Large Lepton Mixing in Seesaw Models*
- Coset-space Family Unification -

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Abstract

We show that the large mixing between ν_μ and ν_τ observed by the SuperKamiokande collaboration is a quite natural prediction in a large class of seesaw models. This large mixing is basically due to the unparallel family structure suggested from the observed mass hierarchies in quark and lepton mass matrices. We show that the unparallel family structure is automatically realized in "coset-space family unification" model based on $E_7/SU(5)\times U(1)^3$. This model also suggests the small angle MSW solution to the solar neutrino problem.

1 Introduction

T. Kajita from the SuperKamiokande collaboration has reported, in this conference, very convincing evidence of neutrino oscillation in their atmospheric neutrino data[1]. It is now clear that the long-standing puzzle of muon neutrino deficit in underground detectors[2] is due to the neutrino oscillation. A remarkable feature of the oscillation is almost maximal mixing between ν_μ and ν_τ ($\sin^2\theta_{23} \geq 0.8$), in sharp contrast to the quark sector for which mixing angles among different generations are all small. At first glance the rule governs the lepton mass matrices seems significantly different from the one relevant for the quark sector. We first show, in this talk, that the large mixing between ν_μ and ν_τ is quite naturally understood in a large class of seesaw models[3].

2 General Consideration in Seesaw Models

We adopt the SU(5) grand unification (GUT) as an example to make our point clearer, in which the lepton doublets belong to $\mathbf{5}^*$ of SU(5) GUT. We also assume supersymmetry(SUSY).

*Talk is given by T. Yanagida

Let us discuss first the up-type quark mass matrix that is given by the following superpotential:

$$W = h_{ij} \mathbf{10}_i \mathbf{10}_j < H(\mathbf{5}) > . \tag{1}$$

The most natural explanation of the mass hierarchy is given by the Froggatt-Nielsen mechanism[4]. We here assume a U(1) symmetry which is broken by a condensation of a superfield ϕ. The observed mass hierarchy,

$$m_t : m_c : m_u \simeq 1 : \epsilon^2 : \epsilon^4, \tag{2}$$

suggests that $\epsilon =< \phi > /M_G \sim 1/20$ and the U(1) charges are 0, 1, 2 and -1 for the third, second, first families of $\mathbf{10}$'s and the ϕ. Here M_G is the gravitational scale $M_G \simeq 2.4 \times 10^{18}$GeV.

The down-type quark/charged lepton mass matrix is given by

$$W = f_{ij} \mathbf{10}_i \mathbf{5}^*_j < \overline{H}(\mathbf{5}^*) > . \tag{3}$$

The observed mass hierarchy,

$$m_b : m_s : m_d = m_\tau : m_\mu : m_e \simeq 1 : \epsilon : \epsilon^3, \tag{4}$$

suggests that the third, second and first families of $\mathbf{5}^*$ have the U(1) charges A, A, and $A+1$, respectively. A crucial point is that the third and the second families of $\mathbf{5}^*$ have the same U(1) charge A. A could be 0 or 1. We take $A = 0$ for simplicity. We should stress here that the observed mass hierarchies in quark and lepton mass matrices already suggest an unparallel family structure in Table 1.

			U(1) charge
$\mathbf{10}_3$	$\mathbf{5}^*_3$	$\mathbf{5}^*_2$	[0]
$\mathbf{10}_2$	$\mathbf{5}^*_1$		[1]
$\mathbf{10}_1$			[2]

Table 1: Unparallel Family Structure

Now, let us discuss the neutrino mass matrix. In a generic seesaw model it is given by the following effective superpotential:

$$W_{eff} = \frac{\kappa_{ij}}{M_{\nu_R}} \mathbf{5}^*_i \mathbf{5}^*_j < H(\mathbf{5}) H(\mathbf{5}) > \tag{5}$$

The U(1) charge assignment for $\mathbf{5}^*_i$ leads to

$$\kappa_{ij} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & \epsilon^2 \end{pmatrix} . \tag{6}$$

SU(5)	$U(1)_1$	$U(1)_2$	$U(1)_3$
$\mathbf{10}_1$	0	0	4
$\mathbf{10}_2$	0	3	-1
$\mathbf{10}_3$	2	-1	-1
$\mathbf{5}^*_1$	0	3	3
$\mathbf{5}^*_2$	2	-1	3
$\mathbf{5}^*_3$	2	2	-2
$\mathbf{1}_1$	0	3	-5
$\mathbf{1}_2$	2	-1	-5
$\mathbf{1}_3$	2	-4	0
$\mathbf{5}$	2	2	2

Table 2: U(1) charges of the NG multiplets. The $U(1)_1$, $U(1)_2$ and $U(1)_3$ are the unbroken U(1)'s of coset-subspaces $E_7/E_6\times U(1)$, $E_6/SO(10)\times U(1)$ and $SO(10)/SU(5)\times U(1)$, respectively.

Notice that the U(1) charges for the superheavy right-handed neutrino ν_R are canceled out in the effective neutrino mass matrix in eq.(5). From eq.(6) we easily see a large mixing close to the maximal between ν_μ and ν_τ. The appearance of the large mixing is originated from the unparallel family structure discussed above.[1] On the contrary to the ν_μ-ν_τ mixing, we have small mixing between ν_e and ν_μ or ν_τ. Thus, the small angle MSW solution [5] to the solar neutrino problem [6] is also a quite natural expectation in a large class of seesaw models.

3 Coset-space Family Unification on $E_7/SU(5)\times U(1)^3$

In this section we show that the unparallel family structure discussed in the previous section is naturally obtained in the coset-space family unification[7] based on E_7.

The $E_7/SU(5)\times U(1)^3$ model[8, 9] contains three families of $\mathbf{10}_i + \mathbf{5}^*_i + \mathbf{1}_i$ ($i = 1-3$) and one $\mathbf{5}$ as NG multiplets. Here, the SU(5) is the usual GUT gauge group. Their quantum numbers under the unbroken subgroup are given in Table 2. Notice that the first family $\mathbf{10_1}$ has non-vanishing charge only for the $U(1)_3$ which means that the $\mathbf{10_1}$ is the NG multiplet for $SO(10)/SU(5)\times U(1)$. Similarly, we find that $\mathbf{10_2}$, $\mathbf{5^*_1}$ and $\mathbf{1_1}$ are NG multiplets for $E_6/SO(10)\times U(1)$ and the remaining fields are NG multiplets for $E_7/E_6\times U(1)$. Thus, it is now clear that the unparallel family structure is an automatic prediction of this coset-space family unification [10].

This model can not be quantized in the original form, since there is a nonlinear-sigma model anomaly[11, 9]. However, this global obstruction is easily removed[9] by introducing a matter multiplet $\mathbf{5}^*$ which is also needed for an

[1] This crucial point is emphasized by T. Yanagida and P. Ramond in this conference.

SU(5) gauge-anomaly cancellation[8]. We assume that some explicit breaking induces an invariant mass for the NG $\mathbf{5}$ and this matter $\mathbf{5}^*$ and we neglect them in our discussion.

In addition to the NG multiplets we introduce a pair of Higgs multiplets $\mathbf{5}_H$ and $\mathbf{5}^*_H$. As long as the global E_7 is exact these Higgs multiplets never have Yukawa couplings to the NG quarks and leptons. Thus, the observed hierarchy in quark-lepton mass matrices is regarded as a consequence of a hierarchy in the explicit breaking of the global E_7. This situation is very similar to that in the QCD, where the mass hierarchy between NG pions and kaons ($m_K^2 \gg m_\pi^2$) is originated from the hierarchy in quark masses ($m_s \gg m_{u,d}$) which are explicit breaking parameters of the chiral $SU(3)_L \times SU(3)_R$.

We consider three steps for the explicit breaking:

$$E_7 \underset{\epsilon_0}{\longrightarrow} E_6 \underset{\epsilon_1}{\longrightarrow} SO(10) \underset{\epsilon_2}{\longrightarrow} SU(5), \tag{7}$$

which leads to the mass hierarchy

$$\begin{aligned} m_t \gg m_c \gg m_u \\ m_b \gg m_s \gg m_d \\ m_\tau \gg m_\mu \gg m_e. \end{aligned} \tag{8}$$

To realize this hierarchy we assume that the global E_7 is broken explicitly by the fundamental representation of $E_7, \mathbf{56}$, which contains six breaking parameters, $\epsilon_0, \bar{\epsilon}_0, \epsilon_1, \bar{\epsilon}_1, \epsilon_2, \bar{\epsilon}_2$ that are all singlets of SU(5). They carry U(1) charges as

$$\begin{aligned} &\epsilon_0(-3,0,0), \quad &\bar{\epsilon}_0(3,0,0) \\ &\epsilon_1(-1,-4,0), \quad &\bar{\epsilon}_1(1,4,0) \\ &\epsilon_2(-1,-1,-5), \quad &\bar{\epsilon}_2(1,1,5) \end{aligned} \tag{9}$$

where the numbers in each parenthesis denote charges of $U(1)_1 \times U(1)_2 \times U(1)_3$. The desired hierarchy in eq.(7) is represented by

$$\epsilon_0 \gg \epsilon_1 \gg \epsilon_2. \tag{10}$$

The structure of Yukawa couplings for the NG quarks and leptons depends on U(1) charges of the Higgs $\mathbf{5}_H$ and $\mathbf{5}^*_H$. To determine them, we consider that the Higgs multiplets $\mathbf{5}_H$ and $\mathbf{5}^*_H$ belong to $\mathbf{27}$ of E_6 in $\mathbf{133}$ of E_7. Then, U(1) charges for the $\mathbf{5}_H$ are given by

$$\mathbf{5}_H \ (2,2,2). \tag{11}$$

The Higgs $\mathbf{5}^*_H$ is a linear combination of two $\mathbf{5}^*$'s in $\mathbf{27}$ of E_6[2] as

$$\mathbf{5}^*_H = \sin\theta \mathbf{5}^*_{16} + \cos\theta \mathbf{5}^*_{10} \tag{12}$$

[2] $\mathbf{27}$ of E_6 is decomposed to $\mathbf{16} + \mathbf{10} + \mathbf{1}$ of SO(10). The $\mathbf{16}$ and $\mathbf{10}$ contain one $\mathbf{5}$ and two $\mathbf{5}^*$ of SU(5).

where U(1) charges for $\mathbf{5}^*_{16}$ and $\mathbf{5}^*_{10}$ are given by[3]

$$\mathbf{5}^*_{16}(2,-1,3) \;\; \text{and} \;\; \mathbf{5}^*_{10}(2,2,-2). \tag{13}$$

We now discuss Yukawa couplings for the quark and lepton multiplets. In general, Yukawa couplings are given in a form $a_n\epsilon^n\psi\psi H$ where ϵ, ψ and H stand for the explicit breaking parameters, the NG multiplets and the Higgs multiplets, respectively. By our choice of the U(1) charges for the explicit breaking parameters and Higgs multiplets, Yukawa couplings take the following form in the leading order of the explicit breaking parameters, ϵ's;

$$\begin{aligned} W &= W_U + W_D + W_E + W_\nu, & (14)\\ W_U &= \sum_{ij} a_{ij} Y_{Uij} \mathbf{10}_i \mathbf{10}_j \mathbf{5}_H, & (15)\\ W_D &= W_E = \sum_{ij} b_{ij} Y_{D/Eij} \mathbf{5}^*_i \mathbf{10}_j \mathbf{5}^*_H, & (16)\\ W_\nu &= \sum_{ij} c_{ij} Y_{\nu ij} \mathbf{5}^*_i \mathbf{1}_j \mathbf{5}_H, & (17) \end{aligned}$$

where W_U, W_D, W_E and W_ν represent superpotentials of Yukawa couplings for up-type quarks, down-type quarks, charged leptons and neutrinos. In these expressions Y's are given by,[4, 5]

$$Y_U \simeq \begin{pmatrix} \epsilon_2^2 & \epsilon_1\epsilon_2 & \epsilon_0\epsilon_2 \\ \epsilon_1\epsilon_2 & \epsilon_1^2 & \epsilon_0\epsilon_1 \\ \epsilon_0\epsilon_2 & \epsilon_0\epsilon_1 & \epsilon_0^2 \end{pmatrix}, \tag{18}$$

$$Y_{D/E} \simeq \begin{pmatrix} \epsilon_1\epsilon_2\cos\theta & \epsilon_1^2\cos\theta & \epsilon_0\epsilon_1\cos\theta \\ \epsilon_0\epsilon_2\cos\theta & \epsilon_0\epsilon_1\cos\theta & \epsilon_0^2\cos\theta \\ \epsilon_0\epsilon_2\sin\theta & \epsilon_0\epsilon_1\sin\theta & \epsilon_0^2\sin\theta \end{pmatrix}, \tag{19}$$

$$Y_\nu \simeq \begin{pmatrix} \epsilon_1^2 & \epsilon_0\epsilon_1 & \epsilon_0\epsilon_2 \\ \epsilon_0\epsilon_1 & \epsilon_0^2 & 0 \\ 0 & 0 & \epsilon_0^2 \end{pmatrix} \tag{20}$$

[3] The orthogonal combination of the $\mathbf{5}^*_{16}$ and $\mathbf{5}^*_{10}$ is assumed to have a GUT scale mass. We also assume that color triplets in $\mathbf{5}_H$ and $\mathbf{5}^*_H$ receive a GUT scale mass after the spontaneous breakdown of the SU(5) GUT. This requires a fine tuning. We do not, however, discuss this fine tuning problem here, since it is beyond the scope of this talk.

[4] One may wonder that in eq.(19) the (3,1) element of $Y_{D/E}$, has a term of $\epsilon_0\epsilon_1$. We do not think that such a term appears there, since in the limit $\epsilon_2 \to 0$, the global SO(10) symmetry becomes exact and the $\mathbf{10}_1$ is the true NG multiplet which has no Yukawa interaction in the superpotential.

[5] Precisely speaking, our coset-space $E_7/SU(5)\times U(1)^3$ contains three dimensional parameters f_0, f_1 and f_2. We assume $f_0 \sim f_1 \sim f_2$ here, for simplicity. However, even if it is not the case, one obtains the same form of Yukawa couplings as in eqs.(18), (19) and (20) by redefining ϵ's as $\epsilon_i = \tilde{\epsilon}_i/f_i$ (i =0,1,2) where $\tilde{\epsilon}_i$ are original dimensional parameters for the explicit E_7 breakings.

We have assumed the E_7 representations for ϵ_i, $\mathbf{5}_H$ and $\mathbf{5}^*_H$ to determine their U(1) charges. However, we consider that this assumption is over statement since the E_7 is already spontaneously broken. What is relevant to our analysis is only their charges of the unbroken subgroup $SU(5)\times U(1)^3$. With this general consideration it is impossible to estimate the coefficients a_{ij}, b_{ij} and c_{ij} in eqs.(15), (16) and (17) and hence we assume that they are of O(1).

From the above Yukawa couplings in eqs.(18) and (19) we easily derive the following mass relations;

$$\begin{aligned}
&\frac{m_u}{m_c} \sim \frac{\epsilon_2^2}{\epsilon_1^2}, \\
&\frac{m_c}{m_t} \sim \frac{\epsilon_1^2}{\epsilon_0^2}, \\
&\frac{m_e}{m_\mu} = \frac{m_d}{m_s} \sim \frac{\epsilon_2}{\epsilon_0}\sin^{-1}\theta, \\
&\frac{m_\mu}{m_\tau} = \frac{m_s}{m_b} \sim \frac{\epsilon_1}{\epsilon_0}\sin\theta\cos\theta.
\end{aligned} \tag{21}$$

These relations describe very well the observed mass relations provided that

$$\frac{\epsilon_1}{\epsilon_0} \sim 0.05, \quad \frac{\epsilon_2}{\epsilon_1} \sim 0.05 \quad \text{and} \quad \tan\theta \sim 1. \tag{22}$$

We see that the Cabibbo-Kobayashi-Maskawa mixing angles for quarks between the 1st and the 2nd, the 2nd and the 3rd, and the 3rd and the 1st family are of the order ϵ_2/ϵ_1, ϵ_1/ϵ_0, and ϵ_2/ϵ_0, respectively. It also describes the observed mixing angles very well provided that the relations in eq.(22) are satisfied.

We do not further mention details of the mass relations since there should be corrections to the mass matrices in eqs.(18) and (19) from some higher dimensional operators which may affect masses for lighter particles significantly. Otherwise, we have a SU(5) GUT relation, $m_d = m_e$, which seems unrealistic[12].

So far, we have discussed the mass matrices for quarks and charged leptons and found that the qualitative global structure of the obtained matrices fits very well the observed mass spectrum for quarks and charged leptons (except for $m_d = m_e$) and mixing angles for quarks if the relations in eq.(22) are satisfied[6].

We are now at the point to discuss neutrino masses and lepton mixings. We assume that Mayorana masses for right-handed neutrinos N_i are induced by SU(5) singlet Higgs multiplets $\bar{s}_i(\mathbf{1})$. We introduce two singlets $\bar{s_1}(\mathbf{1})$ and $\bar{s}_2(\mathbf{1})$ whose U(1) charges[7] are given by

$$\bar{s}_1(1,4,0) \quad \text{and} \quad \bar{s}_2(1,1,5). \tag{23}$$

Their vacuum expectation values, $\langle\bar{s_1}\rangle$ and $\langle\bar{s_2}\rangle$ are expected to be of order of the SU(5) GUT scale $\sim 10^{16}$ GeV.

[6] The observed mass for the strange quark seems somewhat smaller than the SU(5) GUT value[12].

[7] These $\bar{s}_i(\mathbf{1})$ are regarded as SU(5) singlet components of $\mathbf{56}$ of E_7.

Majorana masses for N_i are induced from nonrenormalizable interactions of a form;[8]

$$W_N = \frac{\epsilon^2}{M_G} N_i N_j \bar{s}_k \bar{s}_l. \tag{24}$$

Here, M_G is the gravitational scale $M_G \simeq 2.4 \times 10^{18}$ GeV. Then, the matrix of the Majorana masses takes the following form;[9]

$$M_{\nu_R} = \frac{1}{M_G} \begin{pmatrix} \epsilon_1^2 \bar{s}_2{}^2 & \epsilon_0\epsilon_1 \bar{s}_2{}^2 & \epsilon_0\epsilon_1 \bar{s}_1\bar{s}_2 \\ \epsilon_0\epsilon_1 \bar{s}_2{}^2 & \epsilon_0^2 \bar{s}_2{}^2 & \epsilon_0^2 \bar{s}_1\bar{s}_2 \\ \epsilon_0\epsilon_1 \bar{s}_1\bar{s}_2 & \epsilon_0^2 \bar{s}_1\bar{s}_2 & \epsilon_0^2 \bar{s}_1{}^2 \end{pmatrix}, \tag{25}$$

where all elements are multiplied by undetermined factors of O(1) like in the case for quarks and leptons.

The neutrino masses are given by[3]

$$m_\nu \simeq m_{\nu_D} M_{\nu_R}^{-1} m_{\nu_D}^T, \tag{26}$$

where

$$(m_{\nu_D})_{ij} = c_{ij} Y_{\nu ij} \langle \mathbf{5}_H \rangle. \tag{27}$$

Three eigenvalues of the matrix in eq.(26) are of order, $m_{\nu_1} \sim \epsilon_1^2 M_G \langle \mathbf{5}_H\rangle^2/\langle \bar{s}_2\rangle^2$, $m_{\nu_2} \sim \epsilon_0^2 M_G \langle \mathbf{5}_H\rangle^2/\langle \bar{s}_2\rangle^2$ and $m_{\nu_3} \sim \epsilon_0^2 M_G \langle \mathbf{5}_H\rangle^2/\langle \bar{s}_1\rangle^2$. It is remarkable that for $\langle \mathbf{5}_H\rangle \sim 100$GeV, $\epsilon_0 \sim 1$ and $\langle \bar{s}_i\rangle \sim 10^{16}$GeV we get the desired mass for neutrino $m_{\nu_i} \sim 0.1$ eV.

From the Mikheev-Smirnov-Wolfenstein solution(MSW)[5] to the solar neutrino problem, we have[13, 14]

$$\delta m^2_{\nu_e\nu_\mu} \simeq 10^{-6} - 10^{-5}\mathrm{eV}^2. \tag{28}$$

We see that there are two choices

$$\left(\frac{\langle \bar{s}_1\rangle}{\langle \bar{s}_2\rangle}\right)^2 \sim 10^{-2} - 10^{-1} \text{ or } \left(\frac{\langle \bar{s}_2\rangle}{\langle \bar{s}_1\rangle}\right)^2 \sim 10^{-2} - 10^{-1} \tag{29}$$

to account for atmospheric and solar neutrino anomalies, simultaneously. Thus, all off-diagonal elements of the diagonalization matrix for the neutrino mass matrix in eq.(26) are of O(0.1) in either cases.

However, it is very interesting that the mixing angle for lepton doublets which mixes charged leptons in the second and the third family is of order $\tan\theta$ (see eq.(19)) and hence of the order 1. This means, together with the above result, that the weak mixing angle relevant for ν_μ-ν_τ oscillation can be so large, $\sin^2 2\theta_{\nu_\mu\nu_\tau} \simeq 1$, as required for explaining the observed atmospheric neutrino anomaly. On the other hand, the mixing angle for $\nu_\mu - \nu_e$ oscillation is very small, $\theta_{\nu_\mu\nu_e} \sim$ O(0.1), which may fit the small angle MSW solution[13, 14] to the solar neutrino problem.

[8] Other mass terms such as $\epsilon^2 N_i N_j$ can be forbidden by some chiral symmetry.

[9] The mass term of the form $\epsilon^4 N_i N_j$ may produce a similar form to eq.(25) if $\bar{\epsilon}_0 = 0$ and $\bar{\epsilon}_1, \bar{\epsilon}_2 \neq 0$.

4 Conclusion and Discussion

In this talk we have shown that the coset-space family unification on $E_7/SU(5)\times U(1)^3$ naturally accommodates the large lepton mixing, $\sin^2 2\theta_{\nu_\mu \nu_\tau} \simeq 1$, necessary for explaining the atmospheric neutrino anomaly reported by the SuperKamiokande collaboration[1]. The main reason why we have a large mixing of the SU(2) lepton doublets in the second and the third family is the twisted structure of family. Namely, the $\mathbf{5}^*$'s in the second and the third family both live on the same coset-subspace $E_7/E_6\times U(1)$. On the other hand the **10**'s in the third, the second and the first family live on the separate coset-subspaces, $E_7/E_6\times U(1)$, $E_6/SO(10)\times U(1)$ and $SO(10)/SU(5)\times U(1)$, respectively. This unparallel family structure is an unique feature of the present coset-space family unification.

It is quite natural that the NG multiplets carry no $U(1)_R$ charge. Thus, the dangerous lower ($d = 4, 5$) dimensional operators contributing to proton decays are forbidden by imposing the R-invariance $U(1)_R$. However, the R invariance is broken at the gravitino scale at least and hence we may expect small R-violating $d = 4$ operators.

The existence of approximate global E_7 symmetry is the most crucial assumption in our coset-space family unification. We hope that it is understood by some underlying physics at the gravitational scale. The Horava and Witten M theory[15] will be a hopeful example, since it is known[16] that there appear enhanced global symmetries on the 10 dimensional boundary of 11 dimensional space-time .

References

[1] SuperKamiokande Collaboration, T. Kajita, this proceeding; Y. Fukuda et al, hep-ex/9807003.

[2] Kamiokande Collaboration, K.S. Hirata et al., Phys. Lett. B 205 (1988) 416; Phys. Lett. B 280 (1992) 146; Y. Fukuda *et al.*, Phys. Lett. B 335 (1994) 237;
IMB Collaboration, D. Casper *et al.*, Phys. Rev. Lett. 66 (1991) 2561; R. Becker-Szendy *et al.*, Phys. Rev. D 46 (1992) 3720;
SOUDAN2 Collaboration, T. Kafka, Nucl. Phys. B (Proc. Suppl.) 35 (1994) 427; M. C. Goodman, Nucl. Phys. B (Proc. Suppl.) 38 (1995) 337; W. W. M. Allison et al., Phys. Lett. B 391 (1997) 491.

[3] T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe*, edited by A. Sawada and H. Sugawara, (KEK, Thukuba, Japan, 1979);
M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, edited by F. van Nieuwenhuizen and D. Freedman, (North Holland, 1979).

[4] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B 147 (1979) 277.

[5] L. Wolfenstein, Phys. Rev. D 17 (1978) 2369;
S. P. Mikheev and A. Yu. Smirnov, Sov. J. Nucl. Phys. 42 (1985) 913.

[6] Homestake Collaboration, K. Lande. this proceeding;
SuperKamiokande & Kamiokande Collaboration, Y. Suzuki, this proceeding;
GALLEX Collaboration, T. Kirstenys, this proceeding;
SAGE Collaboration, V. N. Gavri, this proceeding.

[7] W. Buchmuller, R. D. Peccei and T. Yanagida, Nucl. Phys. B 227 (1983) 503.

[8] T. Kugo and T. Yanagida, Phys. Lett. B 134 (1984) 313.

[9] T. Yanagida and Y. Yasui, Nucl. Phys. B 269 (1986) 576.

[10] J. Sato and T. Yanagida, Phys. Lett. B430 (1998) 127.

[11] G. Moore and P. Nelson, Phys. Rev. Lett. 53 (1984) 1519; Commun. Math Phys. 100 (1985) 83;
P. di Vecchia, S. Ferrara and L. Girardello, Phys. Lett. B 151 (1985) 199;
E. Cohen and C. Gomez, Nucl. Phys. B 254 (1985) 235.

[12] Particle Data Group, Phys. Rev. D 54 (1996) 1.

[13] J. N. Bahcall and P. I. Krastev, Phys. Rev. D 53 (1996) 4211;
G. L. Fogli, E. Lisi and D. Montanino, Astropart.Phys.9 (1998) 119.

[14] See, for a review, M. Fukugita and T. Yanagida, in *Physics and Astrophysics of Neutrinos*, edited by M. Fukugita and A. Suzuki (Springer-Verlag, Tokyo, 1994).

[15] P. Horava and E. Witten, Nucl. Phys. B460 (1996) 506; Nucl. Phys. B475 (1996) 94.

[16] E. Sharpe, Nucl. Phys. B523 (1998) 211.