3
Experiment [36]	Liquid vapor		1.24	0.625	0.316	0.107			5.0
	Antiferromagnets		1.25	0.64	0.328	0.112			4.9
Landau approximation			1.0	0.5	0.5	0.0	3.0	0.0	2.0

field expectation value $\bar{\phi}_c$, where the effective potential has its minimum, comes to be zero smoothly as the temperature increases. This indicates that a second-order phase transition takes place in this model correctly [33,34]. The imaginary part of the effective potential below the critical temperature is shown in Fig. 2. One can observe that the magnitude of it increases as the field expectation value decreases; this illustrates that a state with a smaller field expectation value is less stable below the critical temperature. The critical temperature as a function of the coupling constant λ is shown in Fig. 3. This shows a similar behavior to the leading result obtained in Ref. [35], but has a slight difference ($\sim 2\%$). In the remainder of this section we determine some critical exponents β , δ , $\gamma^{+/-}$, and α . The amplitude ratio χ_+/χ_- is also determined. The results are summarized in Table I.

First, we observe the stable point $\bar{\phi}_c$ carefully. Figure 4 shows $\bar{\phi}_c$ as a function of temperature. It decreases monotonically and vanishes smoothly as the temperature increases. We then focus on its behavior near the critical temperature T_c and determine β , which relates the magnetization to the temperature near T_c . This is defined as follows:

$$\phi_c \propto (-\tau)^\beta \quad (\tau \sim 0, T < T_c), \quad (6)$$

where $\tau = (T - T_c)/T_c$. We plot $\log(\bar{\phi}_c)$ against $\log(-\tau)$ in Fig. 5; we fit the data to a linear function and draw it in Fig. 5. We determine β from the gradient of it. We find $\beta = 0.385$.

Next, we determine the exponent δ which is defined as follows:

$$\bar{\phi}_c \propto J^{1/\delta} = \left(\frac{\partial V}{\partial \bar{\phi}} \right)^{1/\delta} \quad (T = T_c). \quad (7)$$

One can derive the following relation from this:

$$V \propto \bar{\phi}^{\delta+1} \quad (T = T_c). \quad (8)$$

We show the effective potential at T_c in Fig. 6. We plot $\log(V)$ against $\log(\bar{\phi})$ in Fig. 7; we fit the data to a linear function and draw it in Fig. 7. We determine δ from the gradient of it. The result is $\delta = 4.0$.

Then, we determine $\gamma^{+/-}$ and χ_+/χ_- . They are defined as follows through the susceptibility:

$$\chi \equiv \left. \frac{\partial \bar{\phi}}{\partial J} \right|_{J=0} \sim \chi_+ \tau^{-\gamma^+} \quad (\tau \sim 0, T > T_c), \quad (9)$$

$$\chi \equiv \left. \frac{\partial \bar{\phi}}{\partial J} \right|_{J=0} \sim \chi_- \tau^{-\gamma^-} \quad (\tau \sim 0, T < T_c). \quad (10)$$

To calculate χ , we relate χ to the curvature using the following identity derived from the definition of the effective potential:

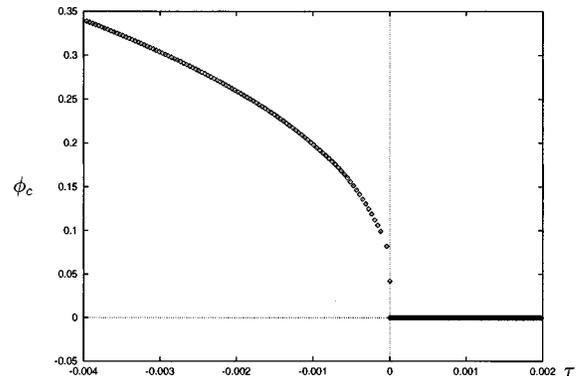


FIG. 4. Stable field expectation value as a function of temperature ($\lambda = 1$). It decreases monotonically and vanishes smoothly as the temperature increases.

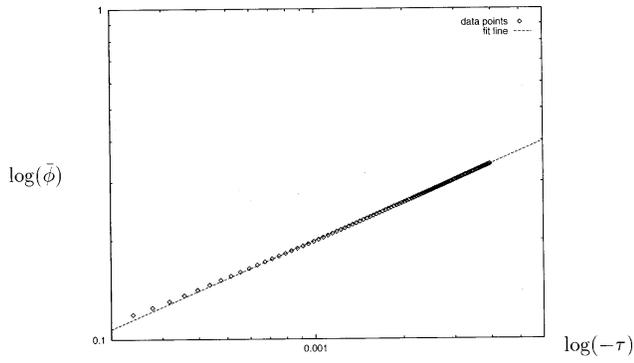


FIG. 5. Plot of $\log(\bar{\phi})$ - $\log(-\tau)$ ($\lambda = 1$). The data points are fit to a linear function. Using its gradient, β is determined.

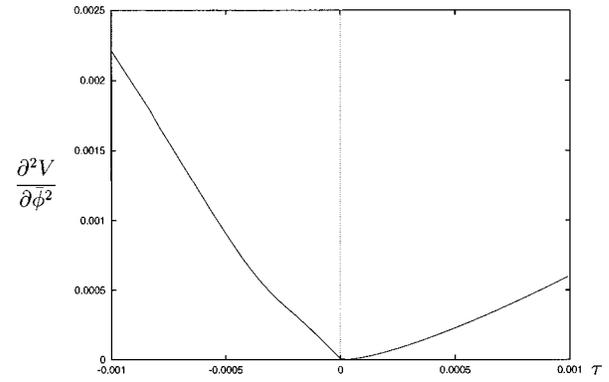


FIG. 8. Curvature at minimum point $\partial^2 V / \partial \phi^2$ as the temperature varies ($\lambda = 1$).

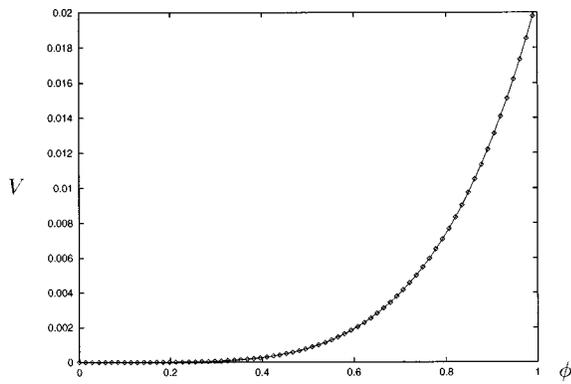


FIG. 6. Effective potential at the critical temperature ($\lambda = 1$).

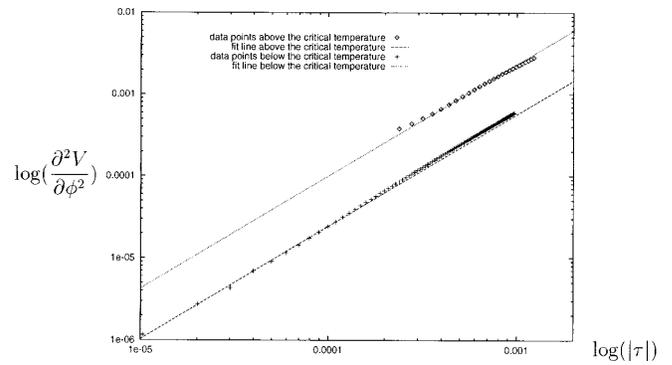


FIG. 9. Plots of $\log(\partial^2 V / \partial \phi^2)$ - $\log(|\tau|)$ ($\lambda = 1$). The data points are fit to linear functions. Using their gradients, $\gamma^{+/-}$ and χ_+ / χ_- are determined.

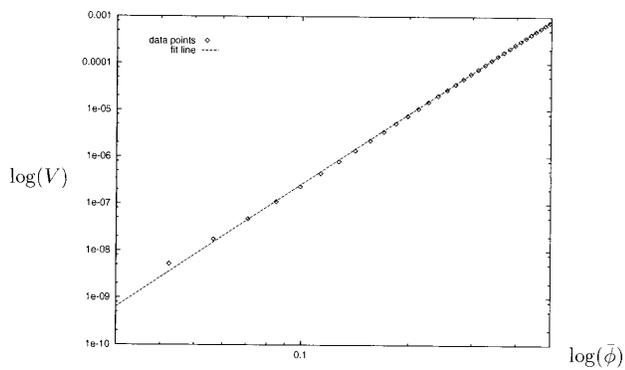


FIG. 7. Plot of $\log(V)$ - $\log(\bar{\phi})$ ($\lambda = 1$). The data points are fit to a linear function. Using its gradient, δ is determined.

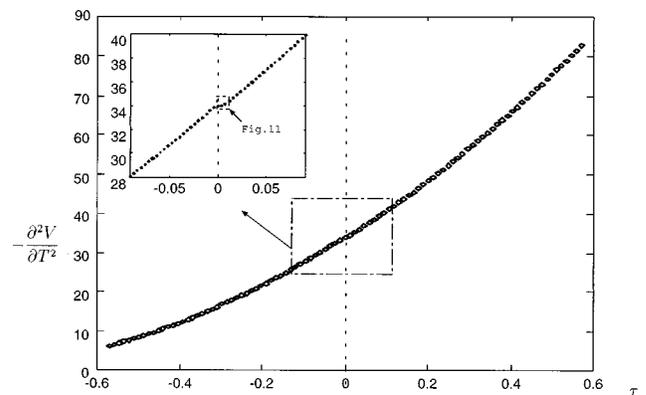


FIG. 10. Second derivative of the effective potential with respect to temperature ($\lambda = 1$).

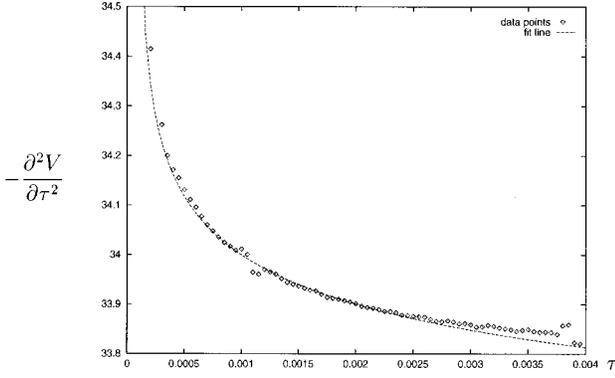


FIG. 11. Specific heat C as a function of τ around the critical temperature. One can observe that it blows up around the critical temperature. One of the critical exponents, α , is determined from this.

$$\left. \frac{\partial \bar{\phi}}{\partial J} \right|_{J=0} = \left(\frac{\partial^2 V}{\partial \phi^2} \right)^{-1} \Big|_{\bar{\phi}=\phi_c}. \quad (11)$$

We show $(\partial^2 V / \partial \phi^2)^{-1} |_{\bar{\phi}=\phi_c}$ as a function of temperature in Fig. 8. We also plot $\log(\partial^2 V / \partial \phi^2)$ against $\log(|\tau|)$ in Fig. 9; we fit the data to linear functions and draw them in Fig. 9. We determine $\gamma^{+/-}$ from the gradient of it and χ_+ / χ_- from the intercepts. We find $\gamma \equiv \gamma^+ = \gamma^- = 1.37$, $\chi_+ / \chi_- = 3.4$.

Finally, we pay attention to the second derivative of the effective potential with respect to the temperature, which is proportional to the specific heat C . The exponent α is defined as follows:²

$$C \propto \frac{\partial^2 V}{\partial \tau^2} \propto \tau^{-\alpha} \quad (\tau \sim 0). \quad (12)$$

This derivative is shown in Fig. 10 as a function of temperature. We focus on its behavior around T_c in Fig. 11 and observe that it blows up there. One of the critical exponents, α , is determined using this. The result is $\alpha=0.12$.

The results are summarized in Table I and compared with results obtained by various methods. A discussion is presented in the next section.

IV. SUMMARY AND DISCUSSION

The critical exponents and the amplitude ratio were determined using the auxiliary-mass method developed in Ref. [27] by the improved numerical method in the Appendix. The results are summarized in Table I. We found that $\lambda \phi^4$ theory shows a second-order phase transition as it should. Though the critical exponents calculated here do not satisfy the scaling relations, they satisfy the inequalities of critical exponents. For example, the inequalities given by Griffiths [38],

²Though the amplitude ratio of the specific heat can also be defined, it is not determined because of numerical reasons.

$$\gamma^- \geq \beta(\delta - 1), \quad (13)$$

$$\gamma^+(\delta + 1) \geq (2 - \alpha)(\delta - 1), \quad (14)$$

are satisfied. In the following we compare our results with others'.

First, the results are compared with the values obtained by perturbative finite-temperature field theory with a daisy resummation. Since a first-order phase transition is indicated at one-loop order [7,28], the critical exponents cannot be determined by perturbation theory. At two-loop order, a second-order phase transition is observed and the critical exponents are the same as those obtained by the Landau approximation.³ In comparison with these values, the results obtained in the present paper are considerably good.

Second, they are compared with the values obtained by the renormalization group and by lattice simulation, which they agree with greatly. In comparison with these accurate values, our results are not very good. These errors are probably caused by the replacement (4). Since this replacement is based on the neglect of momentum dependence in Π , we have to take into account the momentum dependence in order to improve our results [39].

As mentioned in Sec. I, finite-temperature field theory is an optimum theory to investigate phase transitions; it is based on statistical principles and can be applied to both first-order and second-order phase transitions. Perturbation theory, however, often breaks down and it prevents us from using finite-temperature field theory. The auxiliary-mass method enables finite-temperature field theory to be used in various situations.

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APPENDIX: NUMERICAL METHOD

The numerical method, which we use to solve Eq. (5), is explained in this Appendix. The partial differential equation (5) is written as follows:

$$\frac{\partial V}{\partial m^2} = \frac{1}{2} \bar{\phi}^2 + f \left(\frac{\partial^2 V}{\partial \bar{\phi}^2} \right). \quad (A1)$$

Here, $f(x)$ is the integral in Eq. (5). First, we make the lattice shown in Fig. 12. The partial differential equation (5) is, then, differenced as follows [30]:

³We used the two-loop order effective potential calculated in [7]. We determined the critical exponents from this both numerically and analytically.

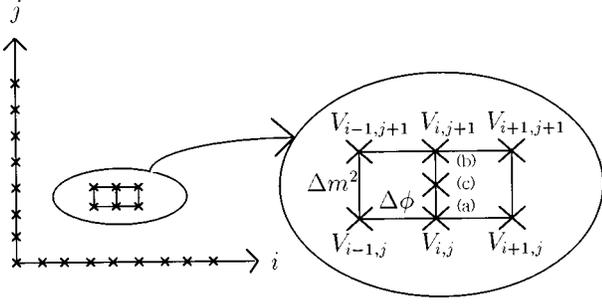


FIG. 12. Lattice used to difference Eq. (5).

$$\frac{V_{i,j+1} - V_{i,j}}{\Delta m^2} = \frac{1}{2} \phi_i^2 + f \left(\alpha \left(\frac{V_{i+1,j+1} - 2V_{i,j+1} + V_{i-1,j+1}}{(\Delta \phi)^2} \right) + (1 - \alpha) \left(\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right) \right). \quad (\text{A2})$$

The parameter α decides where the Laplacian $\partial^2 V / \partial \phi^2$ is evaluated. If $\alpha=0$ is selected, the Laplacian is evaluated at (a) in Fig. 12. The method of this selection is called the explicit method, which we used in [27]. This method is simple, because $V_{x,j+1}$ is determined only by substituting $V_{x,j}$ into the right-hand side. It, however, suffers from a numerical instability, when a smaller mesh $\Delta \phi$ is chosen [30]; therefore, we could not make the mesh small in [27]. If $\alpha=1$ is selected, the Laplacian is evaluated at (b) in Fig. 12. The method of this selection is called the implicit method, which does not suffer from numerical instability at least if $f(x)$ is a linear function [30]—as far as we know, when $f(x)$ is not a linear function as in our case, not many things are known. If $\alpha=1/2$ is selected, the Laplacian is evaluated at (c) in Fig. 12. The method of this selection is called the Crank-Nicholson method, which also does not suffer from numerical instability at least if $f(x)$ is a linear function. What is more, the result converges more rapidly with decreasing Δm^2 using this method [30]. Both the implicit and the Crank-Nicholson methods, however, require us to solve the coupled nonlinear equation (A2); this prevents us from using an established method in the case $f(x) \propto x$.

We developed two methods in order to overcome this difficulty. The first method is based on a Taylor expansion of $f(x)$. Equation (A2) is rewritten as follows:

$$\frac{V_{i,j+1} - V_{i,j}}{\Delta m^2} = \frac{1}{2} \phi_i^2 + f \left(\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} + \alpha \left(\frac{V_{i+1,j+1} - 2V_{i,j+1} + V_{i-1,j+1}}{(\Delta \phi)^2} - \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right) \right). \quad (\text{A3})$$

Since the quantity in the parentheses behind α is the variation of the Laplacian per one step, it is small if Δm^2 is sufficiently small. We then expand $f(x)$ around $(V_{i+1,j} - 2V_{i,j} + V_{i-1,j}) / (\Delta \phi)^2$:

$$\begin{aligned} \frac{V_{i,j+1} - V_{i,j}}{\Delta m^2} &= \frac{1}{2} \phi_i^2 + f \left(\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right) \\ &+ \alpha \left(\frac{V_{i+1,j+1} - 2V_{i,j+1} + V_{i-1,j+1}}{(\Delta \phi)^2} - \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right) \\ &\times f' \left(\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right) \\ &+ \text{higher order terms.} \end{aligned} \quad (\text{A4})$$

This coupled equation is linear with respect to $V_{x,j+1}$ and can be solved easily [30].

The second method is based on an iteration. In order to solve Eq. (A2), we iterate as follows until a solution is found:

$$\begin{aligned} \frac{V_{i,j+1}^{n+1} - V_{i,j}^n}{\Delta m^2} &= \frac{1}{2} \phi_i^2 + f \left(\alpha \left(\frac{V_{i+1,j+1}^{n+1} - 2V_{i,j+1}^n + V_{i-1,j+1}^n}{(\Delta \phi)^2} \right) \right. \\ &\left. + (1 - \alpha) \left(\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right) \right). \end{aligned} \quad (\text{A5})$$

Here, n is the number of the iteration. Note that we cannot replace $V_{i,j+1}^{n+1}$ with $V_{i,j+1}^n$ unlike the Gauss-Seidel method, which is a powerful method if $f(x)$ is a linear function [30]. Next, the relaxation method is used in order to improve the convergence [30]. Since this procedure is identical to the linear case, we only write down the iteration equation without an explanation:

$$\begin{aligned} \frac{V_{i,j+1}^{n+1} - V_{i,j}^n}{\Delta m^2} &= \omega \left[\frac{1}{2} \phi_i^2 + f \left(\alpha \left(\frac{V_{i+1,j+1}^{n+1} - 2V_{i,j+1}^n + V_{i-1,j+1}^n}{(\Delta \phi)^2} \right) \right. \right. \\ &\left. \left. + (1 - \alpha) \left(\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta \phi)^2} \right) \right) \right] + (1 - \omega) V_{i,j}^n. \end{aligned} \quad (\text{A6})$$

Here, the relaxation parameter ω is determined only by experience. The results of the two methods agree greatly. In the present paper, the latter method is used in order to determine the universal quantities.

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