

Impact of light polarization on chaos synchronization of mutually coupled VCSELs

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We demonstrate that dynamical polarization switching in mutually coupled VCSELs has a profound impact on the chaotic dynamics in the system drastically improving the **in-phase (anti-phase) synchronization between the modes of the two VCSELs with the same (orthogonal) polarizations.** Furthermore, we show that an exchange of the leader-laggard role is observed with the higher (lower) frequency VCSEL being the leader for lower (higher) coupling strength. © 2008 Optical Society of America

Synchronization of mutually coupled semiconductor lasers has attracted a lot of interest recently [1–5]. Locking of the relaxation oscillation frequencies has been observed in [1] with one of lasers entraining the other one. Subnanosecond synchronized chaotic dynamics have been demonstrated in [2] together with spontaneous symmetry breaking, i.e. the synchronization happens at a certain time lag between the dynamics of the two lasers with the higher optical frequency laser being the leader. Recently mutually coupled Vertical-Cavity Surface-Emitting Lasers (VCSELs) have been shown to exhibit chaos synchronization [6]. Solitary VCSELs can switch between two orthogonal linearly polarized (LP) fundamental modes when changing the injection current or temperature [7]. Due to these polarization instabilities mutual coupling can lead to a sequence of bistable polarization switchings (PS) when changing the coupling strength or optical phase [8,9]. When the two LP modes are dynamically excited they tend to synchronize antiphase [6,10]. **It has been recently demonstrated that the lower frequency VCSEL becomes the leader and injection locking has been iden-**

tified as responsible mechanism [11]. In this letter we demonstrate numerically that indeed, dynamical PS has a profound impact on the chaotic dynamics of mutually coupled VCSELs and can lead to an exchange of the leader-laggard role when increasing the coupling strength.

In order to account for the polarization properties of VCSEL we implement a phenomenological model that has recently explained several experimentally observed phenomena in VCSELs, such as PS and mode-hopping in a solitary VCSEL [7], multiple PS, delayed oscillations and coherence resonance in VCSELs with long external-cavity feedback [12]. The two-mode rate-equation model for mutually coupled VCSELs working in fundamental transverse mode regime reads:

$$\frac{dE_x^{1,2}}{dt} = \frac{1}{2} (1 + j\alpha_x^{1,2}) \left[\Gamma_x^{1,2} G_x^{1,2} - \frac{1}{\tau_{px}^{1,2}} \right] E_x^{1,2} + \quad (1)$$

$$\kappa_x^{2,1} E_x^{2,1} (t - \tau_c^{2,1}) \exp(-j\omega_{0x} \tau_c^{2,1}) \pm j\Delta\omega_x E_x^{1,2},$$

$$\frac{dE_y^{1,2}}{dt} = \frac{1}{2} (1 + j\alpha_y^{1,2}) \left[\Gamma_y^{1,2} G_y^{1,2} - \frac{1}{\tau_{py}^{1,2}} \right] E_y^{1,2} + \quad (2)$$

$$\kappa_y^{2,1} E_y^{2,1} (t - \tau_c^{2,1}) \exp(-j\omega_{0y} \tau_c^{2,1}) \pm j\Delta\omega_y E_y^{1,2},$$

$$\frac{dN^{1,2}}{dt} = \frac{J^{1,2}}{eV^{1,2}} - \frac{N^{1,2}}{\tau_e^{1,2}} - G_x^{1,2} |E_x^{1,2}|^2 - G_y^{1,2} |E_y^{1,2}|^2. \quad (3)$$

Here $E_x^{1,2}$ and $E_y^{1,2}$ are the slowly varying x- and y- LP components of the electric fields in VCSELs 1 and 2 defined in a symmetric reference frame for each polarization, $\omega_{0x,0y} = (\omega_{x,y}^1 + \omega_{x,y}^2)/2$; $\Delta\omega_{x,y} = (\omega_{x,y}^1 - \omega_{x,y}^2)/2$ with averaged frequencies $\omega_{0x,0y}$ and frequency detunings $\Delta\omega_{x,y}$. $\alpha_{x,y}^{1,2}$ are the linewidth enhancement factors of the two VCSELs, $\Gamma_{x,y}^{1,2}$ are the confinement factors and $\tau_{px,y}^{1,2}$ are the corresponding photon lifetimes which can

also be different for the two orthogonal LP modes. The mutual coupling is characterized by the coupling-trip delays $\tau_c^{1,2}$ and the coupling strength $\kappa_{x,y}^{1,2}$. The coupling-trip delays $\tau_c^{1,2}$ are given by $\tau_c^{1,2} = 2L^{1,2}/c$ with $L^{1,2}$ being the distances for coupling of VCSEL 1 to VCSEL 2 and vice versa and c - the speed of light. The coupling strength, i.e. the rate of the injection from one laser into the other, is $\kappa_{x,y}^{1,2} = \eta_{x,y}^{1,2}(1 - R^{1,2})/(\tau_{in}^{2,1}\sqrt{R^{1,2}})$, $R^{1,2}$ being the reflectivity of the front mirror, which we consider the same for the two orthogonal polarizations and for the two VCSELs; $\eta_{x,y}^{1,2}$ are the coupling efficiencies and $\tau_{in}^{1,2}$ are the photon round-trip times in the cavities taken the same for the two LP modes. In the rate equations for the carrier densities Eqns. (3), $N^{1,2}$ are the carrier densities, $J^{1,2}$ are the injection currents, $V^{1,2}$ are the active region volumes and $\tau_e^{1,2}$ are the carrier lifetimes. The gains for the two LP modes are given by $G_{x(y)}^{1,2} = g_{x(y)}^{1,2} (N^{1,2} - N_{tr}^{1,2}) \left(1 - \epsilon_s^{1,2} |E_{x(y)}^{1,2}|^2 - \epsilon_c^{1,2} |E_{y(x)}|^2 \right)$ where $N_{tr}^{1,2}$ are the carrier densities at transparency which are assumed to be the same for the x- and y- LP modes and $g_{x,y}^{1,2}$ are the differential gains for the x and y LP modes in VCSEL 1/2. The gain compression is taken into account through $\epsilon_{s(c)}^{1,2}$ - the self (cross) gain saturation coefficients. The VCSEL parameters, identical for the two lasers, are fixed as: $\alpha^{1,2} = 3$, $N_{tr}^{1,2} = 4 \times 10^6 \mu m^{-3}$, $\tau_{px,y}^{1,2} = 1.3ps$, $\tau_e^{1,2} = 1ns$, $V^{1,2} \approx 0.3\mu m^3$, $\tau_{in}^{1,2} = 0.045ps$, $\Gamma^{1,2} = 0.06$, $\epsilon_s^{1,2} = \epsilon_c^{1,2}/2 = 2 \times 10^{-5} \mu m^{-3}$.

Fig.1 shows the time traces of the output powers of the two VCSELs for a coupling strengths of $\kappa = 30GHz$ for the case when the polarization mode competition is turned off, i.e. $g_x^{1,2} = 3.54 \times 10^{-3} \mu m^3/ns$ and $g_y^{1,2} = 0$. The injection current is a few times the threshold current, **i.e.** $J = 1.2mA = 3J_{th}$, and the output power is time-averaged at $0.1\tau_c$ ($\tau_c = 3.2ns$)

to account for the typical time response of the detection equipment. As can be seen from this figure the two lasers exhibit well developed chaotic dynamics. Next, we ensure that the two VCSELs are polarization bistable by taking $g_y^{1,2} = g_x^{1,2} = 3.54 \times 10^{-3} \mu m^3 / ns$ and keeping the rest of the parameters the same. The time traces of the polarization resolved output powers of the two VCSELs are shown in Fig. 2. As can be seen from this figure, a profound change of the dynamics is settled by the polarization mode competition - multiple PS are evident with residence time in a certain LP state related to the coupling-trip delay τ_c . Comparing Fig. 2(a) (2(b)) with Fig. 2(c) (2(d)) reveals that the x and y LP modes of VCSEL 1 (VCSEL 2) feature antiphase dynamics. The strong anticorrelation between the VCSELs' LP modes occurs on a slower time-scale than the fast chaotic pulsing in the mode dynamics and the time-averaging of the polarization resolved output powers makes the anticorrelation clearer.

Due to this strong anticorrelation the changes of the total power of each VCSEL are much less than the corresponding changes of the LP mode powers (for the case of Fig.2 the corresponding standard deviations are 0.002 and 0.013). The maximum correlation between VCSEL 1 and VCSEL 2 is found when the time-traces are lagged by $\pm\tau_c$. In order to quantify the amount of synchronization between VCSEL 1 and VCSEL 2 we calculate the cross correlation coefficients $C_{s,p}^{1,2}$ for time lags of $\pm\tau_c$:

$$C_{s,p}^{1,2}(\pm\tau_c) = \frac{\langle P_s^1(t \pm \tau_c) - \bar{P}_s^1 \rangle \langle P_p^2(t) - \bar{P}_p^2 \rangle}{\sigma_s^1 \sigma_p^2}. \quad (4)$$

Here $\langle \dots \rangle$ denotes time average, $\bar{P}_{s,p}^{1,2}$ and $\sigma_{s,p}^{1,2}$ are the mean values and the standard

deviations of s and p LP states, which take values of x or y . For the time trace of the single LP mode VCSELs in Fig.1 the correlation is small $C_{x,x}^{1,2}[\tau_c(-\tau_c)] = 0.27(0.23)$. However, the profound change of the dynamics in Fig.2 as a result of the strong polarization mode competition leads to about threefold (fourfold) increase of the correlation coefficients $C_{x,x}^{1,2}[\tau_c(-\tau_c)] = 0.79(0.83)$. **The correlation coefficients depend slightly on time of averaging (e.g. $C_{x,x}^{1,2}[\tau_c(-\tau_c)] = 0.73(0.78)$ for twice decreased time of averaging).**

Well pronounced antiphase dynamics is observed in Fig. 2 for the orthogonal LP modes: $C_{x,y}^{1,2}[\tau_c(-\tau_c)] = -0.78(-0.83)$. Polarization antiphase dynamics has been experimentally and numerically demonstrated in mutually coupled VCSELs in a regime of low frequency fluctuations [6, 10]. What was not realized however was that polarization dynamics can drastically improve the synchronization properties as we demonstrate hereby for the case of well developed chaotic dynamics at a current high above the threshold.

We now proceed to show that the polarization mode competition drastically impacts the role of leader-laggard in mutually coupled VCSELs. If $C_{s,p}^{1,2}(-\tau_c)$ is larger than $C_{s,p}^{1,2}(\tau_c)$ laser 1 leads the dynamics and viceversa. The correlation coefficients are plotted in Fig.3 as a function of the frequency detuning between the two VCSELs for a fixed coupling strength of $\kappa^{1,2} = 30GHz$. Then, it becomes clear from Fig.3 that the higher frequency VCSEL is, to some extent, the leading one - for positive detuning the correlation is better when VCSEL 1 is advanced by τ_c . This is in agreement with the results on edge emitting lasers [2]. However, increasing the coupling strength to $\kappa = 60GHz$ we observe a clear exchange of the leader-laggard role - see Fig.4. **This rather surprising result is actually in a very**

good agreement with recent experiments on mutually coupled VCSELs [11]. Also in agreement with the experiment is the very high degree of antiphase synchronization for the orthogonal LP states in Fig.4(b). Finally, to better illustrate the exchange of the leader laggard role we show in Fig.5 the dependencies of the correlation coefficients on the coupling strength for a fixed detuning of $\Delta\nu = 10GHz$. The exchange of the leader - laggard role happens at about $\kappa^{1,2} = 36GHz$, however this value slightly increases with the frequency detuning (e.g. $\kappa^{1,2} = 37GHz$ at $\Delta\nu = 15GHz$).

In conclusion we have shown that polarization mode competition dramatically impacts the chaotic dynamics of mutually coupled VCSELs and improves the inphase (antiphase) synchronization quality between the two VCSEL modes with the same (orthogonal) polarizations. A slight tendency of the higher frequency VCSEL to lead the dynamics is observed for low coupling strength, however the lower frequency VCSELs becomes the leader as the coupling strength increases. **The high degree of correlation/anticorrelation and the leader-laggard relationship are preserved for a slight difference in the polarization gains ($< 0.3\%$ for the set of parameters we use), such that the two VCSELs keep emitting in two LP modes.**

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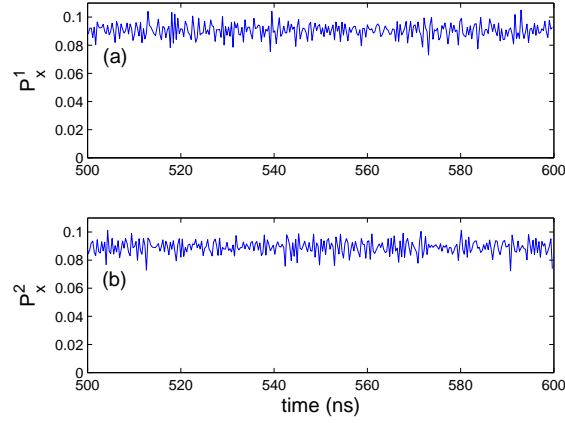


Fig. 1. Time traces of the x LP modes of VCSEL 1 (a) and 2 (b) for injection currents of $J = 1.2mA$, frequency detuning $\Delta\nu = 10GHz$ and coupling strength $\kappa_{1,2} = 30GHz$.

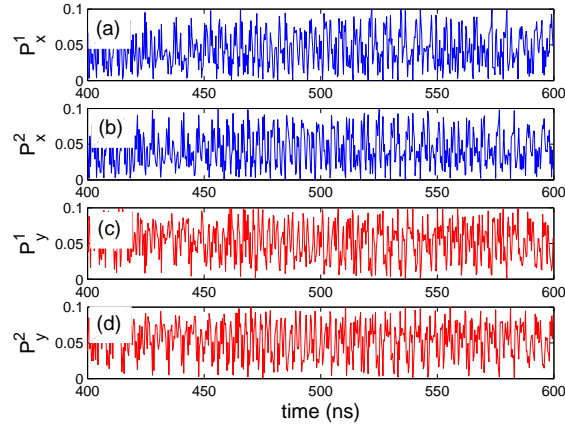


Fig. 2. Time traces of the x LP modes ((a) and (b)) and y LP modes ((c) and (d)) of VCSEL 1 ((a) and (c)) and VCSEL 2 ((b) and (d)) for injection currents of $J = 1.2mA$, frequency detuning $\Delta\nu = 10GHz$ and coupling strength $\kappa_{1,2} = 60GHz$.

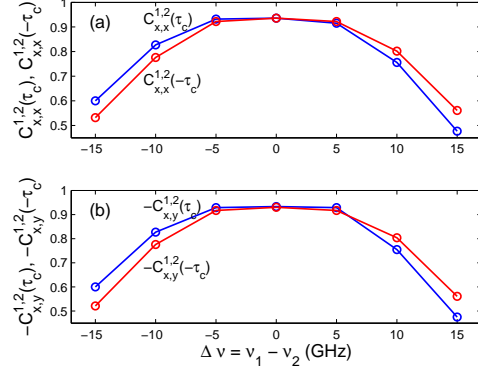


Fig. 3. Correlation coefficients between the x LP states (a) and x and y LP states (b) of VCSELs 1 and 2 as a function of the frequency detuning $\Delta\nu$ between them for a fixed coupling strength of $\kappa^{1,2} = 30 \text{ GHz}$.

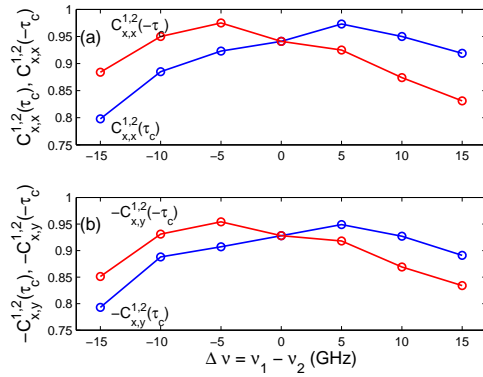


Fig. 4. Same as Fig.3 but for $\kappa^{1,2} = 60 \text{ GHz}$.

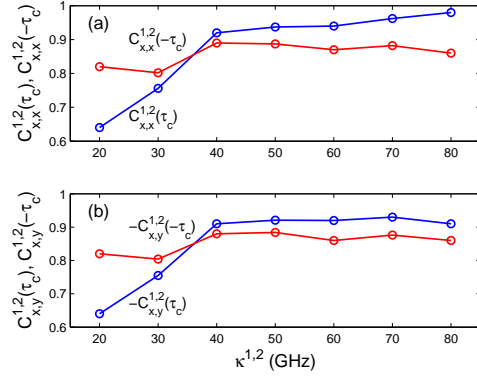


Fig. 5. Correlation coefficients between the x LP states (a) and x and y LP states (b) of VCSELs 1 and 2 as a function of the coupling strength $\kappa^{1,2}$ for a fixed frequency detuning of $\Delta\nu = 10GHz$.