Possible flavor mixing structures of lepton mass matrices

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To search for possible textures of lepton mass matrices, we systematically examine flavor mixing structures which can lead to large lepton mixing angles. We find out 37 mixing patterns are consistent with experimental data, taking into account phase factors in the mixing matrices. Only six of the patterns can explain the observed data without any tuning of parameters, while the others need particular choices for the phase values. It is found that these six mixing patterns are those predicted by the models which have been proposed to account for fermion mass hierarchies. On the other hand, the others may give new flavor mixing structures of lepton mass matrices and therefore new possibilities of model construction.

I. INTRODUCTION

The Super-Kamiokande experiment has confirmed neutrino oscillations in the atmospheric neutrinos, which favors the $\nu_\mu \to \nu_\tau$ process with a large mixing angle $\sin^2 2\theta_{atm} \geq 0.88$ and mass-squared difference $\Delta m^2_{atm} = (1.6-4) \times 10^{-3}$ eV^2 [1]. On the other hand, for the solar neutrino problem [3], the recent data of Super-Kamiokande seem to favor the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution [2], but four solutions are still experimentally allowed: small mixing angle (SMA) MSW [4], LMA-MSW, low Δm^2 (LOW), and vacuum oscillation (VO) solutions [5]. As a result, the neutrino mixing matrix [Maki-Nakagawa-Sakata (MNS) matrix [6]] has two possibilities: one is the matrix with single maximal mixing, which gives the SMA-MSW solution for the solar neutrino problem, and the other with bimaximal mixing [7], which corresponds to the LMA-MSW, LOW, and VO solutions.

Assuming that the neutrino oscillations account for the solar and atmospheric neutrino data, one can consider the prototypes of the MNS mixing matrix U_{MNS} which are written as

$$U_{MNS} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ 0 & -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{pmatrix}, \quad \frac{\Delta m^2_\odot}{\Delta m^2_{atm}} \simeq 10^{-2}, \quad (1)$$

for single maximal mixing, and

$$U_{MNS} \simeq \begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \\ -\dfrac{1}{2} & \dfrac{1}{2} & \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{2} & -\dfrac{1}{2} & \dfrac{1}{\sqrt{2}} \end{pmatrix},$$

$$\frac{\Delta m^2_\odot}{\Delta m^2_{atm}} \simeq \begin{cases} 10^{-2} & \text{(LMA-MSW)}, \\ 10^{-4} & \text{(LOW)}, \\ 10^{-6} & \text{(VO)} \end{cases} \quad (2)$$

for bimaximal mixing. Here $\Delta m^2_\odot$ is the mass-squared difference relevant to the solar neutrino problem.

To clarify the origins of these nearly maximal mixings is one of the most important issues in flavor physics. In constructing the models for fermion masses and mixing, there are some preferred bases given by underlying theories, such as grand unified theories. For the MNS matrix in Eq. (1), the maximal mixing angle may follow from the charged-lepton

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mass matrix, the neutrino mass matrix, or both of them, depending on the models under consideration. In the case of bi-maximal mixing in Eq. (2), the situation is more nontrivial. It is therefore important in light of model construction to search for possible mixing patterns of charged leptons and neutrinos. In this paper, we systematically investigate the mixing patterns where at least one of the mixing matrices has sources of maximal mixing. As we will see, our analyses are independent of particular structures of lepton mass matrices and hence of the mass spectrum of neutrinos. The results are also not concerned with whether the neutrinos are Majorana or Dirac particles, in other words, whether the right-handed neutrinos exist or not. Based on our results, we discuss new possibilities of the forms of lepton mass matrices, which may account for the experimental data.

In Sec. II, we discuss the mixing patterns of charged leptons and neutrinos, and classify them in light of the phenomenological constraints from Super-Kamiokande and long baseline neutrino experiments. In Sec. III, we show several examples of mass matrices of charged leptons and neutrinos that can give the allowed mixing patterns obtained in Sec. II. Sec. IV is devoted to summary and discussions.

II. PHENOMENOLOGY OF MIXING MATRICES

In this section, we study possible flavor mixing structures of leptons, which can lead to large mixing angles. Given the charged-lepton and neutrino mass matrices, the MNS mixing matrix is defined as

$$U_{\text{MNS}} = V_E^\dagger V_\nu, \tag{3}$$

where the 3×3 matrix V's are the mixing matrices which rotate the left-handed fields so that the mass matrices are diagonalized. The matrices V_E and V_ν are generally parametrized as follows:

$$V_E = P U(23) P' U(13) U(12) P'',$$

$$V_\nu = \bar{P} \bar{U}(23) \bar{P}' \bar{U}(13) \bar{U}(12) \bar{P}''. \tag{4}$$

Here $U(ij)$ are the rotation matrices,

$$U(23) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix},$$

$$U(13) = \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix},$$

$$U(12) = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$, and the P's are the phase matrices; $P = \text{diag}(1, e^{ia}, e^{ib})$, $P' = \text{diag}(1, 1, e^{i\delta})$, and $P'' = \text{diag}(e^{ip}, e^{iq}, e^{ir})$. The matrices with overbars, $\bar{U}(ij)$, $\bar{P}$,

$\bar{P}'$, and $\bar{P}''$ in the neutrino side take the same parametrizations as above. In fact, the phase factors in P'' are physically irrelevant in that they can be absorbed with a redefinition of charged-lepton fields. For the phases in $\bar{P}''$, the same prescription can be done in the case of Dirac neutrinos, while for Majorana neutrinos these phases cannot be absorbed into the neutrino fields, and remain physical. Note however that they are irrelevant to the values of mixing angles and hence we can safely drop the phase matrices P'' and $\bar{P}''$ in the following analyses. Now, the MNS matrix is given by

$$U_{\text{MNS}} = U_E^\dagger Q U_\nu, \tag{6}$$

where

$$U_E = U(23) P' U(13) U(12), \quad U_\nu = \bar{U}(23) \bar{P}' \bar{U}(13) \bar{U}(12),$$

$$Q = P^* \bar{P} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}. \tag{7}$$

In U_{MNS}, there are four phase parameters to be considered: α, β, δ_E, and δ_ν. As will be seen below, in our analysis, the phase factors in the matrix Q sometimes play important roles to have phenomenologically viable mixing angles. The mixing matrices $U(ij)$ and $\bar{U}(ij)$ are fixed when the mass matrices of charged leptons and neutrinos are given in a concrete model. On the other hand, from the view of mixing angles, there are six mixing parameters in U_E and U_ν, and it is meaningful to raise a query about which angles are responsible for the observed maximal mixings in U_{MNS}. In order to study this, we phenomenologically analyze the mixing structures of lepton flavor without referring to specific models.

In the first approximation, we assume that mixing angles are zero or large, and examine possible combinations of U_E and U_ν referring to the indications of Super-Kamiokande and long baseline neutrino experiments. Let us consider the following nine types of mixing matrices for U_E and U_ν. The first three types of matrices are given by taking one of the mixing angles as maximal and the others as zero:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ 0 & -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{pmatrix}, \quad \begin{matrix} s_{12} = 0, \\ s_{13} = 0, \\ s_{23} = 1/\sqrt{2}, \end{matrix} \tag{8}$$

$$S = \begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \\ -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \begin{matrix} s_{12} = 1/\sqrt{2}, \\ s_{13} = 0, \\ s_{23} = 0, \end{matrix} \tag{9}$$

$$L = \begin{pmatrix} \dfrac{1}{\sqrt{2}} & 0 & \dfrac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\dfrac{1}{\sqrt{2}} & 0 & \dfrac{1}{\sqrt{2}} \end{pmatrix}, \quad \begin{array}{l} s_{12}=0, \\ s_{13}=1\sqrt{2}, \\ s_{23}=0, \end{array} \tag{10}$$

where we have used the notations A, S, and L for three types of mixing matrices, respectively. The second of three types are described by the matrices with one of the mixing angles being zero and the others being maximal:

$$B = \begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \\ -\dfrac{1}{2} & \dfrac{1}{2} & \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{2} & -\dfrac{1}{2} & \dfrac{1}{\sqrt{2}} \end{pmatrix}, \quad \begin{array}{l} s_{12}=1/\sqrt{2}, \\ s_{13}=0, \\ s_{23}=1/\sqrt{2}, \end{array} \tag{11}$$

$$H = \begin{pmatrix} \dfrac{1}{2} & \dfrac{1}{2} & \dfrac{1}{\sqrt{2}} \\ -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} & 0 \\ -\dfrac{1}{2} & -\dfrac{1}{2} & \dfrac{1}{\sqrt{2}} \end{pmatrix}, \quad \begin{array}{l} s_{12}=1/\sqrt{2}, \\ s_{13}=1/\sqrt{2}, \\ s_{23}=0, \end{array} \tag{12}$$

$$N = \begin{pmatrix} \dfrac{1}{\sqrt{2}} & 0 & \dfrac{1}{\sqrt{2}} \\ -\dfrac{1}{2} & \dfrac{1}{\sqrt{2}} & \dfrac{1}{2} \\ -\dfrac{1}{2} & -\dfrac{1}{\sqrt{2}} & \dfrac{1}{2} \end{pmatrix}. \quad \begin{array}{l} s_{12}=0, \\ s_{13}=1/\sqrt{2}, \\ s_{23}=1/\sqrt{2}. \end{array} \tag{13}$$

The threefold maximal mixing [8] and the unit matrix are also added into our analysis:

$$T = \begin{pmatrix} \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{3}}e^{-i\delta} \\ -\dfrac{1}{2}-\dfrac{1}{2\sqrt{3}}e^{i\delta} & \dfrac{1}{2}-\dfrac{1}{2\sqrt{3}}e^{i\delta} & \dfrac{1}{\sqrt{3}} \\ \dfrac{1}{2}-\dfrac{1}{2\sqrt{3}}e^{i\delta} & -\dfrac{1}{2}-\dfrac{1}{2\sqrt{3}}e^{i\delta} & \dfrac{1}{\sqrt{3}} \end{pmatrix}, \quad \begin{array}{l} s_{12}=1/\sqrt{2}, \\ s_{13}=1/\sqrt{3}, \\ s_{23}=1/\sqrt{2}, \end{array} \tag{14}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad \begin{array}{l} s_{12}=0, \\ s_{13}=0, \\ s_{23}=0. \end{array} \tag{15}$$

In addition to these, the so-called democratic mixing [9] is examined since this mixing pattern is rather different from the above ones and might be derived from well-motivated underlying theories:

$$D = \begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{3}} \\ -\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{3}} \\ 0 & -\dfrac{2}{\sqrt{6}} & \dfrac{1}{\sqrt{3}} \end{pmatrix}, \quad \begin{array}{l} s_{12}=1/2, \\ s_{13}=1/\sqrt{3}, \\ s_{23}=1/\sqrt{2}. \end{array} \tag{16}$$

Note that if one of the matrix elements of U_E (U_ν) is zero, one can take P' ($\bar{P}'$) as a unit matrix without loss of generality. The phase δ_E (δ_ν) can be absorbed into a redefinition of Q and P'' ($\bar{P}''$). This fact is easily understood in view of the Jarlskog parameter [10] which measures the sizes of CP violation: in case that one (or more) matrix element is zero, the Jarlskog parameter is vanished. Accordingly, the phase factors δ_E and δ_ν are included only in the type T matrix.

With the above mixing matrices at hand, we have 81 combinations of matrices for U_{MNS}, in which the phases α, β, δ_E, and δ_ν are taken to be free parameters. We examine the MNS matrices referring to the phenomenological constraints coming from the atmospheric neutrino experiments. The Chooz experiment [11] also provides a useful guide for the classification of mixing matrices, in particular, for the $(U_{\text{MNS}})_{e3}$ element. On the other hand, as we mentioned in the Introduction, the solar neutrino problem may be solved with both large and small mixing angle solutions, and we will deal with it as predictions of each case of 81 combinations for U_{MNS}. In what follows, we take a convention where the mixing between the labels 2 and 3 is relevant to the atmospheric neutrinos and the mixing between the labels 1

TABLE I. The classification of mixing patterns. The numbers denote the categories defined in Sec. II.

$U_E \backslash U_\nu$	A	S	L	B	H	N	T	I	D
A	**4**	**5**	2	**4**	2	2	2	**5**	2
S	2	1	2	2	2	**4**	**4**	1	**4**
L	2	1	1	2	1	**4**	**4**	1	**4**
B	3	2	**4**	3	**4**	3	**4**	2	**4**
H	**4**	2	3	**4**	3	**4**	**4**	2	**4**
N	3	2	**4**	3	**4**	3	**4**	2	**4**
T	**4**	2	**4**	**4**	**4**	**4**	**4**	2	**4**
I	**5**	1	1	**5**	1	2	2	1	2
D	2	**5**	2	2	2	**4**	**4**	**5**	**4**

and 2 to the solar neutrino problem. After all, we find that the 81 mixing patterns are classified into the following five categories: class 1: small mixing for atmospheric neutrinos, class 2: large value of $(U_{\mathrm{MNS}})_{e3}$, class 3: small mixing for atmospheric neutrinos if $(U_{\mathrm{MNS}})_{e3} \ll 1$ by phase tuning, class 4: consistent with the experiments by phase tuning, and class 5: consistent with the experiments independently of phase values.

Only classes 4 and 5 are consistent with the experimental data. Our result of classification is summarized in Table I. We have also numerically checked the "stability" of our classification by allowing the fluctuations of all mixing angles in the region of $\theta_{ij} = \theta_{ij} \pm 5°$, both in the charged-lepton and neutrino sectors. It is found that these fluctuations make no change in the classification table.

In Table I, we first notice that there are several exchanging symmetries. In the neutrino side, the exchanges $A \leftrightarrow B$, $S \leftrightarrow I$, and $L \leftrightarrow H$ do not modify the table. The existence of these symmetries is easily understood. The above exchanges only reverse the predictions for solutions to the solar neutrino problem (from large to small mixing angle and vice versa), and so the classification table remains unchanged. We also have a similar $T \leftrightarrow D$ symmetry. In the charged-lepton side, the exchange $B \leftrightarrow N$ leaves the classification unchanged. This symmetry is a bit curious. Because of the constraint from the CHOOZ experiment, it is usually assumed that a bimaximal mixing matrix takes the form of type B. It is, however, found here that the matrix N, which has a large 1-3 mixing, gives exactly the same results as the matrix B does. Unlike the neutrino side, two types of matrices (B and N) give the same predictions even for the solar neutrino solutions (the 1-2 mixing angles). The difference exists only in the values of phase factors which are tuned. This fact would give a new possibility of model building for the fermion masses and mixing.

Class 5 contains the following six mixing patterns:

$$(U_E, U_\nu) = (A,S), \quad (A,I), \quad (I,A),$$
$$(I,B), \quad (D,S), \quad (D,I). \tag{17}$$

There are essentially only three types of combinations due to the exchanging symmetries stated above. As we will discuss in the next section, it is interesting that these six combina-

tions are certainly predicted by the various models which have been proposed to account for fermion mass hierarchies. Notice, however, that at this stage we do not refer any particular structures of mass matrices but only discuss the combinations of two unitary matrices U_E and U_ν, combined with the phenomenological constraints on the rotation angles. The coincidence of these two approaches might show a profound connection between the mass eigenvalues and mixing angles. Another interesting point we find in Eq. (17) is that the "naturalness," i.e., the absence of parameter tuning indicates that the large 1-2 mixing relevant to the solar neutrino problem must come from the neutrino side (except for the cases of democratic mixing). This is naturally understood in view of the charged-lepton masses, and indeed commonly seen in the literature. That is, in the charged-lepton sector, the mass hierarchy between the first and second generations is too large for the large-angle solar solutions.[1] It should be noticed here that the same result is obtained only from a viewpoint of mixing matrices. This may be again regarded as a sign of deep connections between masses and mixing angles.

In the category of class 4, there are 31 patterns of mixing matrices. These patterns require suitable choices of phase values to be consistent with the experimental data. The result is summarized in Table II, where we present the values of mixing angles for atmospheric neutrinos ($\sin^2 2\theta_{\mathrm{atm}}$) and for solar neutrinos ($\sin^2 2\theta_\odot$) in case where $(U_{\mathrm{MNS}})_{e3}$ is set to be minimum. For each combination, we also show the relevant phases which are tuned to obtain the minimum value of $(U_{\mathrm{MNS}})_{e3}$. In some cases, the mixing angles $\sin^2 2\theta_{\mathrm{atm}}$ and $\sin^2 2\theta_\odot$ have uncertainties since there still exists phase degrees of freedom with the minimized values of $(U_{\mathrm{MNS}})_{e3}$. The mixing patterns in class 4 need various numbers of phase tuning in order to obtain experimentally suitable MNS matrices. For example, the types $(U_E, U_\nu) = (A,A)$ and (A,B), which are often seen in the literature, requires only one phase tuning to fix all the mixing angles in U_{MNS} (see also the next section). Clearly, fewer numbers of parameter tuning are preferable for higher predictability. We find from Table II that the 8 combinations:

$$(U_E, U_\nu) = (S,N), \quad (S,D), \quad (L,N), \quad (L,D),$$

$$(B,L), \quad (H,A), \quad (H,D), \quad (N,L), \tag{18}$$

have the same predictability as (A,A) and (A,B); all the mixing angles can be settled by only one phase tuning. Remarkably, these combinations have not been discussed so far in the literature and would provide new possibilities for constructing models where fermion masses and mixing angles are properly reproduced.

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[1]Note that the democratic mass matrix cannot explain the mass hierarchy between these lighter families. Moreover, small perturbations do not necessarily result in large 1-2 mixing.

TABLE II. The mixing patterns in class 4. The values of mixing angles are shown in the case that $(U_{\mathrm{MNS}})_{e3}$ is minimal (the minimum values are also shown in the table). The last column denotes the (number of) relevant phases which are needed for tuning $(U_{\mathrm{MNS}})_{e3}$. The uncertainties in $\sin^2 2\theta_{\mathrm{atm}}$ and $\sin^2 2\theta_\odot$ are fixed by additional phase tunings.

U_E-U_ν	$\sin^2 2\theta_{\mathrm{atm}}$	$\sin^2 2\theta_\odot$	$(U_{\mathrm{MNS}})_{e3}$	(No. of) phases
$A-A$	$0-1$	0	0	(0)
$A-B$	$0-1$	1	0	(0)
$S-N$	0.73	0.73	0.15	α (1)
$S-T$	$8/9$	$1/4-1$	0	$\alpha+\delta_\nu$ (1)
$S-D$	$8/9$	0	0	α (1)
$L-N$	0.73	0.73	0.15	β (1)
$L-T$	$8/9$	$1/4-1$	0	$\beta+\delta_\nu$ (1)
$L-D$	$8/9$	$3/4$	0	β (1)
$B-L$	0.73	0.73	0.15	β (1)
$B-H$	0.73	$0.23-0.96$	0.15	β (1)
$B-T$	$8/9$	$1/4-1$	0	α, β (2)
$B-D$	$8/9$	$15/16$	0	α, β (2)
$H-A$	0.73	0.73	0.15	$\alpha-\beta$ (1)
$H-B$	0.73	$0.23-0.96$	0.15	$\alpha-\beta$ (1)
$H-N$	1	1	0	α, β (2)
$H-T$	$8/9$	$1/16-1$	0	α, β (2)
$H-D$	$8/9$	$15/16$	0	$\alpha-\beta$ (1)
$N-L$	0.73	0.73	0.15	β (1)
$N-H$	0.73	$0.23-0.96$	0.15	β (1)
$N-T$	$8/9$	$1/4-1$	0	α, β (2)
$N-D$	$8/9$	$15/16$	0	α, β (2)
$T-A$	1	$8/9$	0	$\delta_E, \alpha-\beta$ (2)
$T-L$	1	$8/9$	0	δ_E, β (2)
$T-B$	1	$1/9-1$	0	$\delta_E, \alpha-\beta$ (2)
$T-H$	1	$1/9-1$	0	δ_E, β (2)
$T-N$	$8/9-1$	$8/9$	0	α, β (2)
$T-T$	$0-1$	$0-1$	0	$\alpha+\delta_\nu, \beta+\delta_\nu$ (2)
$T-D$	$0-1$	$0-1$	0	α, β (2)
$D-N$	$1/36-0.96$	0.73	0.15	α (1)
$D-T$	$0-1$	$1/4-1$	0	$\alpha+\delta_\nu$ (1)
$D-D$	$0-1$	0	0	α (1)

III. TEXTURE OF LEPTON MASS MATRICES

The results in the previous section have been obtained independently of any structures of lepton mass matrices and hence of the mass spectrum of neutrinos. In this section, we discuss some implications of the above results for the forms of lepton mass matrices. Note that the mass textures we will discuss below are only examples among various models which can lead to the same mixing patterns. There are indeed infinite possibilities for mass matrices due to the remaining freedom of mixing matrices, mass eigenvalues and their signs, the particle property of neutrinos, etc. In the following discussion we do not want to exhaust possible mass textures but, based on the previous results, to show several examples which may correctly reproduce the experimental data.

First we assume that neutrinos are Majorana particles, for simplicity. The neutrino Majorana mass matrix M_ν is con-

structed by $U_\nu^* M_\nu^{\mathrm{diag}} U_\nu^\dagger$, where M_ν^{diag} is the diagonal neutrino mass matrix but is not fully determined even if the experimental data is given. In most cases below, we adopt a hierarchy in the neutrino masses M_ν^{diag}. Moreover if one assumes the see-saw mechanism for tiny mass scales, M_ν is a low-energy effective-mass matrix and a full mass matrix form may be highly complicated. On the other hand, the charged-lepton mass matrix is constructed by $U_E M_E^{\mathrm{diag}} R_E^\dagger$ where M_E^{diag} is the diagonal charged-lepton mass matrix and R_E is the mixing matrix which rotates the right-handed charged-lepton fields. Since R_E is experimentally unknown, the charged-lepton mass matrix is not uniquely reconstructed. In the following examples, we assume $R_E=I$ or U_E, and take hierarchical mass eigenvalues which may be parametrized by the Cabibbo angle λ.

Let us begin by discussing the mixing patterns in class 5. As noted in the previous section, these mixing patterns have often appeared in the literature. In other words, there are various models of lepton mass matrices which lead to these mixing patterns. We overview the six patterns in class 5. The first is the case $(U_E,U_\nu)=(A,S)$, which predicts bimaximal mixing for the MNS matrix. Assuming $R_E=I$, we obtain a charged-lepton mass matrix M_E and a Majorana mass matrix M_ν as

$$M_E \propto \begin{pmatrix} & & \\ \lambda^2 & 1 & \\ \lambda^2 & 1 & \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} \epsilon & \epsilon & \\ \epsilon & \epsilon & \\ & & 1 \end{pmatrix}, \quad (19)$$

where ϵ is a small parameter and the blanks in the matrices mean smaller entries. Such a type of texture has been derived, for example, in $SO(10)$ grand unified models [12]. These models adopt the see-saw mechanism, and the source of large mixing in M_ν comes from the Dirac-type mass matrix of neutrinos, which is connected with that of down quarks under the grand unified theory (GUT) symmetry.

The next pattern is the case $(U_E,U_\nu)=(A,I)$ that predicts single maximal mixing for the MNS matrix, and it can be derived from, for example,

$$M_E \propto \begin{pmatrix} & & \\ \lambda^2 & 1 & \\ \lambda^2 & 1 & \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}, \quad (20)$$

where we have taken $R_E=I$. These mass matrices are indeed obtained in E_7, E_6, and $SO(10)$ grand unified theories [13]. In the GUT mass textures (19) and (20), the large mixing in M_E is achieved by the mixing among the standard-model fields and extra particles. The third example is $(U_E,U_\nu)=(I,A)$, which gives single maximal mixing for the MNS matrix, and it leads to

$$M_E \propto \begin{pmatrix} \lambda^{4-5} & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} \epsilon & & \\ & 1 & 1 \\ & 1 & 1 \end{pmatrix}, \quad (21)$$

where $R_E = I$ is assumed. This texture has been discussed, for example, in the R-parity violating models [14]. The fourth one is $(U_E, U_\nu) = (I, B)$, which gives bimaximal mixing. Assuming $R_E = I$, it gives

$$M_E \propto \begin{pmatrix} \lambda^{4-5} & & \\ & \lambda^2 & \\ & & 1 \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} & 1 & 1 \\ 1 & & \\ 1 & & \end{pmatrix}. \quad (22)$$

It has been shown that this texture follows from the radiative generation mechanisms for neutrino masses [15].

The fifth and sixth patterns are a bit special since they depend on the democratic lepton mass matrix [9], which usually predicts $R_E = U_E$. The combination $(U_E, U_\nu) = (D, S)$ predicts single maximal mixing for the MNS matrix, and gives

$$M_E \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} 1 & \epsilon' & \\ \epsilon' & 1 & \\ & & 1+\epsilon \end{pmatrix}. \quad (23)$$

On the other hand, $(U_E, U_\nu) = (D, I)$ has nearly bimaximal mixing, and gives

$$M_E \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}. \quad (24)$$

We here again stress that all the above mixing patterns in class 5 are allowed by the experimental data without any tuning of (sometimes unphysical) phases α, β, and $\delta_{E,\nu}$.

Next let us discuss the mixing patterns in class 4, where the presence of phase factors is essential for the MNS matrix to have the right values of mixing angles. The 31 mixing patterns are classified into this category, but only a few mass matrix models with these patterns have been constructed. These patterns thus could provide potentially useful textures of lepton mass matrices.

At first, we discuss the well-known example $(U_E, U_\nu) = (A, A)$ which leads to the following mass matrices by taking $R_E = I$:

$$M_E \propto \begin{pmatrix} \lambda^2 & 1 & \\ \lambda^2 & 1 & \\ & & \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} 1 & 1 & \\ 1 & 1 & \\ & & \end{pmatrix}. \quad (25)$$

This form often appears in the models with $U(1)$ flavor symmetries [16,17]. With this texture, the mixing angles at leading order become

$$\theta_{\text{atm}} = \frac{\beta - \alpha}{2}, \quad \theta_\odot = (U_{\text{MNS}})_{e3} = 0, \quad (26)$$

where α and β are the phase parameters in the matrix Q [see Eq. (7)]. This gives the SMA solution for the solar neutrino problem, and the constraint $(U_{\text{MNS}})_{e3} \ll 1$ is also satisfied. For the atmospheric neutrinos, however, one must tune the phase values so that $\beta - \alpha \simeq \pi/2$. That is, due to the presence

of the phase matrix Q, the cancellation of two large mixing angles from U_E and U_ν can be avoided. Though the Super-Kamiokande result still allows about $10°$ deviations from the maximal mixing angle, it can be explained only in about 20% region of the whole phase parameter space of (α, β). It should be noted we numerically checked that this situation is unchanged even if one includes $\pm 20°$ fluctuations of the mixing angles in U_E and U_ν. This fact means that some amount of tuning of phase parameters is indeed required to have right predictions.

Another pattern, for which the concrete models have been constructed, is the case of $(U_E, U_\nu) = (A, B)$. Assuming $R_E = I$, it leads to the mass matrix form

$$M_E \propto \begin{pmatrix} \lambda^2 & 1 & \\ \lambda^2 & 1 & \\ & & \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} & 1 & 1 \\ 1 & & \\ 1 & & \end{pmatrix}. \quad (27)$$

These textures have been discussed in Ref. [18]. It is also pointed out in Ref. [17] that this mixing pattern can be predicted by the texture in Eq. (25). The mixing angle θ_{atm} is the same as in Eq. (26), and a phase combination $\beta - \alpha$ must be tuned so that one gets the maximal mixing of atmospheric neutrinos.

As we stated in Sec. II, there are several new mixing patterns in class 4 which have not yet been discussed. Like the cases (A, A) and (A, B), the eight new patterns in Eq. (18) only need a single phase tuning for fitting all the experimental data; the solar, atmospheric, and long baseline neutrino experiments. Let us show an example for the case $(U_E, U_\nu) = (S, N)$. This mixing pattern, with $R_E = I$, gives the following form of mass matrices:

$$M_E \propto \begin{pmatrix} \lambda^{4-5} & \lambda^2 & \\ \lambda^{4-5} & \lambda^2 & \\ & & 1 \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} 2 & \sqrt{2} & \sqrt{2} \\ \sqrt{2} & 1+\epsilon & 1-\epsilon \\ \sqrt{2} & 1-\epsilon & 1+\epsilon \end{pmatrix}. \quad (28)$$

In this case, we have

$$\sin^2 2\theta_{\text{atm}} = \sin^2 2\theta_\odot = \left| \frac{1}{2} + \frac{1}{2\sqrt{2}} e^{i\alpha} \right|^2,$$

$$(U_{\text{MNS}})_{e3} = \left| \frac{1}{2} - \frac{1}{2\sqrt{2}} e^{i\alpha} \right|. \quad (29)$$

Here we would like to emphasis that a single phase tuning of α ensures all the mixing angles to be consistent with the experiments. A smaller value of $(U_{\text{MNS}})_{e3}$ tuned by a phase rotation automatically leads to larger mixing angles for solar and atmospheric neutrinos. In this example, the mixing angle $\sin^2 2\theta_{\text{atm}}$ might be a bit smaller than the experimental bound from the atmospheric neutrino anomaly. One can, however, easily get a proper MNS matrix if a few deviations from the rigid values of mixing angles in $U_{E,\nu}$ are taken into account. [Such deviations just correspond to those in the mass matrices (28).] As we mentioned earlier, even with these devia-

tions, the classification is not changed and it is enough to tune only one phase parameter. Notice that since the $(U_{\mathrm{MNS}})_{e3}$ mixing in Eq. (29) is close to the CHOOZ bound, this pattern will be tested in the near future.

For the examples that need more than one phase tuning, we refer to the models in [19] which introduce the following types of mass matrices:

$$M_E \propto \begin{pmatrix} \lambda^{4-5} & \lambda^2 & 1 \\ \lambda^{4-5} & \lambda^2 & 1 \\ \lambda^{4-5} & \lambda^2 & 1 \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (30)$$

This corresponds to the mixing pattern $(U_E, U_\nu) = (T, T)$ or to the special case $(U_E, U_\nu) = (D, D)$, where suitable MNS matrices can also be obtained by phase tuning.

Including the above examples, we find that class 4 contains several possible mixing patterns which no one has discussed so far [see Table II and Eq. (18)]. Model construction utilizing such types of textures may be worth performing.

Before closing this section, we note the connections of the low-energy Majorana neutrino mass matrices discussed above with those at high-energy scale [20,21].[2] To discuss the stability of lepton flavor mixing against quantum corrections, we need to determine the pattern of neutrino masses and Majorana phases [21,23]. For example, the neutrinos which are degenerate in mass with the same phase sign may receive a considerable change of flavor mixing structure [21,24]. The mass matrices in Eqs. (20), (23), and (24), therefore, have a possibility of changing the values of mixing angles during the renormalization-group evolution. In particular, the mixing angles of the democratic-type mass matrix [Eqs. (23) and (24)], which is expected in the models with $S_{3L} \times S_{3R}$ or $O(3)_L \times O(3)_R$ symmetries [9], might receive large quantum modifications [25].

IV. SUMMARY AND DISCUSSION

To study the origins of the nearly maximal mixing of lepton flavor is one of the most important issues in particle physics. We have examined what types of mixing matrices of charged leptons and neutrinos can be consistent with the neutrino experimental results. Our analyses in Sec. II do not depend on any details of underlying models, in particular, of the mass matrix forms of charged leptons and neutrinos. The results are hence independent of the mass spectrum and property of neutrinos, for example, whether they are Dirac or Majorana particles. As typical forms of charged-lepton and neutrino mixing matrices, we have adopted nine types of unitary matrices, which contain sources of large mixing angles and could be induced from some underlying theories. We have then examined 9×9 combinations of mixing matrices and checked whether the resultant MNS mixing matrices satisfy the phenomenological constraints from atmospheric and long baseline neutrino experiments. In our analyses, the phase factors, which cannot be absorbed into redefinitions of lepton fields, play important roles.

As a result, we have found that there are various mixing patterns of charged leptons and neutrinos for the MNS matrix with bimaximal or single maximal mixing. Among them, only six patterns are experimentally allowed without any tuning of phase values. Interestingly, these patterns are indeed derived from the concrete models which have been proposed to account for the fermion mass hierarchy problem. The other patterns can give solutions to the observed neutrino anomalies depending on the choices of phase values. In this class of patterns, physically more significant mixing patterns may be the ones which need fewer numbers of phase tuning to have definite predictions. We have found that 10 combinations satisfy this criterion; only a single phase tuning is required. They have not been studied enough in lepton mass matrix models and will give new possibilities of model construction. Note that the tuned phases are not completely unphysical unlike in the quark sector, but some of them are connected to Majorana phases and CP violation phenomena in the lepton sector. Combined with these effects, the improved measurements of mixing angles $\sin^2 2\theta_\odot$ and $(U_{\mathrm{MNS}})_{e3}$ will be important to select possible flavor mixing structures of leptons.

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