Ambiguities of theoretical parameters and *CP* or *T* violation in neutrino factories

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We study the sensitivity to the *CP*- or *T*-violation search in the presence of ambiguities of the theoretical parameters. Three generations of neutrinos are considered. The parameters whose ambiguities are considered are the differences of the squared masses, the mixing angles, and the density of matter. We first consider the statistics that are sensitive to the genuine *CP*-violation effect originating from the imaginary coupling. No ambiguity of the parameters is considered in this part. It is argued that the widely adopted usual statistics are not necessarily sensitive to the genuine *CP*-violation effect. Two statistics that are sensitive to the imaginary coupling are proposed. The qualitative difference between these statistics and the usual ones are discussed. Next we proceed to the case where the ambiguity of the parameters is present. The sensitivity of the *CP*-violation search is greatly spoiled when the baseline length is longer than about one thousand kilometers, which turns out to be due to the ambiguity of the matter effect. Thus the *CP*-violation search by use of *CP* conjugate channels turns out to require a low energy neutrino and short baseline length. It is also shown that such a loss of sensitivity is avoided by using *T*-conjugate oscillation channels.

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I. INTRODUCTION

The observation of the atmospheric neutrino anomaly by Super-Kamiokande [1] provided us with convincing evidence that neutrinos have nonvanishing masses. There is another indication of neutrino masses and mixings by the solar neutrino deficit [2–6].

These results give us the allowed region and excluded region for the mixing angles and the mass square differences. Let us now assume that there are three flavors of neutrinos and denote the lepton mixing matrix U, which relates the flavor eigenstates ν_α $(\alpha=e,\mu,\tau)$ with the mass eigenstates ν_i with mass m_i $(i=1,2,3)$ as $\nu_\alpha=\Sigma_{i=1}^3 U_{\alpha i}\nu_i$, by

$$U=\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}-s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12}-s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{23}s_{12}-c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12}-c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix}. \tag{1}$$

Here c_{ij} and s_{ij} stand for $\cos\theta_{ij}$ and $\sin\theta_{ij}$, respectively. The observations of the atmospheric neutrino anomaly give us an allowed region for $\sin\theta_{23}$ and the larger mass square difference $(\equiv\delta m_{31}^2)$. The solar neutrino deficit provides allowed regions for $\sin\theta_{12}$ [7] and the smaller mass square difference $(\equiv\delta m_{21}^2)$. On the other hand, the no-oscillation results of reactor and accelerator experiments give us an exclusion region for $\sin\theta_{13}$ (e.g., Ref. [8]). There is no constraint on the *CP*-violating phase δ.

The idea of neutrino factories with muon storage rings was proposed [9] to determine these mixing parameters (and the sign of δm_{31}^2 in addition). It attracted the interest of many physicists [10–20], and the neutrino factories turned out to be a very promising candidate for the next generation neutrino oscillation experiments. We will be able to observe neutrino oscillations even if $\sin\theta_{13}$ is as small as 0.01. We will also be able to detect the *CP*-violation effects in such experiments [21,22]. The possibility to observe *CP* violation through long baseline neutrino oscillation experiments was discussed in Refs. [23–28] and many papers followed these works.

By what observation can we insist that we measure the *CP* violation? *CP* violation is characterized by the intrinsic imaginary part of a coupling in a Lagrangian. The presence

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of an imaginary part of a coupling gives different properties to particles and antiparticles, and it is observed to be *CP* or *T*-violation effects. Hence we need to discuss *CP* or *T* violation using a quantity which is sensitive to the imaginary part of a coupling. We have to be very careful to construct such a quantity since there is an indirect sensitivity to the *CP*-violating phase (which constitute not an imaginary part of couplings but a real part) through a unitarity [29]. As for lepton *CP* or *T* violation in the long baseline neutrino oscillation experiments, one of such quantities is the difference of event rate between *CP* or *T* conjugate channels. We must take care of matter effect [30] for *CP* conjugate channels since it gives difference to the event rate too. Therefore we must take into consideration ambiguities of the parameters as the matter effect can mimic the genuine *CP* violation partially. We will show that the ambiguities of parameters spoil the experimental sensitivity.[1]

We formulate the treatment of the ambiguity of parameters within the statistical method. It is important to build a statistical quantity that is sensitive to the imaginary part of coupling. We propose a proper statistics, and show explicitly that the sensitivity to *CP*-violation effect changes by taking the ambiguities into account. To this end we estimate how large exposure (proportional to number of muons and detector size) is required to observe *CP*-violation effect by the neutrino factory experiment. The optimal experimental setup (muon energy E_μ and baseline length L) is shown through such considerations assuming three generations of neutrinos which account for the solar neutrino deficit and the atmospheric neutrino anomaly.

This paper is organized as follows. In Sec. II we consider statistical quantities which are proper to search for a genuine *CP*-violation effect. There we assume that the parameters such as θ_{ij}'s and δm_{ij}^2's are known without ambiguities. We will discuss in Sec. III the case where the ambiguities of the parameters are taken into account. We present the requirement on the number of muons and the mass of a detector to observe genuine *CP*-violation effect through measurements of *CP*-conjugate oscillation channels. It is shown that the introduction of the ambiguities of parameters greatly changes the sensitivity to the *CP*-violation search. In Sec. IV we investigate *CP*-violation search using *T*-conjugate oscillation channels. Ambiguities of parameters are taken into account also in this section. The sensitivity in this case is far better compared to the case using *CP*-conjugate channels, showing that this case is ideal (preferable) to search for *CP*-violation effect. Finally a summary and discussion are given in Sec. V.

II. DIRECT OBSERVATION OF *CP*-VIOLATION EFFECT

Let us first discuss the physical quantity which characterizes the presence of *CP* or *T* violation in the long baseline neutrino oscillation experiments. Such quantities must be sensitive to the imaginary part of the coupling. We need to be particularly careful when we make use of *CP*-conjugate channels in the presence of the matter effect.

Let us first recall how the imaginary part of the lepton coupling gets into the oscillation probabilities. We use the oscillation in the vacuum as a simplest example. The oscillation probability from ν_α to ν_β in the vacuum is given by

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta; E, L) &= \sum_i \left| U_{\beta i} e^{-i\delta m_i^2 L/(2E)} U_{\alpha i}^* \right|^2 \\ &= \sum_{i,j} U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^* e^{-i\delta m_{ij}^2 L/(2E)} \\ &= \sum_{i,j} \mathrm{Re}\, U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^* \cos \frac{\delta m_{ij}^2 L}{2E} \\ &\quad + \sum_{i,j} \mathrm{Im}\, U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^* \sin \frac{\delta m_{ij}^2 L}{2E}, \end{aligned} \tag{2}$$

where E and L are the energy of neutrinos and the baseline length, respectively. The unitarity of U leads to

$$\begin{aligned} \mathrm{Im}\, U_{\alpha 1}^* U_{\alpha 2} U_{\beta 1} U_{\beta 2}^* &= \mathrm{Im}\, U_{\alpha 2}^* U_{\alpha 3} U_{\beta 2} U_{\beta 3}^* \\ &= \mathrm{Im}\, U_{\alpha 1}^* U_{\alpha 3} U_{\beta 3} U_{\beta 1}^* \equiv J, \end{aligned} \tag{3}$$

which allows us to write

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta; E, L) &= \sum_{i,j} \mathrm{Re}\, U_{\alpha i}^* U_{\alpha j} U_{\beta i} U_{\beta j}^* \cos \frac{\delta m_{ij}^2 L}{2E} \\ &\quad + J \sum_{i,j} \sin \frac{\delta m_{ij}^2 L}{2E}. \end{aligned} \tag{4}$$

The Jarlskog parameter J defined by Eq. (3) vanishes when all the U_{ij}'s are real, and the second term of Eq. (4) also vanishes. The existence of imaginary part of lepton coupling gives nonvanishing J, and thus we need to observe the quantity which is sensitive to J (including its sign) to search directly for the *CP*-violation effect. Note that this statement is independent of the parametrization of U such as in Eq. (1).

Now we consider the quantity which is sensitive to the Jarlskog parameter J in the presence of matter on the baseline. We put some assumptions for simplicity to discuss this point. Suppose that we have same initial energy spectra for neutrinos and antineutrinos. Also we suppose that the antineutrinos have about twice larger data size so that the expected event numbers for neutrinos and antineutrinos are equal in no-oscillation case. Then the oscillation event numbers of neutrinos $N(\nu_\alpha \to \nu_\beta)$ and that of antineutrinos $N(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$ are expected to be equal in vacuum if *CP* is conserved. The two event numbers are in practice different due to *CP* violation (if any) and matter on the baseline. The difference of the two event numbers,

[1]It is also important to consider experimental systematic errors and backgrounds, but we do not consider them in this paper. We assume that we can determine all the quantities such as particle energy. The only error taken into account is statistical ones.

$$\Delta N(\delta) \equiv N(\nu_\alpha \to \nu_\beta;\delta) - N(\overline{\nu}_\alpha \to \overline{\nu}_\beta;\delta) \tag{5}$$

is intuitively sensitive to the genuine *CP* violation. Here the *CP*-violating angle δ is explicitly written. This quantity does not vanish due to matter effect, even in the absence of genuine *CP* violation. We thus consider

$$\Delta N(\delta) - \Delta N(\delta_0), \tag{6}$$

where $\delta=\delta_0 \in \{0,\pi\}$ corresponds to the *CP* conserving case.

We stress here that the quantity

$$N(\nu_\alpha \to \nu_\beta;\delta) - N(\nu_\alpha \to \nu_\beta;\delta_0) \tag{7}$$

is not necessarily sensitive to the genuine *CP* violation. To compare Eqs. (5) and (7), let us consider the oscillation probability $P(\nu_\alpha \to \nu_\beta;\delta)$, which is related to $N(\nu_\alpha \to \nu_\beta;\delta)$ roughly by

$$N(\nu_\alpha \to \nu_\beta;\delta) \sim \frac{E^3}{L^2} P(\nu_\alpha \to \nu_\beta;\delta). \tag{8}$$

It can be shown in the high energy limit[2] [31]

$$\begin{aligned} P(\nu_\mu \to \nu_e;\delta,a) = {} & \left(\frac{\delta m_{31}^2 L}{4E}\right)^2 \left[B + \frac{\delta m_{21}^2}{\delta m_{31}^2}(j\cos\delta - 2B\sin^2\theta_{12})\right]\left[1 - \frac{1}{3}\left(\frac{aL}{4E}\right)^2\right] \\ & + \left(\frac{\delta m_{31}^2 L}{4E}\right)^3 \frac{aL}{4E}\left\{\frac{2}{3}B\cos 2\theta_{13} + \frac{\delta m_{21}^2}{\delta m_{31}^2}\left[\frac{1}{3}j\cos\delta(2\cos 2\theta_{13}-1) - 2B\cos 2\theta_{13}\sin^2\theta_{12}\right]\right\} \\ & - \left(\frac{\delta m_{31}^2 L}{4E}\right)^3 \frac{\delta m_{21}^2}{\delta m_{31}^2} j \sin\delta + \mathcal{O}\left(\frac{1}{E^4}\right), \end{aligned} \tag{9}$$

where

$$j \equiv \sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}, \tag{10}$$

$$B \equiv |U_{e3}|^2|U_{\mu 3}|^2 = \sin^2\theta_{23}\sin^2 2\theta_{13}, \tag{11}$$

$$a \equiv 2\sqrt{2}G_F n_e E, \tag{12}$$

and n_e in Eq. (12) is the average electron number density in the matter on the baseline. Recall that the Jarlskog parameter J defined by Eq. (3) is expressed under the parametrization Eq. (1) by

$$J = \sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta, \tag{13}$$

which is related to j in Eq. (10) by

$$J = j\sin\delta. \tag{14}$$

Thus the third term of Eq. (9) is the contribution from the genuine *CP*-violation effect. Note again that this statement is also independent of the parametrization.

We obtain from Eq. (9)

$$P(\nu_\mu \to \nu_e;\delta,a) - P(\nu_\mu \to \nu_e;\delta_0,a) = \left(\frac{\delta m_{31}^2 L}{4E}\right)^2 \frac{\delta m_{21}^2}{\delta m_{31}^2} j(\cos\delta \mp 1)\left[1 - \frac{1}{3}\left(\frac{aL}{4E}\right)^2\right] + \mathcal{O}\left(\frac{1}{E^3}\right) \tag{15}$$

($-$ sign for $\delta_0=0$ and $+$ for $\delta_0=\pi$) and

$$\begin{aligned} P(\nu_\mu \to \nu_e;\delta,a) - P(\overline{\nu}_\mu \to \overline{\nu}_e;\delta,a) &= P(\nu_\mu \to \nu_e;\delta,a) - P(\nu_\mu \to \nu_e;-\delta,-a) \\ &= 2\left(\frac{\delta m_{31}^2 L}{4E}\right)^3 \left(\frac{aL}{4E}\left\{\frac{2}{3}B\cos 2\theta_{13} + \frac{\delta m_{21}^2}{\delta m_{31}^2}\left[\frac{1}{3}j\cos\delta(2\cos 2\theta_{13}-1)\right.\right.\right. \\ &\quad \left.\left.\left. - 2B\cos 2\theta_{13}\sin^2\theta_{12}\right]\right\} - \frac{\delta m_{21}^2}{\delta m_{31}^2} j\sin\delta\right) + \mathcal{O}\left(\frac{1}{E^4}\right). \end{aligned} \tag{16}$$

[2] The limit is valid when $E_\nu \gtrsim \delta\tilde{m}_{31}^2 L/4$, where $\delta\tilde{m}_{31}^2 \equiv \sqrt{(\delta m_{31}^2\cos 2\theta_{13} - a)^2 + (\delta m_{31}^2\sin 2\theta_{13})^2}$.

We can observe in Eq. (15) that the leading term of $P(\nu_\mu \to \nu_e;\delta,a) - P(\nu_\mu \to \nu_e;\delta_0,a)$ does not contain the genuine *CP*-violation term. This leading term can be canceled by taking the difference between the probabilities of neutrinos and antineutrinos. Equation (16) is indeed sensitive to the $j \sin\delta$ term, though it contains an unavoidable matter effect term in addition.

Our viewpoint is that the *CP*-violation search must be carried out directly by observing the contribution of the $j \sin\delta$ term in Eq. (9), which originates from the *imaginary* part of the coupling as in Eqs. (3), (13), and (14). This term is not the leading term at least in the high energy region, but it can be picked up as a leading term by taking the difference between neutrinos and antineutrinos as in Eq. (16). Applying this consideration also to event rates, we regard that Eq. (6) is a better quantity than Eq. (7) to pursue the possibility of direct *CP*-violation search. An analysis using Eq. (7) is a usual parameter fitting method. It does not take into careful consideration whether or not $N(\nu_\alpha \to \nu_\beta;\delta)$ is sensitive to imaginary part of the coupling. Even if one obtains a result that $N(\delta) \neq N(\delta_0)$ in an experiment which is sensitive only to the real part, there remains a possibility to build a certain Lagrangian with totally *real* coupling which can reproduce $N(\delta)$. In this respect $N(\delta) \neq N(\delta_0)$ cannot be the definite clue of the presence of *CP* violation.

Let us further exemplify the difference between Eqs. (6) and (7). We consider the following toy setup of an experiment. A source of neutrino beam is N_μ muons which decay into neutrinos at a muon ring. The neutrinos extracted from the ring are detected at a detector if their energy E_ν is larger than a threshold energy $E_{\rm th}$. The detector has mass $M_{\rm detector}$ and contains $N_{\rm target}$ target atoms, which are related as

$$N_{\rm target} = 6.02 \times 10^{34} \frac{M_{\rm detector}}{[100\ \rm kt]}. \tag{17}$$

We assume the neutrino-nucleon cross section σ is proportional to neutrino energy as

$$\sigma = \sigma_0 E_\nu , \tag{18}$$

where

$$\sigma_0 = \begin{cases} 0.67 \times 10^{-38}\ \rm cm^2/GeV & \text{for neutrinos,} \\ 0.34 \times 10^{-38}\ \rm cm^2/GeV & \text{for antineutrinos.} \end{cases} \tag{19}$$

The expected number of appearance events in the energy bin $E_{j-1} < E_\nu < E_j$ $(j = 1,2,\dots,n)$ is then given by

$$N_j(\nu_\alpha \to \nu_\beta;\delta) \equiv \frac{N_\mu N_{\rm target} \sigma_0}{\pi m_\mu^2} \frac{E_\mu^2}{L^2} \int_{E_{j-1}}^{E_j} E_\nu f_{\nu_\alpha}(E_\nu) \times P(\nu_\alpha \to \nu_\beta;\delta) \frac{dE_\nu}{E_\mu}, \tag{20}$$

where m_μ is the muon mass, and $f_{\nu_\alpha}(E_\nu)$ is the neutrino flux that is concretely given by Eqs. (68) and (69). We define

$$C \equiv \frac{\sigma_0}{\pi m_\mu^2} \frac{N_{\rm target}}{M_{\rm detector}} \tag{21}$$

and

$$R_j(\nu_\alpha \to \nu_\beta;\delta) \equiv \int_{E_{j-1}}^{E_j} E_\nu f_{\nu_\alpha}(E_\nu) P(\nu_\alpha \to \nu_\beta;E_\nu,\delta) \frac{dE_\nu}{E_\mu}, \tag{22}$$

so that

$$\begin{aligned} N_j(\nu_\alpha \to \nu_\beta;\delta) &= N_\mu M_{\rm detector} \frac{E_\mu^2}{L^2} C R_j(\nu_\alpha \to \nu_\beta;\delta) \\ &= \begin{Bmatrix} 1.14 \\ 0.58 \end{Bmatrix} \times 10^3 \frac{N_\mu}{[10^{21}]} \frac{M_{\rm detector}}{[100\ \rm kt]} \\ &\quad \times \left(\frac{E_\mu/[\rm GeV]}{L/[1000\ \rm km]} \right)^2 \times R_j(\nu_\alpha \to \nu_\beta;\delta) \end{aligned} \tag{23}$$

(1.14 for neutrinos and 0.58 for antineutrinos). A quantity $N_\mu M_{\rm detector}$ is normalized in unit of $[10^{21} \times 100\ \rm kt]$ in Eq. (23). The value of unity in this unit is a quite optimistic one compared to the presently discussed values [13]. The requirement on $N_\mu M_{\rm detector}$ in this normalization must be about unity or less so that we can observe the *CP* -violation effect experimentally.

The widely adopted χ^2 statistical quantity based on Eq. (7) is defined by

$$\begin{aligned} \chi_1^2(\delta_0) &\equiv \sum_{j=1}^n \frac{[N_j(\delta) - N_j(\delta_0)]^2}{N_j(\delta)} + \sum_{j=1}^n \frac{[\overline{N}_j(\delta) - \overline{N}_j(\delta_0)]^2}{\overline{N}_j(\delta)} \\ &= N_\mu M_{\rm detector} \frac{E_\mu^2}{L^2} C \left\{ \sum_{j=1}^n \frac{[R_j(\delta) - R_j(\delta_0)]^2}{R_j(\delta)} \right. \\ &\quad \left. + \sum_{j=1}^n \frac{[\overline{R}_j(\delta) - \overline{R}_j(\delta_0)]^2}{\overline{R}_j(\delta)} \right\}, \end{aligned} \tag{24}$$

where

$$N_j(\delta) \equiv N_j(\nu_\alpha \to \nu_\beta;\delta), \quad \overline{N}_j(\delta) \equiv N_j(\overline{\nu}_\alpha \to \overline{\nu}_\beta;\delta), \tag{25}$$

$$R_j(\delta) \equiv R_j(\nu_\alpha \to \nu_\beta;\delta), \quad \overline{R}_j(\delta) \equiv R_j(\overline{\nu}_\alpha \to \overline{\nu}_\beta;\delta) \tag{26}$$

and n is the number of bins. Similarly we define based on Eq. (6) as[3]

[3]This quantity depends on a certain model through subtraction of the matter effect. However, if we can observe this asymmetry significantly, then we would be able to conclude that there is a genuine *CP*-violation effect in the real Lagrangian, even if the real theory is not the model that we assume.

$$\chi_2^2(\delta_0)\equiv\sum_{j=1}^{n}\frac{[\Delta N_j(\delta)-\Delta N_j(\delta_0)]^2}{N_j(\delta)+\overline{N}_j(\delta)}$$
$$=N_\mu M_{\text{detector}}\frac{E_\mu^2}{L^2}C\sum_{j=1}^{n}\frac{[\Delta R_j(\delta)-\Delta R_j(\delta_0)]^2}{R_j(\delta)+\overline{R}_j(\delta)},\qquad(27)$$

where

$$\Delta R_j(\delta)\equiv R_j(\delta)-\overline{R}_j(\delta).\qquad(28)$$

We need both $\delta\neq 0$ and $\delta\neq\pi$ to ascertain that *CP* violation is present. We thus define

$$\chi_1^2\equiv\min_{\delta_0\in\{0,\pi\}}\chi_1^2(\delta_0),\qquad(29)$$

$$\chi_2^2\equiv\min_{\delta_0\in\{0,\pi\}}\chi_2^2(\delta_0).\qquad(30)$$

and require

$$\chi_1^2>\chi_{90\%}^2(2n),\qquad(31)$$

$$\chi_2^2>\chi_{90\%}^2(n)\qquad(32)$$

to claim that the *CP*-violation effect is observable at 90% confidence level in the method with n energy bins; more details on statistics are found in the Appendix. Equations (31) and (32) are equivalent to

$$N_\mu M_{\text{detector}}>(N_\mu M_{\text{detector}})_{\min;1,90\%}\equiv\frac{1}{C}\frac{L^2}{E_\mu^2}\frac{\chi_{90\%}^2(2n)}{\min\limits_{\delta_0\in\{0,\pi\}}\left\{\sum_{j=1}^{n}\frac{[R_j(\delta)-R_j(\delta_0)]^2}{R_j(\delta)}+\sum_{j=1}^{n}\frac{[\overline{R}_j(\delta)-\overline{R}_j(\delta_0)]^2}{\overline{R}_j(\delta)}\right\}}\qquad(33)$$

and

$$N_\mu M_{\text{detector}}>(N_\mu M_{\text{detector}})_{\min;2,90\%}\equiv\frac{1}{C}\frac{L^2}{E_\mu^2}\frac{\chi_{90\%}^2(n)}{\min\limits_{\delta_0\in\{0,\pi\}}\left\{\sum_{j=1}^{n}\frac{[\Delta R_j(\delta)-\Delta R_j(\delta_0)]^2}{R_j(\delta)+\overline{R}_j(\delta)}\right\}},\qquad(34)$$

respectively.

We present example plots of $(N_\mu M_{\text{detector}})_{\min;1,90\%}$ and $(N_\mu M_{\text{detector}})_{\min;2,90\%}$ in Figs. 1 and 2. We adopt only single-bin method here because to divide the energy region into some bins does not make the sensitivity better, especially in the case without ambiguities of the parameters. We will explain the reason in detail in Sec. III B. The parameters are taken as follows so that they are consistent with the present experimental limit:

$$\sin\theta_{13}=0.1,\quad \sin\theta_{23}=\frac{1}{\sqrt{2}},\quad \sin\theta_{12}=0.5,\qquad(35)$$

$$\delta m_{31}^2=3\times10^{-3}\ \text{eV}^2,\quad \delta m_{21}^2=1\times10^{-4}\ \text{eV}^2.\qquad(36)$$

CP-violating angle δ is taken to be $\pi/2$. Matter effect a is approximated to be constant on the baseline, but its value depends on the baseline length since the longer baseline gets deeper in the Earth. The Preliminary Reference Earth Model [32] is adopted to estimate matter density as in Fig. 3 [31].

There is a qualitative difference between Figs. 1 and 2. Figure 1 shows that the sensitivity in terms of $N(\delta)-N(\{0,\pi\})$ enhances as the muon energy gets larger. This does not hold for $\Delta N(\delta)-\Delta N(\{0,\pi\})$ as seen in Fig. 2. There is a sweet spot in this case in terms of muon energy and baseline length that optimizes the sensitivity to the *CP*-violation effect.

We discuss the relation between χ_1^2 and χ_2^2. In the *CP*- or *T*-violation search, one compares $[N_j(\delta),\overline{N}_j(\delta)]$ with $[N_j(\delta_0),\overline{N}_j(\delta_0)]$. One can equivalently compare $[N_j^{\text{Total}}(\delta),\Delta N_j(\delta)]$ with $[N_j^{\text{Total}}(\delta_0),\Delta N_j(\delta_0)]$, where $N_j^{\text{Total}}(\delta)\equiv N_j(\delta)+\overline{N}_j(\delta)$. A χ^2 statistics defined by

$$\chi_{1'}^2(\delta_0)\equiv\sum_{j=1}^{n}\frac{[N_j^{\text{Total}}(\delta)-N_j^{\text{Total}}(\delta_0)]^2}{N_j^{\text{Total}}(\delta)}+\sum_{j=1}^{n}\frac{[\Delta N_j(\delta)-\Delta N_j(\delta_0)]^2}{N_j^{\text{Total}}(\delta)}\qquad(37)$$

obviously corresponds to $\chi_1^2(\delta_0)$. The second term of Eq. (37) is $\chi_2^2(\delta_0)$ itself. Hence we focus on the first term in order to understand the relation between χ_1^2 and χ_2^2. We note that $N^{\text{Total}}(\delta)$ is insensitive to *CP*-violation effect, since the magnitude of genuine *CP*-violation effect for neutrinos and antineutrinos are identical with opposite sign. The term we are discussing is thus insensitive to the imaginary coupling. Our aim was a direct *CP*- or *T*-violation search or a search for an imaginary coupling, and thus we dropped this term to obtain χ^2. On the other hand, the dependence on the energy of N^{Total} and ΔN in the high energy region is given by

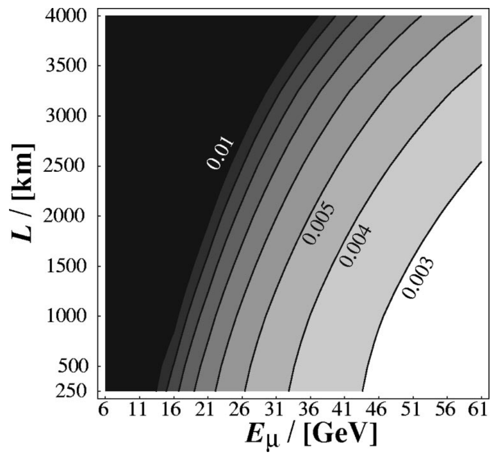

FIG. 1. A contour plot of the required data size to observe the genuine *CP*-violation effect. A quantity $(N_\mu M_{\rm detector})_{\rm min;1,90\%}$ defined in Eq. (33) is plotted in unit of $[10^{21}\times 100\ {\rm kt}]$ as a function of muon energy and baseline length. Smaller value of this value means the higher sensitivity. Here $E_{\rm th}=1$ GeV, and the case of $\delta=\pi/2$ is presented. Other parameters are taken as shown in Eqs. (35) and (36). The smaller value of $(N_\mu M_{\rm detector})_{\rm min;1,90\%}$ means the higher sensitivity of the *CP* violation search. The use of χ_1^2 leads to the higher sensitivity as E_μ gets larger.

$$N^{\rm Total}\sim E_\mu, \tag{38}$$

$$\Delta N\sim E_\mu^0, \tag{39}$$

which follows from Eqs. (8), (15), and (16). Hence the first term gets larger as energy gets larger, and dominates the right hand side of Eq. (39) in the high energy region; total fit gets better in spite of a poor fit of the imaginary coupling. The high sensitivity obtained by use of χ_1^2 and shown in Fig. 1 was achieved in this way.

The sweet spot seen in Fig. 2 can be intuitively understood. The *CP*-violation effect appears when the number of generations is more than three [33]. On one hand the heaviest state decouples from the oscillation at the low energy region such that $E_\mu \ll \delta m_{21}^2 L$, and on the other hand the lighter two generations are effectively degenerate in high energy regions such that $E_\mu > \delta m_{31}^2 L$. Thus the suitable energy region to observe *CP*-violation effect is roughly given by $\delta m_{21}^2 L \lesssim E_\mu \lesssim \delta m_{31}^2 L$. The sweet spot exactly lies in this region reflecting that χ_2^2 is indeed sensitive to the imaginary coupling.

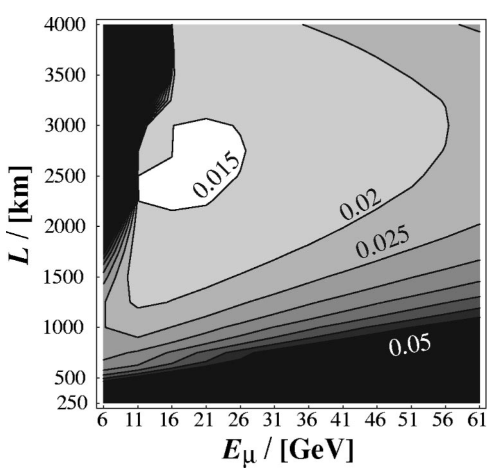

FIG. 2. A contour plot of $(N_\mu M_{\rm detector})_{\rm min;2,90\%}$ in unit of $[10^{21}\times 100\ {\rm kt}]$. The parameters are the same as in Fig. 1. A larger data sample is needed compared to Fig. 1. Optimum muon energy and baseline length makes a sweet spot in the graph.

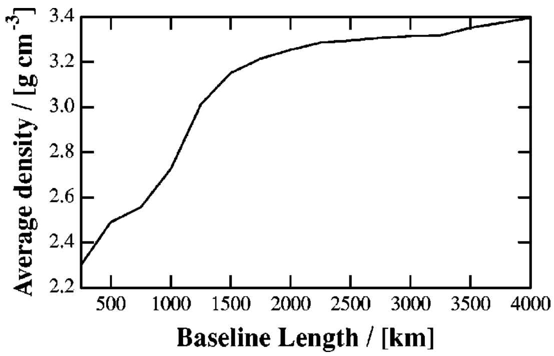

FIG. 3. Approximated matter density as a function of baseline length, calculated from the Preliminary Reference Earth Model [32].

We mention here that the sensitivity for the case $\delta=\pi/2$ and for $\delta=-\pi/2$ is not very different, contrary to the discussion by other authors [13]. The previous works compared $N(\delta)$ with $N(\delta_0=0)$ alone, but we compared $N(\delta)$ with both $N(\delta_0=0)$ and $N(\delta_0=\pi)$. One should keep in mind that *CP* symmetry is conserved not only in the $\delta=0$ case but in the $\delta=\pi$ case; the imaginary coupling is absent in both two cases. The real coupling is different for these two cases, and thus we need to distinguish an experimental result with both of them. We took these points into account by the definition Eqs. (29) and (30). We present in Figs. 4 and 5 the sensitivity plot similar to Figs. 1 and 2, but this case for $\delta=-\pi/2$. We observe indeed no qualitative difference between Figs. 1 and 4 and between Figs. 2 and 5, respectively.

We have seen so far that we can extract the imaginary coupling by constructing the χ^2 statistical quantity as in Eq. (30). The construction was motivated by taking the differ-

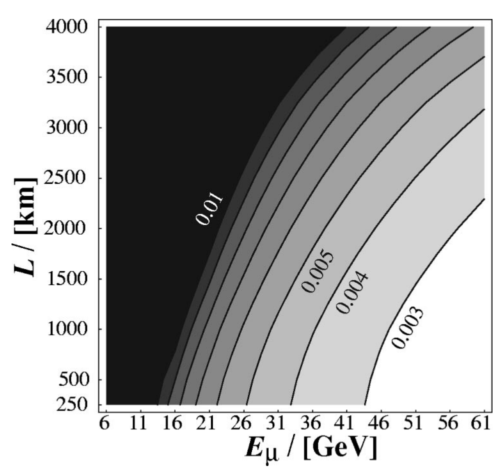

FIG. 4. Same as Fig. 1, but here $\delta=-\pi/2$.

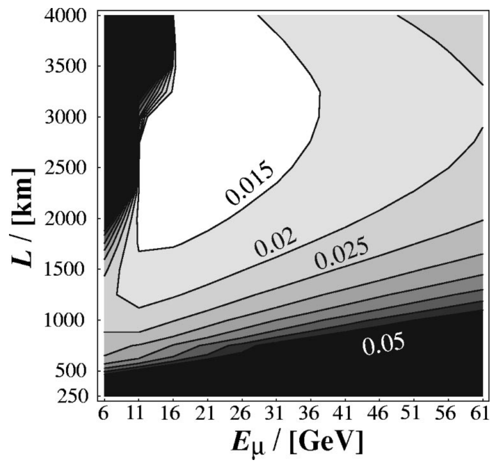

FIG. 5. Same as Fig. 2, but here $\delta=-\pi/2$.

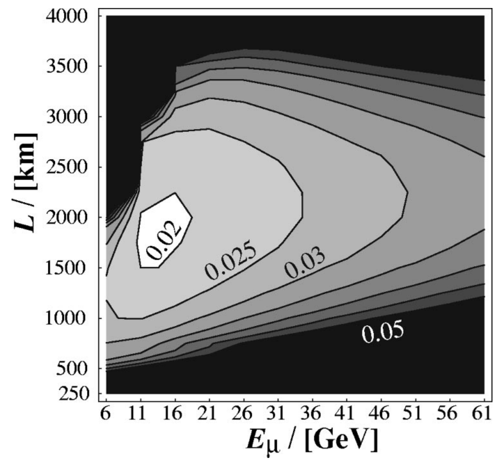

FIG. 6. A contour plot of $(N_\mu M_{\text{detector}})_{\min;3,90\%}$ in unit of $[10^{21}\times 100\text{ kt}]$. The parameters are the same as in Fig. 1. A sweet spot is seen in this figure as was also seen in Figs. 2 and 5.

ence of event rates of neutrinos and antineutrinos. We can build another quantity along this idea as

$$N(\delta)-\frac{N(\delta_0)}{\overline{N}(\delta_0)}\overline{N}(\delta), \tag{40}$$

which vanishes when $\delta=\delta_0$. A χ^2 statistics for this quantity is given by

$$\chi_3^2(\delta_0)=\sum_{j=1}^{n}\frac{[\overline{N}(\delta_0)N(\delta)-N(\delta_0)\overline{N}(\delta)]^2}{\overline{N}(\delta_0)^2N(\delta)+N(\delta_0)^2\overline{N}(\delta)}. \tag{41}$$

The sensitivity condition with 90% confidence level in the n-bin method is

$$\chi_3^2>\chi_{90\%}^2(n), \tag{42}$$

or

$$N_\mu M_{\text{detector}}>(N_\mu M_{\text{detector}})_{\min;3,90\%}$$
$$\equiv\frac{1}{C}\frac{E_\mu^2}{L^2}\frac{\chi_{90\%}^2(n)}{\min\limits_{\delta_0\in\{0,\pi\}}\left\{\sum_{j=1}^{n}\frac{[\overline{R}(\delta_0)R(\delta)-R(\delta_0)\overline{R}(\delta)]^2}{\overline{R}(\delta_0)^2R(\delta)+R(\delta_0)^2\overline{R}(\delta)}\right\}}. \tag{43}$$

Figure 6 shows an example plot of $(N_\mu M_{\text{detector}})_{\min;3,90\%}$ for the parameters given by Eqs. (35) and (36). The graph is similar to Figs. 2 and 5, which are obtained from χ_2^2. The quantity χ_3^2 is thus also sensitive to the imaginary coupling and suitable as a statistics for the direct *CP*-violation search.

III. *CP*-VIOLATION SEARCH IN PRESENCE OF AMBIGUITIES OF THE PARAMETERS

A. Sensitivity to *CP*-violation effect in presence of ambiguities of the parameters

We used $N(\delta_0)$ and $\overline{N}(\delta_0)$ in the definitions of χ_2^2 and χ_3^2. Exact values of mixing angles and δm^2's are required to obtain $N(\delta_0)$ and $\overline{N}(\delta_0)$, but they are not known in practice. We discuss how the ambiguities of parameters spoil the sensitivity to the *CP* violation.

The ambiguity of parameters is especially important when we make use of *CP* conjugate oscillation channels. It is because the genuine *CP*-violation effect in this case is contaminated by the matter effect. An estimation of matter effect is required, and the ambiguity of parameters is an obstacle to the estimation. For better understanding, let us get back to $P(\nu_\mu\to\nu_e)-P(\overline{\nu}_\mu\to\overline{\nu}_e)$ in the high energy region given by Eq. (16). It consists of two parts, the matter effect part ΔP_{Matter} and the *CP*-violation effect part ΔP_{CPV}:

$$\Delta P_{\text{Matter}}\equiv 2\left(\frac{\delta m_{31}^2 L}{4E}\right)^3\frac{aL}{4E}\left\{\frac{2}{3}B\cos 2\theta_{13}+\frac{\delta m_{21}^2}{\delta m_{31}^2}\left[\frac{1}{3}j\cos\delta(2\cos 2\theta_{13}-1)-2B\cos 2\theta_{13}\sin^2\theta_{12}\right]\right\}, \tag{44}$$

$$\Delta P_{\text{CPV}}\equiv -2\left(\frac{\delta m_{31}^2 L}{4E}\right)^3\frac{\delta m_{21}^2}{\delta m_{31}^2}j\sin\delta. \tag{45}$$

There is an ambiguity in ΔP_{Matter} due to the ambiguities in δm^2's, θ's and a. A sensitivity to *CP*-violation part ΔP_{CPV} is lost if the ambiguity of ΔP_{Matter} is larger than ΔP_{CPV} itself. Ambiguity of all the parameters contributes to the ambiguity of ΔP_{Matter}. It is thus important to take into account ambiguities of all the parameters. It is expected that the ambiguity of ΔP_{Matter} is large when ΔP_{Matter} itself is large. Recalling that ΔP_{Matter} is proportional to baseline length L [due to the factor $aL/(4E)$], one should obtain a poor sensitivity in the long baseline region. It is important to consider the sensitivity to the *CP*-violation effect when the parameters are not precisely known.

We improve the discussion given in the previous section and formulate how to take ambiguities of parameters into account in estimating the sensitivity to the *CP*-violation effect. Suppose that one uses the parameters $\{\tilde{x}_i\} \equiv \{\tilde{\theta}_{12}, \tilde{\theta}_{23}, \tilde{\theta}_{13}, \delta\tilde{m}^2_{21}, \delta\tilde{m}^2_{31}, \tilde{a}\}$, which are different from the true values $\{x_i\} \equiv \{\theta_{12}, \theta_{23}, \theta_{13}, \delta m^2_{21}, \delta m^2_{31}, a\}$, to calculate $N_j(\delta_0)$ and $\overline{N}_j(\delta_0)$. One will estimate

$$\tilde{N}_j(\delta_0) = N_j(\nu_\alpha \to \nu_\beta; \{\tilde{x}_i\}, \delta_0) \tag{46}$$

instead of $N_j(\delta_0)$, where $N_j(\nu_\alpha \to \nu_\beta; \{\tilde{x}_i\}, \delta_0)$ is evaluated from Eq. (20). Then matter effect can be overestimated and hence the sensitivity to the *CP* violation can be spoiled.

First we see this using χ^2_2. In this case one will estimate the fake *CP* violation due to matter as follows:

$$\Delta\tilde{N}_j(\delta_0) = N_j(\nu_\alpha \to \nu_\beta; \{\tilde{x}_i\}, \delta_0) - N_j(\overline{\nu}_\alpha \to \overline{\nu}_\beta; \{\tilde{x}_i\}, \delta_0), \tag{47}$$

instead of $\Delta N_j(\delta_0)$. We then obtain

$$\tilde{\chi}^2_2(\delta_0) \equiv \sum_{j=1}^{n} \frac{[\Delta N_j(\delta) - \Delta\tilde{N}_j(\delta_0)]^2}{N_j(\delta) + \overline{N}_j(\delta)} \tag{48}$$

instead of $\chi^2_2(\delta_0)$ defined in Eq. (27). Adjusting the parameters $\{\tilde{x}_i\}$ within the ambiguities, one can provide a better fit to the expected values in no *CP*-violation case by minimizing χ^2. One can nevertheless infer *CP* violation is present in 90% confidence level if one cannot make the value of χ^2 smaller than $\chi^2_{90\%}$. We thus generalize Eq. (30) and define

$$\tilde{\chi}^2_2 \equiv \min_{\delta_0 \in \{0,\pi\}; \{\tilde{x}_i\}} \tilde{\chi}^2_2(\delta_0). \tag{49}$$

A criterion that *CP*-violation effect is observable for 90% confidence level in the *n*-bin method is given similar to Eq. (32) as

$$\tilde{\chi}^2_2 > \chi^2_{90\%}(n), \tag{50}$$

which can be rewritten in terms of $N_\mu M_{\text{detector}}$ as

$$N_\mu M_{\text{detector}} > (N_\mu M_{\text{detector}})_{\text{min;2,amb,90\%}} \equiv \frac{1}{C} \frac{L^2}{E^2_\mu} \frac{\chi^2_{90\%}(n)}{\min\limits_{\delta_0 \in \{0,\pi\}; \{\tilde{x}_i\}} \left\{ \sum_{j=1}^{n} \frac{[\Delta R_j(\{x_i\}, \delta) - \Delta R_j(\{\tilde{x}_i\}, \delta_0)]^2}{R_j(\{x_i\}, \delta) + \overline{R}_j(\{x_i\}, \delta)} \right\}}. \tag{51}$$

We present $(N_\mu M_{\text{detector}})_{\text{min;2,amb,90\%}}$ in Fig. 7 to observe the *CP*-violation effect at the 90% confidence level. All the parameters $\{x_i\}$ are assumed to have ambiguities of 10%, and their central values are taken as in Eqs. (35), (36), and Fig. 3, so that

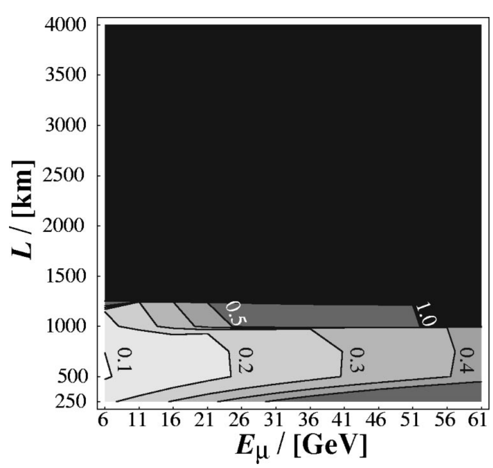

FIG. 7. A contour plot of $(N_\mu M_{\text{detector}})_{\text{min;2,amb,90\%}}$ in unit of $[10^{21} \times 100 \text{ kt}]$. The parameters are the same as in Fig. 2. The required data size is much larger than the no-ambiguity case shown in Fig. 2. The sensitivity is rapidly lost when the baseline length gets longer than about one thousand kilometers.

$$0.09 < \sin\tilde{\theta}_{13} < 0.11,$$

$$\frac{0.9}{\sqrt{2}} < \sin\tilde{\theta}_{23} < \frac{1.1}{\sqrt{2}},$$

$$0.45 < \sin\tilde{\theta}_{12} < 0.55,$$

$$2.7 \times 10^{-3}\ \text{eV}^2 < \delta\tilde{m}^2_{31} < 3.3 \times 10^{-3}\ \text{eV}^2,$$

$$0.9 \times 10^{-4}\ \text{eV}^2 < \delta\tilde{m}^2_{21} < 1.1 \times 10^{-4}\ \text{eV}^2,$$

$$0.9a < \tilde{a} < 1.1a. \tag{52}$$

It is seen in both figures that genuine *CP*-violation effect is difficult to be observed when the baseline length is longer than about one thousand kilometers. An estimation of the matter effect is obscured by the ambiguity when the baseline length is long, and the *CP* violation effect cannot be separated from matter effect. This result can be understood qualitatively by the following rough estimation. *CP*-violation effect is hidden by the ambiguity of the matter effect when the matter effect is large enough. We require

$$\left|\frac{\Delta P_{\mathrm{CPV}}}{\Delta P_{\mathrm{Matter}}}\right| \lesssim 1 \tag{53}$$

as a rough estimation to observe the *CP*-violation effect.[4] Putting Eqs. (44) and (45) into Eq. (53), one obtains a condition on L as

$$L \lesssim \frac{4E_\nu}{a}\,\frac{3(\delta m_{21}^2/\delta m_{31}^2) j \sin\delta}{2\sin^2\theta_{23}\sin^2 2\theta_{13}\cos 2\theta_{13}}. \tag{54}$$

Applying our test parameters Eqs. (35), (36), and Fig. 3 to Eq. (54), one obtains

$$L \lesssim 1250 \text{ km}, \tag{55}$$

which is consistent with Fig. 7.

Next we illustrate using χ_3^2 that the sensitivity to the *CP* violation is lost in the presence of ambiguities of parameters. The correspondent of Eq. (51) in this case is given by

$$N_\mu M_{\mathrm{detector}} > (N_\mu M_{\mathrm{detector}})_{\mathrm{min;3,amb,90\%}} \equiv \frac{1}{C}\frac{E_\mu^2}{L^2}\,\frac{\chi^2_{90\%}(n)}{\displaystyle\min_{\delta_0\in\{0,\pi\};\{\tilde{x}_i\}}\left\{\sum_{j=1}^{n}\frac{[R_j(\{\tilde{x}_i\},\delta_0)\overline{R}_j(\{x_i\},\delta)-\overline{R}_j(\{\tilde{x}_i\},\delta_0)R_j(\{x_i\},\delta)]^2}{R_j(\{\tilde{x}_i\},\delta_0)^2\overline{R}_j(\{x_i\},\delta)+\overline{R}_j(\{\tilde{x}_i\},\delta_0)^2R_j(\{x_i\},\delta)}\right\}}. \tag{56}$$

We present a plot of $(N_\mu M_{\mathrm{detector}})_{\mathrm{min;3,amb,90\%}}$ in Fig. 8. The ambiguity of parameters makes the sensitivity worse also in this case, as we see by comparing Figs. 6 and 8. It is seen, however, that the sensitivity shown in Fig. 8 is better than that shown in Fig. 7, which means that χ_3^2 avoids the ambiguity of the matter effect better than χ_2^2. One can understand the better sensitivity of χ_3^2 as a cancellation of ambiguity of $\sin\theta_{13}$ when the high energy limit applies. The dominant parts of R's are given in the high limit by

$$\begin{aligned}
R_j(\{x_i\},\delta) &= B(S+T)+V,\\
\overline{R}_j(\{x_i\},\delta) &= B(S-T)-V,\\
R_j(\{\tilde{x}_i\},\delta) &= \tilde{B}(S+T),\\
R_j(\{\tilde{x}_i\},\delta) &= \tilde{B}(S-T),
\end{aligned} \tag{57}$$

where

$$S \equiv \int_{E_{j-1}}^{E_j} E_\nu f_{\nu_\alpha}(E_\nu)\left(\frac{\delta m_{31}^2 L}{4E_\nu}\right)^2 \frac{dE_\nu}{E_\mu},$$

$$T \equiv \frac{2}{3}\cos 2\tilde{\theta}_{13}\int_{E_{j-1}}^{E_j} E_\nu f_{\nu_\alpha}(E_\nu) \times\left(\frac{\delta m_{31}^2 L}{4E_\nu}\right)^3\left(\frac{aL}{4E_\nu}\right)\frac{dE_\nu}{E_\mu}, \tag{58}$$

$$V \equiv j\sin\delta\int_{E_{j-1}}^{E_j} E_\nu f_{\nu_\alpha}(E_\nu) \times\left(\frac{\delta m_{31}^2 L}{4E_\nu}\right)^3\left(\frac{\delta m_{21}^2}{\delta m_{31}^2}\right)\frac{dE_\nu}{E_\mu}. \tag{59}$$

Only the ambiguity of $\sin\theta_{13}$ is taken into account here, and thus $\tilde{B}\equiv\sin\theta_{23}\sin^2 2\tilde{\theta}_{13}$. Using Eqs. (57), we obtain

$$\chi_3^2 \simeq \frac{(\tilde{B} j\sin\delta)^2}{2\tilde{B}^2 B} = \frac{(j\sin\delta)^2}{2B}. \tag{60}$$

Note here that $\tilde{B}$ vanishes in Eq. (60). The ambiguity of $\sin\theta_{13}$ is thus canceled away in the high energy limit. We also expect that the ambiguity does not spoil the sensitivity to the *CP*-violation effect even in the lower energy.

The sensitivity can be enhanced by a construction of a good statistics such as χ_3^2, but in general the sensitivity to the imaginary part of the coupling is smaller as the baseline length becomes longer. We confirm it for a couple of parameter sets

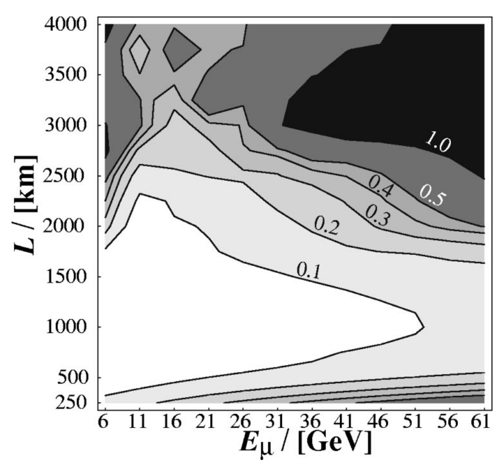

FIG. 8. A contour plot of $(N_\mu M_{\mathrm{detector}})_{\mathrm{min;3,amb,90\%}}$ in unit of $[10^{21}\times 100\ \mathrm{kt}]$. The parameters are taken to be the same as Fig. 2. Note that the sensitivity is enhanced compared to Fig. 7.

[4] One should actually use as a denominator the ambiguity of $\Delta P_{\mathrm{Matter}}$, not $\Delta P_{\mathrm{Matter}}$ itself. We tentatively use Eq. (53) to give a rough estimation, however.

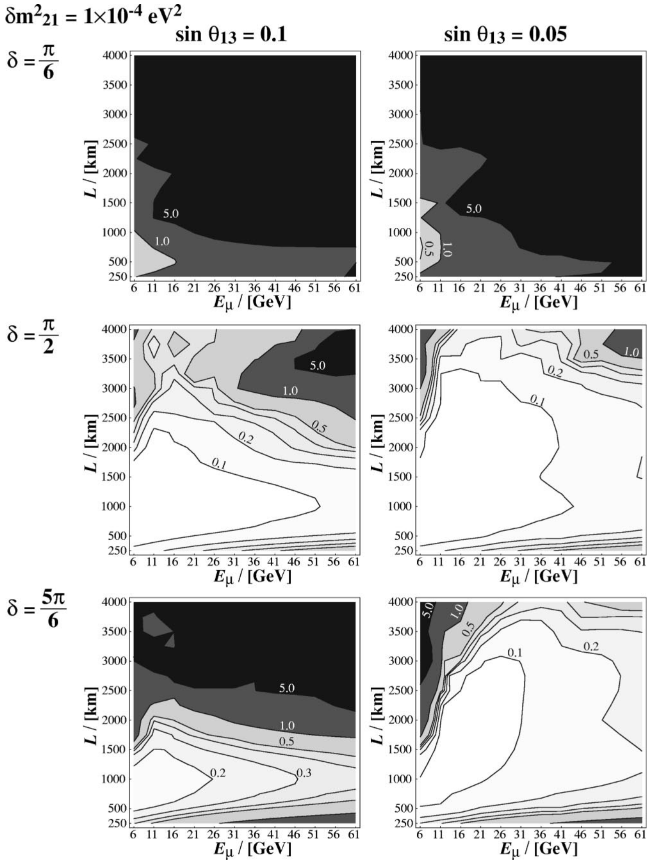

FIG. 9. Same as Fig. 8, but for different parameters. All the graphs presented here are for $\delta m^2_{21}=1\times10^{-4}$ eV^2. The graphs in left column are for $\sin\theta_{13}=0.1$ while the ones in right column are for $\sin\theta_{13}=0.05$. The top two graphs are for $\delta=\pi/6$, the second two graphs are for $\delta=\pi/2$, and the bottom two graphs are for $\delta=5\pi/2$. Parameters not presented here are taken to be same as Fig. 2. The difference of the sensitivity for $\delta=\pi/6$ and for $\delta=5\pi/6$ is due to the difference of matter effect.

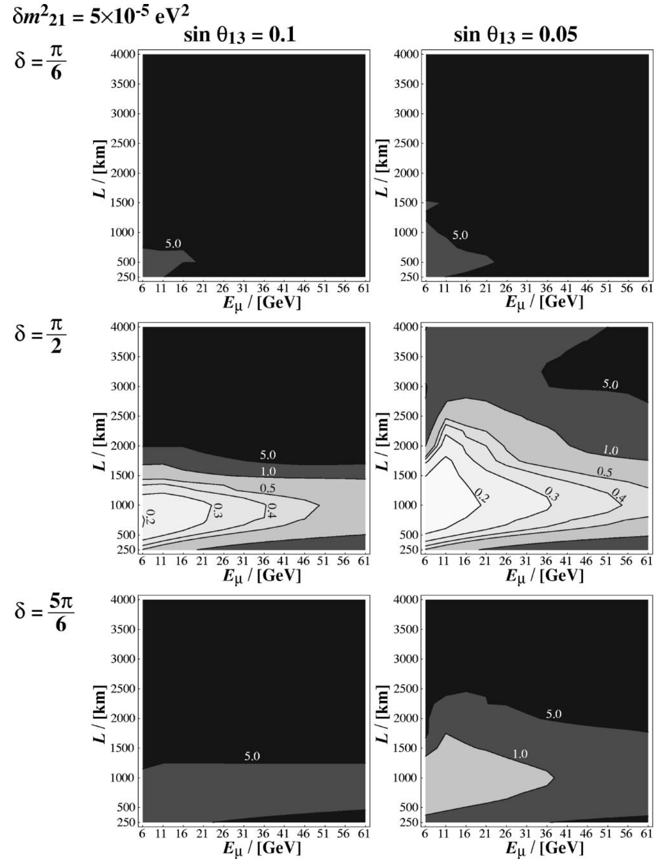

FIG. 10. Same as Fig. 9, but for $\delta m^2_{21}=5\times10^{-5}$ eV^2.

by presenting Figs. 9 and 10. It is also seen in Fig. 9 that the sensitivity for $\delta=\pi/6$ and for $\delta=5\pi/6$ is quite different. The genuine *CP* violation has a same magnitude for both of these two cases. On the other hand, the term proportional to $\cos\delta$, which is contained in the matter effect term [see, e.g., Eq. (44)], has an opposite sign. The magnitude of the matter effect contamination is thus different, and it leads to the difference of the sensitivity according to our discussion that the sensitivity to *CP*-violation effect is controlled by the magnitude of the matter effect.

We have been discussing the direct observation of *CP*-violating effect. One can verify our results by use of a quantity similar to χ^2_1 defined in Eq. (24). Equation (24) focuses on the difference between $N(\nu_\alpha\to\nu_\beta;\delta)$ and $N(\nu_\alpha\to\nu_\beta;\delta_0)$, and χ^2_1 has little sensitivity to the genuine *CP*-violation effect as a result. Instead we define

$$\chi^2_{1\mathrm{asym}}(\delta_0)\equiv\sum_{j=1}^{n}\frac{[N_j(\delta)-N_j(-\delta)]^2}{N_j(\delta)}+\sum_{j=1}^{n}\frac{[\overline{N}_j(\delta)-\overline{N}_j(-\delta)]^2}{\overline{N}_j(\delta)}$$
$$=N_\mu M_{\mathrm{detector}}\frac{E_\mu^2}{L^2}C\left\{\sum_{j=1}^{n}\frac{[R_j(\delta)-R_j(-\delta)]^2}{R_j(\delta)}+\sum_{j=1}^{n}\frac{[\overline{R}_j(\delta)-\overline{R}_j(-\delta)]^2}{\overline{R}_j(\delta)}\right\},\tag{61}$$

which is sensitive to the genuine *CP*-violation effect [34].[5] A quantity analogous to $(N_\mu M_{\mathrm{detector}})_{\mathrm{min;1,amb,90\%}}$ is defined by

$$(N_\mu M_{\mathrm{detector}})_{\mathrm{min;1asym,amb,90\%}}\equiv\frac{1}{C}\frac{L^2}{E_\mu^2}\frac{\chi^2_{90\%}(2n)}{\displaystyle\min_{\{\tilde{x}_i\}}\left\{\sum_{j=1}^{n}\frac{[R_j(\{x_i\},\delta)-R_j(\{\tilde{x}_i\},-\delta)]^2}{R_j(\{x_i\},\delta)}+\sum_{j=1}^{n}\frac{[\overline{R}_j(\{x_i\},\delta)-\overline{R}_j(\{\tilde{x}_i\},-\delta)]^2}{\overline{R}_j(\{x_i\},\delta)}\right\}}.\tag{62}$$

[5]This quantity requires both *CP*- and *T*-conjugate channels; we consider this quantity just to verify the discussions so far.

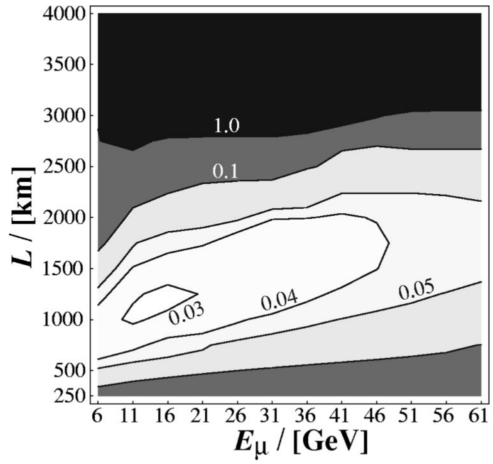

FIG. 11. A contour plot for $(N_\mu M_{\mathrm{detector}})_{\mathrm{min;1asym,amb,90\%}}$ defined by Eq. (62). The sensitivity is lost also when the baseline length is longer than about 2750 km.

Figure 11 shows a contour plot of $(N_\mu M_{\mathrm{detector}})_{\mathrm{min;1asym,amb,90\%}}$. It is seen that the sensitivity is lost when the baseline length is longer than about two thousand kilometers, which is qualitatively consistent to the results obtained in this section.

B. Energy dependence

We discuss the binning of the neutrino energy in a search of *CP* violation. We recall that the genuine *CP* violation in terms of oscillation probability is given by

$$J \sin\frac{\delta m_{21}^2 L}{4E_\nu} \sin\frac{\delta m_{32}^2 L}{4E_\nu} \sin\frac{\delta m_{13}^2 L}{4E_\nu}. \tag{63}$$

Applying $\delta m_{21}^2 \ll \delta m_{31}^2$ and $\delta m_{21}^2 L/(4E_\nu) \ll 1$, Eq. (63) is rewritten to be

$$-J\frac{\delta m_{21}^2 L}{4E_\nu}\sin^2\frac{\delta m_{31}^2 L}{4E_\nu}. \tag{64}$$

It can be seen from Eq. (64) that the genuine *CP*-violation effect has a definite sign as a function of E_ν. It is also applicable to the event rate $N(\delta)$. Dividing the event rates into energy bins is thus meaningless and unnecessary to search for the *CP*-violation effect, when the matter effect is absent. All one need do is to observe the total counts of neutrinos. This is of practical importance for experimental studies since the determination of neutrino energy is very challenging.

On the other hand, a single-bin analysis does not necessarily remain advantageous once the matter effect is taken into account; the multibin analysis is required to remove the matter in such a case that the considered χ^2 is sensitive not only to genuine *CP*-violation effect but to matter effect (see Fig. 12). The number of events per bin is sacrificed by the bin dividing, and each bin has a relatively small number of events compared to the single-bin analysis. As a result, the best-fit point of multibin analysis is less robust than the single-bin analysis, i.e., the best fit point of multibin analysis easily lies far away from the true parameter point.

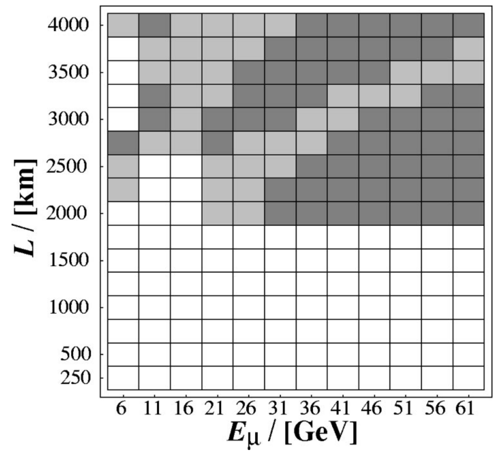

FIG. 12. The most effective binning method to observe the *CP* violation effect with χ_3^2. This corresponds to Fig. 8. Single-, 3-, 5-bin analyses are compared. White, light gray, and dark gray regions mean where single-, 3-, 5-bin analyses are most effective, respectively. This shows that multibin analysis is required in long baseline range.

We conclude as follows from the above considerations. Experiments to search for the *CP*-violation effect should be made with the setup where the single-bin analysis gives the best sensitivity, which means that the matter effect contamination is small. Such a setup has another practical advantage in addition: measurements of the neutrino energy are not required in single-bin analyses. One need not take care of the correlation between bins due to finite energy resolution, which makes the sensitivity to the *CP* violation in the experiment worse.

C. Dependence on sin θ_{13} of the sensitivity to the *CP*-violation effect

We finally discuss the correlation between parameters on the sensitivity to the direct *CP*-violation search.

The magnitude of *CP* violation is determined a single parameter, namely the Jarlskog parameter $j \sin\delta$. The correlation between δ and other parameters such as θ_{13} is often discussed, but it is heavily dependent on the parametrization. The presence or absence of *CP* or *T* violation can be determined without any correlations to the mixing angles.[6]

We present Figs. 13 and 14 to show the correlation between $\sin\theta_{13}$ and δ. They are sensitivity plots using χ_3^2, where δ and $\sin\theta_{13}$ are varied while E and L are fixed. Figure 13 is a test plot for $E=10$ GeV and $L=1000$ km, and Fig. 14 is for $E=20$ GeV and $L=2000$ km. A direct *CP*-violation search is expected to be possible with this setup, as seen in

[6]The real part of coupling is in fact another intrinsic parameter which is independent of the parametrization. The correlation between the real part of the coupling and the imaginary part will be present. This is the only possible correlation for the *CP*-violation effect.

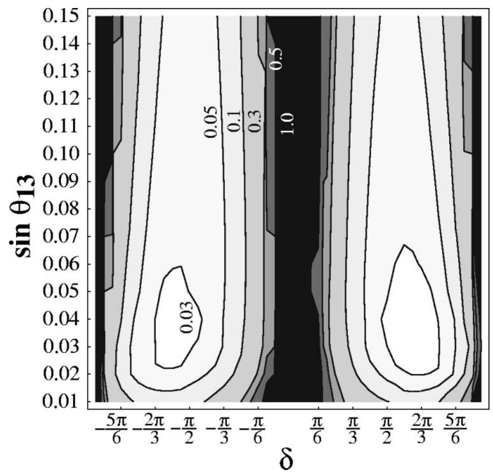

FIG. 13. A similar plot to Fig. 7, but this time $E_\mu = 10$ GeV and $L = 1000$ km are fixed while $\sin\theta_{13}$ and δ are varied. The contour is nearly vertical, which reflects the fact that the value of $\sin\theta_{13}$ is not important to consider the sensitivity to *CP*-violation searches.

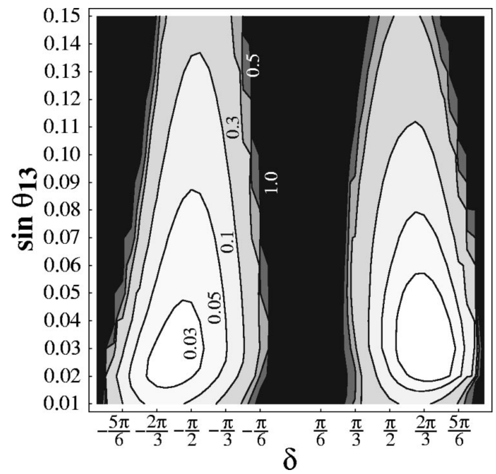

FIG. 14. A similar plot to Fig. 13, but this time $E_\mu = 20$ GeV and $L = 2000$ km. Matter effect is larger in this parameter compared to Fig. 13, which leads to the dependence of the sensitivity upon $\sin\theta_{13}$.

Figs. 9 and 10. It is expected in these figures that the sensitivity scarcely depends on θ_{13} if the statistics is correctly sensitive to the genuine *CP*-violation effect. It is illustrated by a rough estimation of χ_3^2 in such a case:

$$\chi_3^2 \sim \frac{(\Delta P_{\rm CPV})^2}{P(\nu_\mu \to \nu_e)}$$

$$\sim \frac{L^2}{E} \frac{[(\delta m_{21}^2/\delta m_{31}^2) j \sin\delta]^2}{B}$$

$$= \frac{L^2}{E} \left(\frac{\delta m_{21}^2}{\delta m_{31}^2}\right)^2 \frac{(\sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos\theta_{13})^2}{\sin^2\theta_{23} \sin^2 2\theta_{13}}$$

$$\sim \frac{L^2}{E} \left(\frac{\delta m_{21}^2}{\delta m_{31}^2}\right)^2 \sin^2 2\theta_{12} \cos^2\theta_{23} \cos^2\theta_{13}. \tag{65}$$

Equation (65) depends on θ_{13} only through $\cos\theta_{13}$, which is almost unity for $\theta_{13} \ll 1$. The dependence on θ_{13} should be thus quite small if the statistics is sensitive to $j\sin\delta$. We compare Fig. 13 with Fig. 14 to confirm the above discussion. It can be seen in Fig. 8 that the parameter for Fig. 13 is more sensitive than that for Fig. 14. We see that the dependence of the sensitivity upon $\sin\theta_{13}$ in Fig. 13 is quite small, while the dependence is larger in Fig. 14. This is indeed consistent with the above discussion; the larger matter effect gives a sizable contribution to the numerator of χ_3^2 in the latter case, and the estimation given in Eq. (65) does not apply. We thus conclude that $L \sim 1000$ km and $E \sim 10$ GeV is the optimum setup to search for a direct *CP*-violation search with use of the statistics given by Eq. (41).[7]

IV. *T*-VIOLATION SEARCH IN PRESENCE OF AMBIGUITIES OF PARAMETERS

We have discussed in the previous section that the ambiguity of matter effect spoils the sensitivity to the *CP*-violation effect. One can then expect that one can avoid the loss of sensitivity by use of *T*-conjugate oscillation channels, which is free from the matter effect [35].

It is convenient to redefine

$$N_j(\{x_i\},\delta) \equiv N(\nu_\alpha \to \nu_\beta; \{x_i\},\delta) \tag{66}$$

and

$$\overline{N_j}(\{x_i\},\delta) \equiv N(\nu_\beta \to \nu_\alpha; \{x_i\},\delta), \tag{67}$$

where $\alpha, \beta = e, \mu, \tau$, and ν denotes neutrinos and antineutrinos collectively. We are to consider initial neutrinos of different flavors, and we must take into account the difference of the energy spectra of the neutrino beam. For example, the flux of ν_e and ν_μ obtained from the decay of muons with energy E_μ is given in terms of $x \equiv E_\nu / E_\mu$ by

$$f_{\nu_e}(x) = 12x^2(1-x) \tag{68}$$

and

$$f_{\nu_\mu}(x) = 2x^2(3-2x), \tag{69}$$

respectively. The quantity χ_3^2 defined by Eq. (41) is suitable in such a case. We define $(N_\mu M_{\rm detector})_{\rm min;T,amb,90\%}$ in the same manner as the right-most side of Eq. (56) so that

[7] An optimum setup should change if one can find other better statistics, since the sensitivity itself depends on the adopted statistics. The difference of Figs. 7 and 8 is an example.

$$(N_\mu M_{\mathrm{detector}})_{\mathrm{min;T,amb},90\%} \equiv \frac{1}{C}\frac{E_\mu^2}{L^2}\frac{\chi^2_{90\%}(n)}{\min\limits_{\delta_0\in\{0,\pi\};\{\tilde{x}_i\}}\left\{\sum_{j=1}^{n}\frac{[R_j(\{\tilde{x}_i\},\delta_0)\overline{R}_j(\{x_i\},\delta)-\overline{R}_j(\{\tilde{x}_i\},\delta_0)R_j(\{x_i\},\delta)]^2}{R_j(\{\tilde{x}_i\},\delta_0)^2\overline{R}_j(\{x_i\},\delta)+\overline{R}_j(\{\tilde{x}_i\},\delta_0)^2R_j(\{x_i\},\delta)}\right\}}, \tag{70}$$

while this time $R_j(\delta)=R_j(\nu_\alpha\to\nu_\beta;\delta)$ and $\overline{R}_j(\delta)=R_j(\nu_\beta\to\nu_\alpha;\delta)$. T-violation effect is considered to be observable when

$$N_\mu M_{\mathrm{detector}} > (N_\mu M_{\mathrm{detector}})_{\mathrm{min;T,amb},90\%} \tag{71}$$

is satisfied.

We present in Fig. 15 a test plot of $(N_\mu M_{\mathrm{detector}})_{\mathrm{min;T,amb},90\%}$. The parameters are taken as shown in Eq. (52). It can be seen that Fig. 15 is qualitatively similar to a plot in absence of the ambiguity of parameters, which is presented in Fig. 6. A sweet spot, which is expected from the naive estimation in terms of oscillation probability, still remains in Fig. 15; thus we find that the *CP*- or *T*-violation search via *T*-conjugate channels is robust to the ambiguity of the parameters.

Longer baseline length is in general preferable when *T*-conjugate channels are available, since Eq. (65) applies without being troubled with matter effect contamination. This is in contrast to the *CP*-conjugate case, where the matter effect obscures the genuine *CP*-violation effect when the baseline gets too long.

V. SUMMARY AND DISCUSSION

We discussed the optimum experimental setup and the optimum analysis to see the *CP*-violation effect.

We first discussed the difference between the direct measurement and the indirect measurement [29]. Genuine *CP*-violation effect is characterized by an imaginary part of couplings in the Lagrangian and hence quantities sensitive to this imaginary part should be used to measure the *CP* violation. To see this we introduced two statistical quantities, χ^2_1 [Eq. (29)] and χ^2_2 [Eq. (30)].

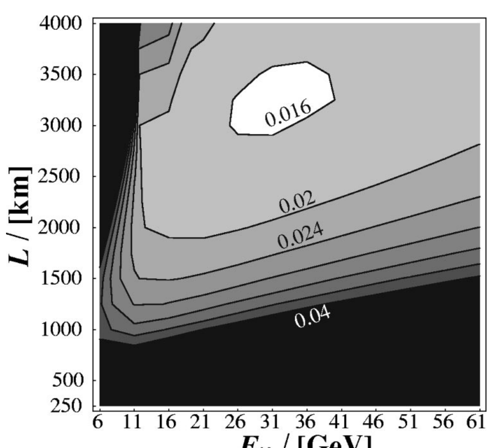

FIG. 15. A contour plot of the required data size to observe the *T*-violation effect. A quantity $(N_\mu M_{\mathrm{detector}})_{\mathrm{min;T,amb},90\%}$ defined in Eq. (70) is plotted in unit of $[10^{21}\times 100\ \mathrm{kt}]$. The parameters are the same as in Fig. 2. This figure is similar to Fig. 6, and it is seen that the use of *T*-conjugate channels is robust to the ambiguities of parameters.

Usually χ^2_1 is used in analyses of neutrino factories. We can test using this whether the data can be explained by the hypothetical data calculated assuming no *CP*-violation effect. We saw, however, that this quantity is not necessarily sensitive to the *CP*-violation part of the coupling. In the high energy region it is sensitive almost only to the *CP*-conserved part of the oscillation probability. We can tell about genuine *CP*-violation effect only through unitarity relation of three generations. The sensitivity to the *CP*-violation effect is often indirect. Thus we concluded that we should not use it to measure *CP* violation since it often measures the *CP*-violation effect only indirectly.

On the other hand, we can test with χ^2_2 whether the asymmetry of oscillation probabilities of neutrinos and antineutrinos exists. We have seen that χ^2_2 is sensitive to the *CP*-violating part of the oscillation probability, and thus it is a more suitable quantity to measure the *CP*-violation effect.

We saw the relation between χ^2_1 and χ^2_2 and found that to pick up an imaginary part of couplings we need to see the difference between particle and antiparticle. In this sense we also introduced a statistics χ^2_3 [Eq. (41)]. This statistics gives better sensitivity to measure the *CP*-violation effect directly when we consider the ambiguities of the theoretical parameters.

Then we investigated the influence of the ambiguities of the theoretical parameters on χ^2_2 and χ^2_3. Since the matter effect causes the difference between the oscillation probabilities of neutrinos and antineutrinos, we have to estimate fake asymmetry to search for the *CP*-violation effect. However, we will always "overestimate" the fake *CP* violation because of the ambiguity of the theoretical parameters, and hence we will always estimate the genuine *CP*-violation effect too small. The matter effect increases as baseline length increases, and we will lose the sensitivity to the asymmetry due to the genuine *CP*-violation effect in longer baseline such as several thousand km.

The sensitivity of χ^2_2 to genuine *CP* violation is lost much more than that of χ^2_3. This is due to the partial cancellation of the ambiguity by $\sin\theta_{13}$. The ambiguity of estimation of matter effect is partially canceled in the numerator. We found that χ^2_3 is better statistics to see *CP*-violation effect directly.

Using χ^2_3 we studied the correlation between $\sin\theta_{13}$ and δ. Comparing Figs. 13 and 14, we found that in general that we have better sensitivity to *CP* violation with baseline length 1000 km than 2000 km. Moreover, if the statistics is only sensitive to the imaginary part of the couplings, the Jarlskog parameter J, it has no dependence on $\sin\theta_{13}$.[8] Indeed in Fig. 13 we see this behavior while in Fig. 14 we see strong dependence on $\sin\theta_{13}$ of the sensitivity. In this sense we also understand that baseline length 1000 km is better to see CP violation directly. Furthermore, Fig. 9 and Fig. 10 show that around 1000 km is the optimal baseline length for various parameter sets.

Taking the statistics which is sensitive to the imaginary part of the lepton couplings, we first showed that there is a sweet spot in terms of E_μ and L when the ambiguities of the parameters are not considered. We have then taken the ambiguities of all the parameters to be 10% and showed that the sweet spot survives in such a case. We expect that the sweet spot also survives when we adopt the more realistic values of the ambiguities. We optimistically expect that other parameters will be determined with ambiguities less than 10% except for θ_{13} in the future. The large ambiguity of θ_{13} is seemingly enough to wash out the sweet spot. We have mentioned in Sec. III A, however, that the ambiguity of θ_{13} is canceled by use of the statistics χ_3^2. We thus conclude that the experimental setup of $E_\mu\sim 10$ GeV and $L\sim 1000$ km is desirable even in the real experiment.

We finally studied T asymmetry. There is no fake asymmetry due to environmental effects such as the matter effect. We found that the naive expectation on CP-violation phenomena is indeed realized.

It is required to find another way to see CP-violation effects if we can observe only appearance events of $\nu_e\to\nu_\mu$ and $\overline{\nu}_e\to\overline{\nu}_\mu$. Otherwise we cannot observe the CP-violation effect in neutrino factories with long baseline ($\geqslant$1000 km) as the asymmetry between neutrinos and antineutrinos. On the other hand, we can observe the CP-violation effect as the T asymmetry very well. Therefore it is very important to establish a way to observe this asymmetry experimentally.

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APPENDIX: STATISTICS

We explain a detail of the statistics used in this paper to estimate how many events we need to tell the existence of the genuine CP-violation effect. To state the feasibility of the experiment we consider not only how well we can distinguish two theories (two parameter sets) but also how well the best fit point lie in the true value. For example, even if in nature $\delta=\pi/2$ is realized, we are not sure that the best fit point for δ sit there. We will have the best fit point value other than $\delta=\pi/2$ and hence we have to take care of this possibility to state the feasibility of the experiment.

To estimate it, we employed the concept of the "power of test." In the test, we set up "null hypothesis," H_0, which should be rejected and its "alternative hypothesis" against H_0,H_1. In this paper we were interested in whether we can insist on the existence of the CP-violation effect, and hence we set the null hypothesis,

$$H_0:\quad \delta=\delta_0 \tag{A1}$$

and "alternative hypothesis" against H_0,

$$H_1:\quad \delta\neq\delta_0. \tag{A2}$$

We also define "test statistics" to give a criterion to reject H_0 for a real data set $N_i^{\rm ex}$. In this paper we examined three test statistics corresponding to χ_1^2, χ_2^2, and χ_3^2:

$$T_1(\delta_0)\equiv\sum_i^n \frac{[N_i^{\rm ex}-N_i^{\rm th}(\delta_0)]^2}{N_i^{\rm th}(\delta_0)}+\sum_i^n \frac{[\overline{N}_i^{\rm ex}-\overline{N}_i^{\rm th}(\delta_0)]^2}{\overline{N}_i^{\rm th}(\delta_0)},$$

$$T_1\equiv\min_{\delta_0\in\{0,\pi\}} T_1(\delta_0), \tag{A3}$$

$$T_2(\delta_0)\equiv\sum_i^n \frac{[\{N_i^{\rm ex}-\overline{N}_i^{\rm ex}\}-\{N_i^{\rm th}(\delta_0)-\overline{N}_i^{\rm th}(\delta_0)\}]^2}{N_i^{\rm th}(\delta_0)+\overline{N}_i^{\rm th}(\delta_0)},$$

$$T_2\equiv\min_{\delta_0\in\{0,\pi\}} T_2(\delta_0), \tag{A4}$$

$$T_3(\delta_0)\equiv\sum_i^n \frac{[\overline{N}_i^{\rm th}(\delta_0)\times N_i^{\rm ex}-N_i^{\rm th}(\delta_0)\times\overline{N}_i^{\rm ex}]^2}{\{\overline{N}_i^{\rm th}(\delta_0)\}^2N_i^{\rm ex}+\{N_i^{\rm th}(\delta_0)\}^2\overline{N}_i^{\rm ex}},$$

$$T_3\equiv\min_{\delta_0\in\{0,\pi\}} T_3(\delta_0), \tag{A5}$$

where $N_i^{\rm th}$ is the event number assumed by the theory with δ. Hereafter we use an example χ_1^2 for the explanation. Furthermore for simplicity we abbreviate T_1 (and accordingly χ_1^2) as

$$T_1=\sum_i^n \frac{[N_i^{\rm ex}-N_i^{\rm th}(\delta_0)]^2}{N_i^{\rm th}(\delta_0)}. \tag{A6}$$

To reject H_0 at α "level of significance," we require

$$T_1>\chi_\alpha^2(2n). \tag{A7}$$

Then the question is how well the inequality (A7) is satisfied for a given value δ. This probability is called "power"

$$\beta_1(\delta)\equiv P_\delta(T_1>\chi_\alpha^2(2n)). \tag{A8}$$

This is the probability that we succeed in seeing the CP-violation effect in the experiment. Thus we have to require that this probability should be larger than γ, which is almost 1.

To estimate the probability, often we generate event sets with a given event rate and check whether H_0 is indeed rejected according to the inequality (A7) with the probability

[8]As long as the $\sin\theta_{13}$ term has a dominant contribution.

γ.[9] Instead of doing so, here we make the following approximation. First, we approximate T_1 as

$$T_1=\sum_i^n \frac{[N_i^{\mathrm{ex}}-N_i^{\mathrm{th}}(\delta_B)]^2}{N_i^{\mathrm{th}}(\delta_B)}+\sum_i^n \frac{[N_i^{\mathrm{th}}(\delta_B)-N_i^{\mathrm{th}}(\delta_0)]^2}{N_i^{\mathrm{th}}(\delta_B)}, \tag{A9}$$

where $N_i^{\mathrm{th}}(\delta_B)$ is "the maximum likelihood estimator," i.e.,

$$\sum_i^n \frac{[N_i^{\mathrm{ex}}-N_i^{\mathrm{th}}(\delta_B)]^2}{N_i^{\mathrm{th}}(\delta_B)}\leqslant\sum_i^n \frac{[N_i^{\mathrm{ex}}-N_i^{\mathrm{th}}(\delta)]^2}{N_i^{\mathrm{th}}(\delta)}, \tag{A10}$$

for all δ. Equation (A9) holds well if $|N_i^{\mathrm{ex}}-N_i^{\mathrm{th}}(\delta_B)|\leqslant O(\sqrt{N_i^{\mathrm{th}}(\delta_B)})$, i.e., the fit for the data N_i^{ex} by $N_i^{\mathrm{th}}(\delta_B)$ is good enough, and N_i^{th} does not vary so rapidly around δ_B. We also assume that the estimator is almost the true value, i.e. $\delta_B\simeq\delta$. Thus

$$\begin{aligned} T_1&=\sum_i^n \frac{[N_i^{\mathrm{ex}}-N_i^{\mathrm{th}}(\delta)]^2}{N_i^{\mathrm{th}}(\delta)}+\sum_i^n \frac{[N_i^{\mathrm{th}}(\delta)-N_i^{\mathrm{th}}(\delta_0)]^2}{N_i^{\mathrm{th}}(\delta)}\\ &=\sum_i^n \frac{[N_i^{\mathrm{ex}}-N_i^{\mathrm{th}}(\delta)]^2}{N_i^{\mathrm{th}}(\delta)}+\chi_1^2. \end{aligned} \tag{A11}$$

With this approximation we calculate the power (A8) as follows:

$$\beta_1(\delta)=P_\delta\left(\sum_i^n \frac{[N_i^{\mathrm{ex}}-N_i^{\mathrm{th}}(\delta)]^2}{N_i^{\mathrm{th}}(\delta)}>\chi_\alpha^2(2n)-\chi_1^2\right). \tag{A12}$$

The left-hand side in parentheses of P_δ follows the χ^2 distribution with $2n$ degree of freedom so the requirement that the power $\beta_1(\delta)$ should be larger than γ is equivalent to the condition

$$\chi_1^2>\chi_\alpha^2(2n)-\chi_\gamma^2(2n-f), \tag{A13}$$

where f means the number of parameters included in the theory. For example, if we take the 0.1 level of the significance and require the power to be 0.99 level, then

$$\chi_1^2\geqslant\chi_{0.1}^2(2n)-\chi_{0.99}^2(2n-f). \tag{A14}$$

Since in general if $\gamma\simeq1$ then $\chi_\gamma^2(2n-f)$ is very small for small n, it is omitted in this paper.[10] Thus we required[11]

$$\chi_1^2\geqslant\chi_\alpha^2(2n). \tag{A15}$$

[9]We have to generate enough event sets to conclude that H_0 is rejected with the probability γ.

[10]In other words we required the perfect power, i.e., $\gamma=1$.

[11]Since the significance level α corresponds naively to the confidence level $(1-\alpha)\times100\%$, we denote $\chi_\alpha^2(n)$ by $\chi^2_{(1-\alpha)\times100\%}(n)$ in this paper.

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