

# Effects of Matter Density Fluctuation in Long Baseline Neutrino Oscillation Experiments

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## Abstract

The effects of matter density fluctuation in long baseline neutrino oscillation experiments are studied. Effects of short wavelength fluctuations are in general irrelevant. Effects of long wavelength fluctuations must be checked on a case-by-case basis. As an example we checked the fluctuation effects and showed its irrelevance in a case of K2K experiments.

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Neutrino oscillation search is one of the most promising way to go beyond the Standard Model. The existence of neutrino oscillation is suggested by the lack of solar neutrino flux [1, 2, 3, 4] and the anomalies of atmospheric neutrinos [5, 6, 7]<sup>1</sup>. Further confirmation of its existence is now close at hand owing to the long baseline neutrino oscillation experiments, such as K2K experiments [10] and Minos experiments [11]. The possible effects of  $CP$  and  $T$  violation in such experiments have been discussed in the literatures [12, 13, 14, 15, 16, 17]. There the matter density is approximated to be constant<sup>2</sup>. We now study effects of the matter density

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<sup>1</sup>Some experiments have not observed the atmospheric neutrino anomaly [8, 9].

<sup>2</sup>We have checked numerically in Ref. [17] that the constant density approximation works well for some parameter space of neutrino's in K2K experiments. However it does not mean the approximation works well for all the parameter space. Another example of numerical calculations is presented in Ref. [18].

fluctuation and clarify when the constant density approximation is valid. We also verify that this approximation works well for the K2K experiment.

Let  $\nu = (\nu_e, \nu_\mu, \nu_\tau)$  and  $\nu' = (\nu_1, \nu_2, \nu_3)$  be the eigenstates of flavor and mass in vacuum, respectively. They are related through the mixing matrix  $U$  as

$$\nu = U\nu'. \quad (1)$$

Defining  $\delta m_{ij}^2 \equiv m_i^2 - m_j^2$  with the mass eigenvalues  $m_i (i = 1, 2, 3)$ , the evolution equation of flavor eigenstates in matter is given by

$$i \frac{d\nu(x)}{dx} = H(x)\nu(x), \quad (2)$$

$$H(x) \equiv \frac{1}{2E} \left\{ U \begin{pmatrix} 0 & & \\ & \delta m_{21}^2 & \\ & & \delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a(x) & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\}. \quad (3)$$

Here

$$a(x) \equiv 2\sqrt{2}G_F n_e(x)E = 7.56 \times 10^{-5} \text{eV}^2 \frac{\rho(x)}{\text{g cm}^{-3}} \frac{E}{\text{GeV}}, \quad (4)$$

$n_e(x)$  is the electron density,  $\rho(x)$  is the matter density and  $E$  is the neutrino energy. The evolution equation (2) is solved as

$$\nu(L) = \text{T exp} \left( -i \int_0^L dx H(x) \right) \nu(0) \equiv S(L)\nu(0), \quad (5)$$

giving the oscillation probability from  $\nu_\alpha$  to  $\nu_\beta (\alpha, \beta = e, \mu, \tau)$  as

$$P(\nu_\alpha \rightarrow \nu_\beta; L, E) = |S(L)_{\beta\alpha}|^2. \quad (6)$$

We assume  $\delta m_{31}^2 \sim (10^{-2} \sim 10^{-3})\text{eV}^2$  and  $\delta m_{21}^2 \sim (10^{-5} \sim 10^{-4})\text{eV}^2$  allowing for atmospheric neutrino anomaly and solar neutrino deficit. Along with  $a \sim 10^{-4}\text{eV}^2$  (see eq.(4)), we see

$$\delta m_{21}^2, a \ll \delta m_{31}^2. \quad (7)$$

We hence separate as  $H(x) = H_0 + H_1(x)$ , where

$$H_0 \equiv \frac{1}{2E} U \begin{pmatrix} 0 & & \\ & 0 & \\ & & \delta m_{31}^2 \end{pmatrix} U^\dagger \quad (8)$$

and

$$H_1(x) \equiv \frac{1}{2E} \left\{ U \begin{pmatrix} 0 & & \\ & \delta m_{21}^2 & \\ & & 0 \end{pmatrix} U^\dagger + \begin{pmatrix} a(x) & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\}, \quad (9)$$

and treat  $H_1(x)$  as a perturbation. Taking up to the lowest order in  $H_1(x)$ , we obtain

$$\begin{aligned} S(L) &\simeq e^{-iH_0L} + e^{-iH_0L} (-i) \int_0^L dx \left[ e^{iH_0x} H_1(x) e^{-iH_0x} \right] \\ &\equiv S_0(L) + S_1(L). \end{aligned} \quad (10)$$

The fluctuation of matter density  $\rho(x)$ , or equivalently that of  $a(x)$ , affects  $S_1(L)$  alone. We separate that effect from  $S_1(L)$  in the following.

Letting

$$\bar{a} \equiv \frac{1}{L} \int_0^L dx a(x) \quad (11)$$

and

$$\delta a(x) \equiv a(x) - \bar{a}, \quad (12)$$

we separate the contribution of  $\delta a(x)$  from  $H_1(x)$  by defining

$$\bar{H}_1(x) \equiv \frac{1}{2E} \left\{ U \begin{pmatrix} 0 & & \\ & \delta m_{21}^2 & \\ & & 0 \end{pmatrix} U^\dagger + \begin{pmatrix} \bar{a} & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\} \quad (13)$$

and

$$\delta H_1(x) \equiv \frac{1}{2E} \begin{pmatrix} \delta a(x) & & \\ & 0 & \\ & & 0 \end{pmatrix}. \quad (14)$$

Defining accordingly

$$\bar{S}_1(L) = -i \int_0^L dx \left[ e^{-iH_0(L-x)} \bar{H}_1(x) e^{-iH_0x} \right] \quad (15)$$

and

$$\delta S_1(L) = -i \int_0^L dx \left[ e^{-iH_0(L-x)} \delta H_1(x) e^{-iH_0x} \right], \quad (16)$$

$\bar{S}_1$  is calculated to be [14]

$$\begin{aligned} \bar{S}_1(x)_{\beta\alpha} &= -i U_{\beta 2} U_{\alpha 2}^* \cdot \frac{\delta m_{21}^2 x}{2E} \\ &\quad -i(\delta_{\beta e} - U_{\beta 3} U_{e3}^*)(\delta_{\alpha e} - U_{e3} U_{\alpha 3}^*) \cdot \frac{\bar{a} x}{2E} \\ &\quad + U_{\beta 3} U_{e3}^* (\delta_{\alpha e} - U_{e3} U_{\alpha 3}^*) \left[ \exp\left(-i \frac{\delta m_{31}^2 x}{2E}\right) - 1 \right] \cdot \frac{\bar{a}}{\delta m_{31}^2} \\ &\quad + (\delta_{\beta e} - U_{\beta 3} U_{e3}^*) U_{e3} U_{\alpha 3}^* \left[ \exp\left(-i \frac{\delta m_{31}^2 x}{2E}\right) - 1 \right] \cdot \frac{\bar{a}}{\delta m_{31}^2} \\ &\quad -i U_{\beta 3} U_{\alpha 3}^* |U_{e3}|^2 \exp\left(-i \frac{\delta m_{31}^2 x}{2E}\right) \cdot \frac{\bar{a} x}{2E}. \end{aligned} \quad (17)$$

To evaluate  $\delta S_1(x)_{\beta\alpha}$  we expand  $\delta a(x)$  as

$$\delta a(x) \equiv \sum_{n=-\infty}^{\infty} a_n e^{-in\pi x/L} \quad (18)$$

and carry out integration for each Fourier component. Note that  $a_0 = 0$  from the definition (12). We then find

$$\begin{aligned}
\delta S_1(x)_{\beta\alpha} &= \delta_{\alpha e} U_{\beta 3} U_{e3}^* \left[ \exp\left(-i\frac{\delta m_{31}^2 x}{2E}\right) - 1 \right] \cdot \sum_{n \neq 0} \frac{a_n}{\delta m_{31}^2} \left(1 - \frac{2n\pi}{\delta m_{31}^2 x/2E}\right)^{-1} \\
&+ \delta_{\beta e} U_{e3} U_{\alpha 3}^* \left[ \exp\left(-i\frac{\delta m_{31}^2 x}{2E}\right) - 1 \right] \cdot \sum_{n \neq 0} \frac{a_n}{\delta m_{31}^2} \left(1 + \frac{2n\pi}{\delta m_{31}^2 x/2E}\right)^{-1} \\
&- U_{\beta 3} U_{\alpha 3}^* |U_{e3}|^2 \left[ \exp\left(-i\frac{\delta m_{31}^2 x}{2E}\right) - 1 \right] \times \\
&\quad \sum_{n \neq 0} \frac{a_n}{\delta m_{31}^2} \left[ \left(1 - \frac{2n\pi}{\delta m_{31}^2 x/2E}\right)^{-1} + \left(1 + \frac{2n\pi}{\delta m_{31}^2 x/2E}\right)^{-1} \right]. \quad (19)
\end{aligned}$$

We now see the order of magnitude of  $\bar{S}_1$  and  $\delta S_1(x)$  ((17) and (19)). Here we assume that the products of  $U$ 's and  $\delta m_{31}^2 L/2E$  are all  $O(1)$  so that we can observe the neutrino oscillation. In this case we can see from (17) that

$$\bar{S}_1 = O\left(\frac{\delta m_{21}^2}{\delta m_{31}^2} \quad \text{or} \quad \frac{\bar{a}}{\delta m_{31}^2}\right). \quad (20)$$

On the other hand all the three terms of  $\delta S_1(x)$  contains factors  $(a_n/\delta m_{31}^2)$  and also  $\{1 - 2n\pi/(\delta m_{31}^2/2E)\}^{-1} \sim 1/n$ . Hence

$$\delta S_1(x) = O\left(\frac{1}{n} \frac{a_n}{\bar{a}} \bar{S}_1\right). \quad (21)$$

We can see from (21) that the long wavelength fluctuation (i.e. small  $n$ ) of the matter density is important; short wavelength fluctuations (i.e. large  $n$ ) of matter density is in general irrelevant due to the factor  $1/n$ . This means we do not need to survey detailed profile of the matter density distribution on the baseline. On the other hand we must check on a case-by-case basis whether the long wavelength fluctuation is relevant or not. This check can be done by considering the magnitudes of  $a_n/\bar{a}$ .

Let us carry out a check for the K2K experiments as an example. Figure 1 shows the density profile between KEK and Kamioka [19]. The first several Fourier coefficients divided by the mean density ( $= a_n/\bar{a}$ ) for this profile is given in Fig. 2. We can see that  $a_n/\bar{a}$  is small, mostly much less than 0.1. This justifies the constant matter density approximation for K2K experiments.

We have shown that short wavelength fluctuation of matter density is irrelevant and only first several Fourier coefficients of fluctuation may be important for long baseline neutrino oscillation experiments. Since we can expect that such fluctuations are much smaller than the mean density in crust we can approximate the matter density to be constant. This is indeed the case for K2K experiments. We could check the approximation works well independent of the parameters relevant to neutrino oscillation.

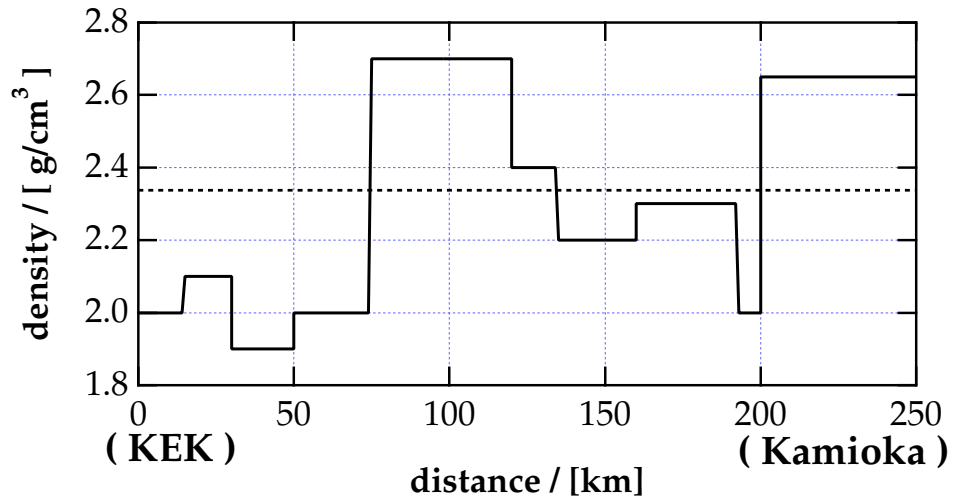


Figure 1: The density profile between KEK and Kamioka.

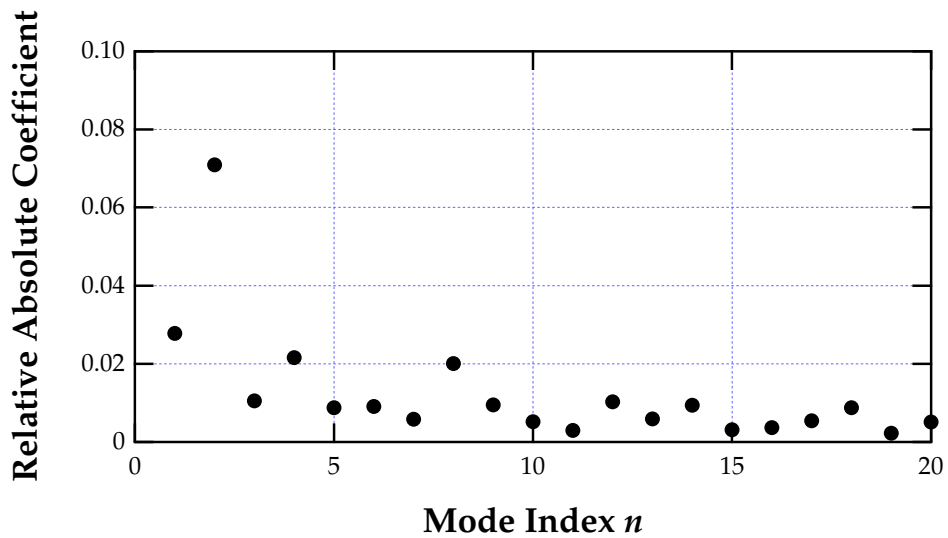


Figure 2: The values of  $a_n/\bar{a}$  for first several  $n$ 's.

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