

# CP and T violation test in neutrino oscillation

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## Abstract

I examine how large violation of CP and T is allowed in long base line neutrino experiments. When we attribute both the atmospheric neutrino anomaly and the solar neutrino deficit to neutrino oscillation we may have a sizable T violation effect proportional to the ratio of two mass differences; it is difficult to see CP violation since we can't ignore the matter effect. I give a simple expression for T violation in the presence of matter.

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# 1 Introduction

The CP and T violation is a fundamental and important problem of the particle physics and cosmology. We have examined CP violation in the quark sector. Is there CP violation in the lepton sector?

The answer is 'Yes' if the neutrinos have masses and complex mixing angles in the electroweak theory.

The neutrino oscillation search is a powerful experiment which can examine masses and mixing angles of the neutrinos. In fact the several underground experiments have shown lack of the solar neutrinos[1, 2, 3, 4] and anomaly in the atmospheric neutrinos[5, 6, 7, 8, 9], implying that the neutrinos have masses. The solar neutrino deficit implies the mass difference of  $10^{-5} - 10^{-4} \text{ eV}^2$ . atmospheric neutrino anomaly suggests mass difference around  $10^{-3} \sim 10^{-2} \text{ eV}^2$ [10, 11, 12].

The latter fact encourages us to make long base line neutrino experiments. Recently such experiments are planned and will be operated in the near future[13, 14]. It seems necessary for us to examine whether there is a chance to observe not only the neutrino oscillation but also the CP or T violation by long base line experiments. Here I show such possibilities taking account of the atmospheric neutrino experiments and considering the solar neutrino experiments. I consider T violation mainly. This talk is based on the work with J. Arafune[15].

## 2 Formulation of CP and T violation in neutrino oscillation

### 2.1 Assumptions

First I show assumptions in this letter.

I assume there are three generations of neutrinos  $\nu_e, \nu_\mu$  and  $\nu_\tau$ . I do not assume the existence of a sterile neutrino.

I also assume that the solar neutrino deficit and the atmospheric neutrino anomaly are attributed to neutrino oscillation, that is one of the two mass square difference is around  $10^{-5} - 10^{-4} \text{ eV}^2$  and the other is in the range of  $10^{-3} - 10^{-2} \text{ eV}^2$ . I denote the smaller mass square difference by  $\delta m_{21}^2$  and the larger by  $\delta m_{31}^2$ .

$$\begin{aligned}\delta m_{21}^2 &\sim 10^{-5} - 10^{-4} \text{ eV}^2 \\ \delta m_{31}^2 &\sim 10^{-3} - 10^{-2} \text{ eV}^2\end{aligned}\tag{1}$$

## 2.2 Brief review

I briefly review CP and T violation in vacuum oscillation[16, 17, 18].

I will use the parametrisation for mixing matrix  $U$  by Chau and Keung[19, 20, 21],

$$\begin{aligned}
 U &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\psi & s_\psi \\ 0 & -s_\psi & c_\psi \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} c_\phi & 0 & s_\phi \\ 0 & 1 & 0 \\ -s_\phi & 0 & c_\phi \end{pmatrix} \begin{pmatrix} c_\omega & s_\omega & 0 \\ -s_\omega & c_\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 &\equiv U_\psi \Gamma U_\phi U_\omega,
 \end{aligned} \tag{2}$$

which is used by Particle Data Group.

The T violation gives the difference between the transition probability of  $\nu_\alpha \rightarrow \nu_\beta$  and that of  $\nu_\beta \rightarrow \nu_\alpha$ [22]:

$$\begin{aligned}
 &P(\nu_\alpha \rightarrow \nu_\beta; E, L) - P(\nu_\beta \rightarrow \nu_\alpha; E, L) \\
 &= 4 \sin \omega \cos \omega \sin \psi \cos \psi \sin \phi \cos^2 \phi \sin \delta (\sin \Delta_{21} + \sin \Delta_{32} + \sin \Delta_{13}) \\
 &\equiv 4Jf,
 \end{aligned} \tag{3}$$

where

$$\begin{aligned}
 \Delta_{ij} &\equiv \delta m_{ij}^2 \frac{L}{2E} = 2.54 \frac{(\delta m_{ij}^2 / 10^{-2} \text{eV}^2)}{(E / \text{GeV})} (L / 100 \text{km}), \\
 J &\equiv \sin \omega \cos \omega \sin \psi \cos \psi \sin \phi \cos^2 \phi \sin \delta \\
 f &\equiv (\sin \Delta_{21} + \sin \Delta_{32} + \sin \Delta_{13}).
 \end{aligned}$$

$\delta m_{ij}^2$  is mass square difference.  $L$  is the distance between the production point and the detection point.  $E$  is the neutrino energy.

In the vacuum the CPT theorem gives the relation between the transition probability of anti-neutrino and that of neutrino,

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta; E, L) = P(\nu_\beta \rightarrow \nu_\alpha; E, L), \tag{4}$$

which relates CP violation to T violation:

$$\begin{aligned}
 &P(\nu_\alpha \rightarrow \nu_\beta; E, L) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta; E, L) \\
 &= P(\nu_\alpha \rightarrow \nu_\beta; E, L) - P(\nu_\beta \rightarrow \nu_\alpha; E, L).
 \end{aligned} \tag{5}$$

## 2.3 CP and T violation in long base line experiments

Let's consider how large the T(CP) violation can be in long base line experiments with the assumed mass square differences eq.1. In long baseline experiments neutrinos have energy of several GeV's and the distance of several hundreds km.  $|\Delta_{31}| \sim O(1)$  and  $|\Delta_{21}| = |\epsilon\Delta_{31}| \ll 1$  in this case, where  $\epsilon \equiv \frac{\delta m_{21}}{\delta m_{31}}$ . The oscillatory part  $f$  becomes  $O(\epsilon)$ :

$$\begin{aligned} f(\Delta_{31}, \epsilon) &= \sin \Delta_{21} + \sin \Delta_{32} + \sin \Delta_{13} \\ &= \sin(\epsilon\Delta_{31}) + \sin\{(1 - \epsilon)\Delta_{31}\} - \sin \Delta_{31} \end{aligned} \quad (6)$$

$$= \epsilon\Delta_{31}(1 - \cos \Delta_{31}) + O(\epsilon^2\Delta_{31}^2). \quad (7)$$

Fig.1 shows the graph of  $f(\Delta_{31}, \epsilon = 0.03)$ . The approximation eq.7 works very well up to  $|\epsilon\Delta_{31}| \sim 1$ . In the following we will use eq.7 instead of eq.6. We see many peaks of  $f(\Delta_{31}, \epsilon)$  in fig.1. In practice, however, we do not see such sharp peaks but observe the value averaged around there, for  $\Delta_{31}$  has a spread due to the energy spread of neutrino beam ( $|\delta\Delta_{31}/\Delta_{31}| = |\delta E/E|$ ). In the following we will assume  $|\delta\Delta_{31}/\Delta_{31}| = |\delta E/E| = 20\%$ [23] as a typical value.

Table 1 gives values of  $f(\Delta_{31}, \epsilon)/\epsilon$  at the first several peaks and the averaged values around there.

We see the T violation effect,

$$\langle P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha) \rangle_{20\%} = 4J \langle f \rangle_{20\%} = J\epsilon \times \begin{cases} 25.9 \\ 56.0 \\ 62.4 \\ \vdots \end{cases} \text{ for } \Delta_{31} = \begin{cases} 3.67 \\ 9.63 \\ 15.8 \\ \vdots \end{cases} \quad (8)$$

at peaks for neutrino beams with 20 % of energy spread. Note that the averaged peak values decrease with the spread of neutrino energy.

Which peak we can reach depends on  $\delta m_{31}^2, L$  and  $E$ . The first peak  $\Delta_{31} = 3.67$  is reached, for example by  $\delta m_{31}^2 = 10^{-2} \text{ eV}^2$ ,  $L = 250 \text{ km}$  (for KEK-Kamiokande long base line experiment) and neutrino energy  $E = 1.73 \text{ GeV}$ .

## 3 T violation

Neutrinos, however, go through not vacuum but matter, the surface of the earth. There is an effect by matter. The matter effect is calculated like this[24, 25]:

$$2\sqrt{2}G_F n_e E \sim 2 \times 10^{-4} \text{ eV}^2 \left( \frac{E}{\text{GeV}} \right) \left( \frac{n}{3\text{g/c.c}} \right), \quad (9)$$

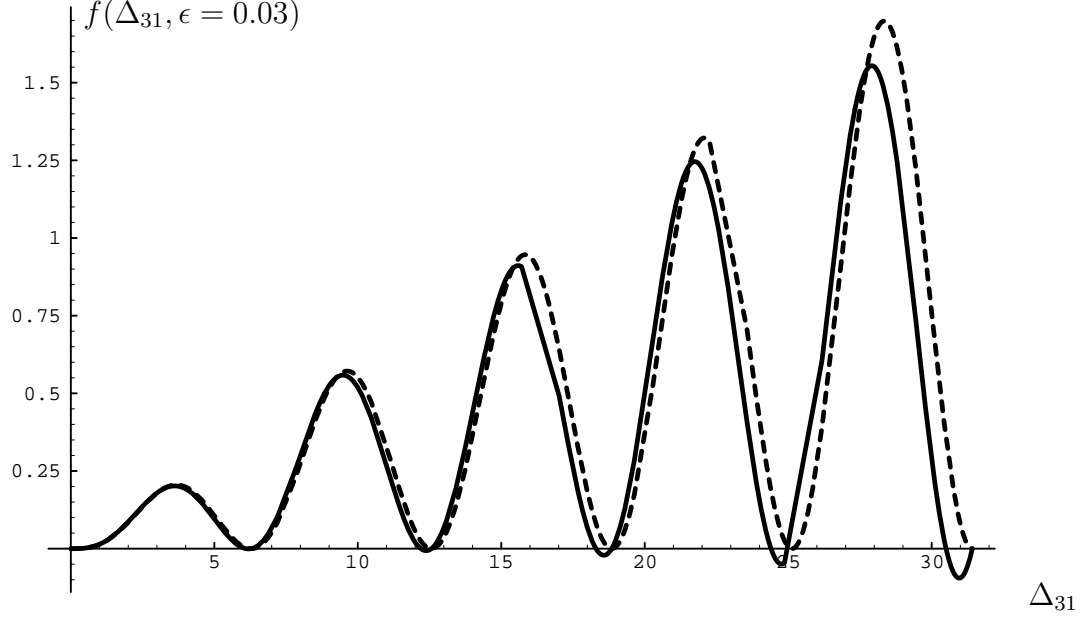


Figure 1: Graph of  $f(\Delta_{31}, \epsilon)$  for  $\epsilon = 0.03$ . The solid line and the dashed line represent the exact expression eq.6 and the approximated one eq.7, respectively. The approximated  $f$  has peaks at  $\Delta_{31} = 3.67, 9.63, 15.8, \dots$  irrespectively of  $\epsilon$ .

$\Delta_{31}$	$f/\epsilon$	$\langle f/\epsilon \rangle_{10\%}$	$\langle f/\epsilon \rangle_{20\%}$
3.67	6.84	6.75	6.48
9.63	19.1	17.6	14.0
15.8	31.5	25.7	15.6
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Table 1: The peak values of  $f(\Delta_{31}, \epsilon)/\epsilon$  and the corresponding averaged values. Here  $\langle f/\epsilon \rangle_{20\%(10\%)}$  is a value of  $f(\Delta_{31}, \epsilon)/\epsilon = \Delta_{31}(1 - \cos \Delta_{31})$ (see eq.7) averaged over the range  $0.8\Delta_{31} \sim 1.2\Delta_{31}$  ( $0.9\Delta_{31} \sim 1.1\Delta_{31}$ ).

where  $n_e$  is the electron number density of the earth and  $n$  is the matter density of the surface of the earth. For neutrino with several GeV energy the matter effect is much greater than the smaller mass square difference  $\delta m_{21}^2$ , though it will be much smaller than the larger mass square difference. Thus we cannot see pure CP violating effect. It requires to subtract such effect in order to deduce the pure CP violation effect[26]. In principle it is possible, because the matter effect is proportional to  $E$  while  $\delta m_{21}^2$  is constant.

In the matter with constant density<sup>1</sup>, we have a pure T violation effect  $P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha)$ , though we do not observe a pure CP violation effect because of an apparent CP violation due to matter. The pure T violation effect is given by the eq.3 with mixing matrix and mass square differences in matter.

### 3.1 T violation in matter

To calculate how large the T violation is, we have to know the mixing matrix and mass square differences in matter. When a neutrino is in matter, its matrix of effective mass squared  $M_m^2$  for weak eigenstates is[20, 21]

$$M_m^2 = U \begin{pmatrix} 0 & & \\ & \delta m_{21}^2 & \\ & & \delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix}, \quad (10)$$

where  $a = 2\sqrt{2}G_F n_e E$  and  $U$  is given by eq.2. This is diagonalized by a mixing matrix  $U_m$  as  $M_m^2 = U_m \text{diag}(\tilde{m}_1^2, \tilde{m}_2^2, \tilde{m}_3^2) U_m^\dagger$ .

Up to the leading order of  $\delta m_{21}^2$ , the the eigenvalues of effective mass square( $\Lambda_i$ ) and the mixing matrix  $U_m$  are given by,

$$\Lambda_1 = \frac{(a + \delta m_{31}^2) - \sqrt{(a + \delta m_{31}^2)^2 - 4a\delta m_{31}^2 \cos^2 \phi}}{2}, \quad (11)$$

$$\Lambda_2 = 0, \quad (12)$$

$$\Lambda_3 = \frac{(a + \delta m_{31}^2) + \sqrt{(a + \delta m_{31}^2)^2 - 4a\delta m_{31}^2 \cos^2 \phi}}{2}, \quad (13)$$

and

$$U_m = U_\psi \Gamma U_\phi \tilde{U}$$

where

$$\begin{aligned} \tan 2\phi' &= \frac{\delta m_{31}^2 \sin 2\phi}{\delta m_{31}^2 \cos 2\phi - a}, \\ \tilde{U} &= \delta_{ij} + \tilde{U}', \tilde{U}'^T = -\tilde{U}', \end{aligned}$$

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<sup>1</sup> Note that the time reversal of  $\nu_\alpha \rightarrow \nu_\beta$  requires the exchange of the production point and the detection point and the time reversal of  $P(\nu_\alpha \rightarrow \nu_\beta)$  in matter is in general different from  $P(\nu_\beta \rightarrow \nu_\alpha)$ [20].

with

$$\begin{aligned}
(\tilde{U}')_{12} &= \frac{\delta m_{21}^2}{\Lambda_1} \cos(\phi - \phi') \cos \omega \sin \omega \\
(\tilde{U}')_{13} &= \frac{\delta m_{21}^2}{\Lambda_3 - \Lambda_1} \cos(\phi - \phi') \sin(\phi - \phi') \sin^2 \omega \\
(\tilde{U}')_{23} &= \frac{\delta m_{21}^2}{\Lambda_3} \sin(\phi - \phi') \cos \omega \sin \omega.
\end{aligned}$$

Using them,  $J$  in matter,  $J_m$ , takes the form.

$$J_m = -\frac{\delta m_{21}^2}{a} \frac{\delta m_{31}^2}{\{(\delta m_{31}^2 + a)^2 - 4\delta m_{31}^2 a \cos^2 \phi\}^{1/2}} \sin \omega \cos \omega \sin \psi \cos \psi \sin \phi \sin \delta. \quad (14)$$

### 3.2 Most likely case: $\delta m_{21}^2 \ll a \ll \delta m_{31}^2$

For example we consider this case because as mentioned earlier this case seems most likely to be realized. I show the result calculated up to the leading order of two small parameter  $\epsilon_1 \equiv \frac{a}{\delta m_{31}^2}$  and  $\epsilon_2 \equiv \frac{\delta m_{21}^2}{a}$ .

Then we have the effective masses

$$\begin{aligned}
\tilde{m}_1^2 &\simeq \Lambda_1 \simeq a \cos^2 \phi, \\
\tilde{m}_2^2 &\simeq \Lambda_2 \simeq 0, \\
\tilde{m}_3^2 &\simeq \Lambda_3 \simeq \delta m_{31}^2 + a \sin^2 \phi.
\end{aligned} \quad (15)$$

and “mass square difference ratio” in matter

$$\epsilon_m = \frac{\tilde{m}_2^2 - \tilde{m}_1^2}{\tilde{m}_3^2 - \tilde{m}_2^2} \simeq -\frac{a \cos^2 \phi}{\delta m_{31}^2}. \quad (16)$$

Note that  $|\epsilon_m| \ll 1$ .

$J_m$  is given by,

$$J_m \sim -\frac{\delta m_{21}^2}{a} \sin \omega \cos \omega \sin \psi \cos \psi \sin \phi \sin \delta.$$

The key quantity  $J\epsilon$  in matter(see eq.8) is then,

$$J_m \epsilon_m = J\epsilon, \quad (17)$$

same as that of vacuum.

Using the argument similar to that used to derive equation for T violation in vacuum, we obtain the T violation effect

$$\langle P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha) \rangle_{20\%} = J_m \epsilon_m \times \begin{Bmatrix} 25.9 \\ 56.0 \\ 62.4 \\ \vdots \end{Bmatrix} = J\epsilon \times \begin{Bmatrix} 25.9 \\ 56.0 \\ 62.4 \\ \vdots \end{Bmatrix}, \quad (18)$$

at peaks, where we choose the mean neutrino energy  $E$  to satisfy (see Table 1)

$$\Delta_{31} = \delta m_{31}^2 \frac{L}{2E} = 3.67, 9.63, 15.8 \dots \quad (19)$$

The size of T violation in matter is same as that in vacuum. Incidentally I may remark that there is no correction of  $O(\epsilon_2)$  to this value because the limit  $a \rightarrow 0$  is smooth.

In the case of large mixing angles [11],  $J/\sin \delta \sim 0.06$  and  $\epsilon \sim 10^{-2}$  are allowed<sup>2</sup> for example. Then the size of T violation

$$\langle P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha) \rangle_{20\%} = \left( \frac{J/\sin \delta}{0.06} \right) \left( \frac{\epsilon}{10^{-2}} \right) \sin \delta \times \begin{Bmatrix} 0.015 \\ 0.033 \\ 0.037 \\ \vdots \end{Bmatrix}. \quad (20)$$

We may see a few % of T violation.

Numerically it is also confirmed with slowly changing density, which satisfies the condition  $a \ll \delta m_{31}^2$ , up to  $O(\epsilon_1)$ . I assumed the density takes the following form:

$$a = a_0 + a_1 \cos k_1(x - x_1) + a_2 \cos k_2(x - x_2),$$

with  $|a_0 + a_1 + a_2| \ll \delta m_{31}^2$  and various  $a_i, k_i$  and  $x_i$ .

## 4 Summary

We have examined mainly T violation in the long base line experiment with the assumption that  $\delta m_{21}^2$  is much smaller than the matter effect “ $a$ ” and  $\delta m_{31}^2$ , that is both the solar neutrino deficit and atmospheric neutrino anomalies are attributed to the neutrino oscillation violation effect. In this case we cannot see pure CP violation because of matter. A few % of T violation is, however, observable depending on the parameters for neutrino. More exactly we may see T violation of  $O(\frac{\delta m_{21}^2}{\delta m_{31}^2})$ .

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<sup>2</sup> Here  $\sin \omega \sim 1/2$ ,  $\sin \psi \sim 1/\sqrt{2}$  and  $\sin \phi = \sqrt{0.1}$ .



The reason of  $O(\frac{\delta m_{21}^2}{\delta m_{31}^2})$  is as follows: The probability of CP and T violation effect should vanish for  $\delta m_{21}^2 \rightarrow 0$ , and therefore be proportional to  $\delta m_{21}^2/\delta m_{31}^2$ ,  $\delta m_{21}^2/(E/L)$  or  $\delta m_{21}^2/a$  by the dimensional analysis. We expected an enhancement for T violation, that is, it is proportional to not  $O(\frac{\delta m_{21}^2}{\delta m_{31}^2})$  but  $O(\frac{\delta m_{21}^2}{a})$ . We have, however, CP and T violation of  $O(\frac{\delta m_{21}^2}{\delta m_{31}^2})$  because in the limit that  $a \rightarrow 0$  they must be smoothly connected to the vacuum case.

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