# Phase-Locked Loops for the Stabilization of Active Magnetic Suspensions*

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A new active magnetic suspension system is proposed which has a displacement sensor of frequency type and uses phase-locked loop techniques for stabilization. This is termed the phase-locked magnetic suspension system (PLMS), and is studied theoretically with its basic model. It is shown that a PLMS can be stabilized with a loop filter of second or higher order. In the designed feedback system, the control input is produced from the difference in phase between the reference and feedback signals. Since phase is the integration of angular frequency, the open-loop transfer function automatically contains an ideal integration. This results in a steady-state position error of zero for step disturbances. It is also pointed out that a PLMS of self-sensing type can be realized with a switching power amplifier whose switching frequency varies with the position of the suspended object.

***Key Words***: Magnetic Bearing, Magnetic Levitation, Phase-Locked Loop, Frequency Modulation, Self-Sensing Magnetic Bearing, Sensorless Magnetic Bearing

## 1. Introduction

Sensors for detecting the position of a suspended object play an important role in active magnetic suspension systems. Most of these systems use an analog type of sensor that converts changes in the gap between the suspended object and the sensor pick-up into changes in the amplitude of an output signal. This type of sensor often uses amplitude modulation techniques. In a conventional inductive sensor, for example, high-frequency AC excitation called carrier is applied to the coil, and the variation in its amplitude due to the change in position is demodulated and filtered.

The frequency modulation as well as the amplitude modulation are most commonly used in communications technology. These can be applied to sensor technology also. For example, an LC oscillator with one of its inductors or capacitors subjected to change with change of the gap, produces a signal whose frequency varies with the position of the suspended object. A magnetic suspension system with the frequency type of sensor can be stabilized by directly feeding back the frequency modulated signal. The phase-locked loops (PLLs) is applied for this purpose in this paper. Such a suspension system is referred to as phase-locked active magnetic suspension system (PLMS).

The PLLs are widely used in frequency synthesizers[1], telecommunication receivers[2], and precise control of motor speed[3]. A PLL contains three basic components: a phase detector (PD), a loop filter, and a voltage-controlled oscillator (VCO). In a PLMS the amplifier-mass-sensor system works as a VCO because an input voltage to the power amplifier changes the position of the suspended object through electromagnetic force, which in turn changes the output frequency of the sensor.

In PLLs, the control input is produced from the difference in phase between the reference and feedback signals. Thus, an ideal integration is automatically contained in the loop. This results in a theoretical steady-state position error of zero for step disturbances.

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One method of reducing the size and cost of active magnetic suspension systems is to use the coils of electromagnets as sensor and force coils[(4)~(8)]. This type of magnetic suspension system is called self-sensing or sensorless. It is pointed out that a PLMS of self-sensing type can be realized with a switching power amplifier whose switching frequency varies with the position of the suspended object.

## 2. Basic Model

The basic model of a PLMS in the one-sided case is shown in Fig. 1. It consists of an object to be suspended, an electromagnet, an amplifier, a displacement sensor of frequency type, a reference oscillator, a phase detector and a loop filter. The equation of motion is given by

$$m\frac{d^2x}{dt^2}=F_e-F_s \tag{1}$$

where

$F_e$ : electromagnetic force acting on the suspended object.

$F_s$ : stationary force acting on the suspended object.
(A typical example is gravity force.)

The stationary value of $F_e$ is set to balance $F_s$. Assuming that the force-displacement-current relation of the electromagnet is linear, Eq. (1) becomes

$$m\frac{d^2x}{dt^2}=K_sx+K_ii \tag{2}$$

where

$K_s$, $K_i$ : coefficients of the electromagnet.

The transfer function $G_p(s)$ is defined by

$$G_p(s)=\frac{x(s)}{i(s)}=\frac{b}{s^2-a} \tag{3}$$

where

$$a=\frac{K_s}{m},\ b=\frac{K_i}{m}.$$

Next, the components in the feedback loop are studied. The angular frequency $\omega$ of the sensor output is a function of the displacement of the suspended object :

$$\omega=\omega(x). \tag{4}$$

It is assumed that Eq. (4) may be linearized as

$$\omega(x)=\omega_r+\Delta\omega(x)$$
$$=\omega_r+K_\omega x \tag{5}$$

where the nominal frequency $\omega_r$ corresponds to the desired position of the suspended object. The reference oscillator produces a signal of the same frequency. Since frequency is the time derivative of phase, $\Delta\omega$ can be represented as

$$\Delta\omega=\frac{d\theta}{dt} \tag{6}$$

where

$\theta$ : the phase of the sensor signal.

Although a phase detector usually has nonlinear characteristics, it can be accurately modeled as a linear device when the loop is in lock[(1)]. Then it is assumed that the phase detector output voltage is proportional to the difference in phase between its inputs ; that is

$$\nu_d=K_d(\theta_r-\theta) \tag{7}$$

where

$K_d$ : phase detector gain factor

$\theta_r$ : the phase of reference signal $\left(\omega_r=\frac{d\theta_r}{dt}\right)$.

The loop filter will be used to stabilize the closed-loop system in the next section. When $F(s)$ is the transfer function of the loop filter, the voltage at the output of the low-pass filter can be described as

$$\nu_e(s)=F(s)\nu_d(s). \tag{8}$$

When the power amplifier is of current-controlled type, the amplifier changes the electromagnet current in proportion to its input ; that is

$$i(t)=K_a\nu_d(t). \tag{9}$$

With these assumptions, the PLMS in the one-sided case can be represented by a linear model shown in Fig. 2.

From Eqs. (3), (5), and (9), the frequency deviating from the reference frequency is related to the filter output as

$$\Delta\omega(s)=K_\omega G_p(s)K_a\nu_e(s). \tag{10}$$

From Eqs. (6), (7), (8), and (10), the linear transfer function relating $\theta_r$ and $\theta$ is

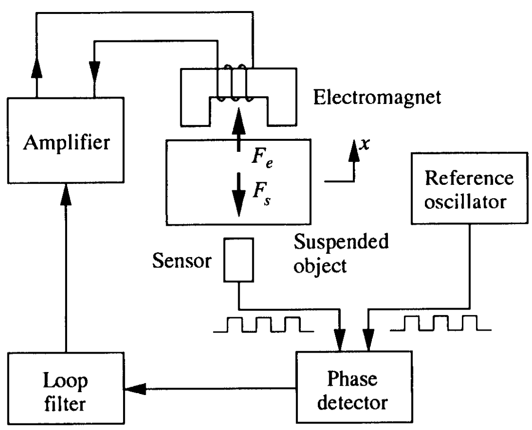


Fig. 1 Schematic diagram of PLMS in case of one-sided operation

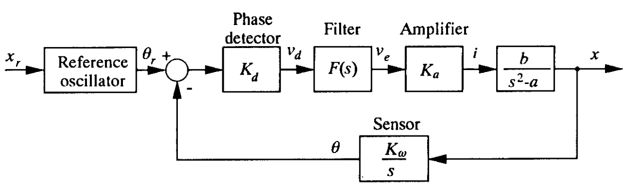


Fig. 2 Block diagram of PLMS using a linearized model in case of one-sided operation

$$T(s)=\frac{\theta(s)}{\theta_r(s)}=\frac{G(s)}{1+G(s)} \tag{11}$$

where

$$G(s)=\frac{K_aK_dK_\omega G_p(s)F(s)}{s}. \tag{12}$$

A PLMS with a pair of two opposite electromagnets is shown in Fig. 3. In this case a reference oscillator can be omitted. The equation of motion is given by

$$m\frac{d^2x}{dt^2}=F_1-F_2. \tag{13}$$

The magnetic forces are approximately given by

$$F_1=F_0+K_sx+K_ii_1 \tag{14}$$

$$F_2=F_0-K_sx+K_ii_2 \tag{15}$$

where

$i_1$, $i_2$ : control current.

It is assumed that the control currents satisfy

$$i_1=-i_2(=i). \tag{16}$$

Substituting Eqs. (14), ..., (16) into Eq. (13) gives

$$m\frac{d^2x}{dt^2}=2K_sx+2K_ii. \tag{17}$$

Then the transfer function $G_p(s)$ is defined by

$$G_p(s)=\frac{x(s)}{i(s)}=\frac{b}{s^2-a} \tag{18}$$

where

$$a=\frac{2K_s}{m},\ b=\frac{2K_i}{m}.$$

The frequency outputs are assumed to be given by

$$\omega_n(x)=\omega_r+\Delta\omega_n(x)\quad(n=1, 2) \tag{19}$$

$$\Delta\omega_1=-\Delta\omega_2=K_\omega x. \tag{20}$$

The two sensor output signals are compared in phase by the PD ; that is

$$\nu_d=K_d(\theta_2-\theta_1) \tag{21}$$

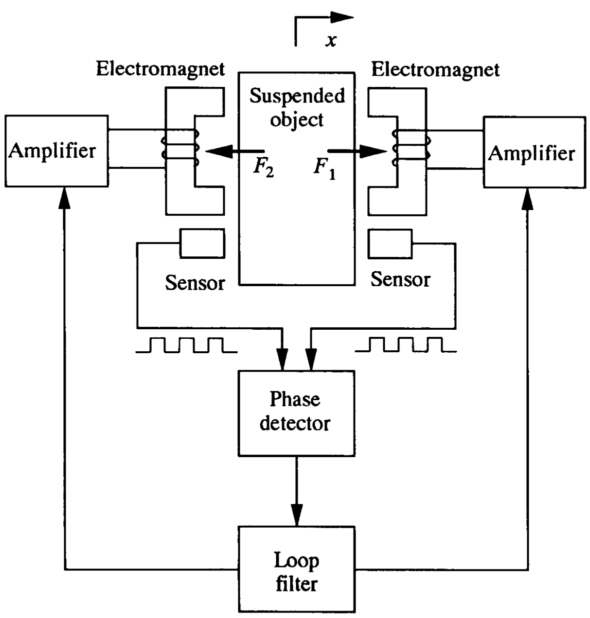


Fig. 3 Schematic diagram of PLMS in case of differential operation

where $\theta_n$ is the phase of the signal of the sensor $n$. Then, $\Delta\omega_n$ can be represented as

$$\Delta\omega_n=\frac{d\theta_n}{dt}\quad(n=1, 2) \tag{22}$$

For theoretical discussion, it is convenient to image the reference signal defined by

$$\theta_r=\frac{\theta_1+\theta_2}{2}\ \left(\Delta\omega_r=\frac{d\theta_r}{dt}\right). \tag{23}$$

Then, Eq. (21) becomes

$$\nu_d=2K_d(\theta_r-\theta_1)=-2K_d(\theta_r-\theta_2). \tag{24}$$

With these assumptions, the PLMS can be represented by the linear model shown in Fig. 4. Then the transfer function of the PLMS with a pair of magnets is obtained as

$$T(s)=\frac{\theta_n(s)}{\theta_r(s)}=\frac{G(s)}{1+G(s)}\quad(n=1, 2) \tag{25}$$

where

$$G(s)=\frac{2K_aK_dK_\omega G_p(s)F(s)}{s}. \tag{26}$$

Although the PLMS in differential operation is described by very nearly the same equations as those in one-sided operation, it has several advantages in practical use. One of them is that a hardware of reference oscillator is unnecessary. Another is suppression of drift of the suspended object due to some difference in frequency change with temperature between the reference oscillator and the sensor.

## 3. Stabilization

In order to design the loop filter $F(s)$, several performance criteria should be taken into consideration. These are stability, steady-state properties, transient response, pull-in characteristics of the PLL, robustness to parameter variations, and noise suppression. In this paper, however, attention is mostly paid to stabilization because of its importance in active magnetic suspension systems.

The loop filter must be selected to stabilize the

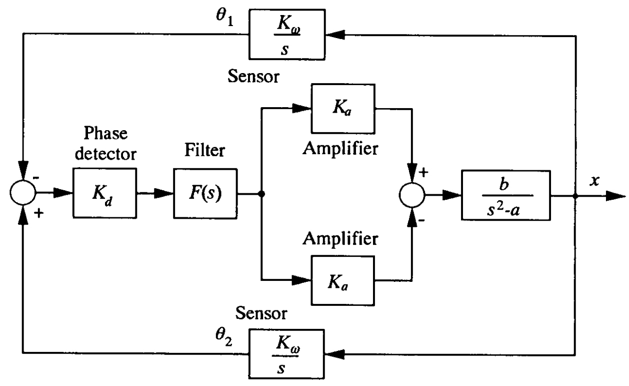


Fig. 4 Block diagram of PLMS using a linearized model in case of differential operation

closed-loop system because the open-loop system is unstable. Assuming that $F(s)$ is proper because of its physical realization and noise problems, a second- or higher-order filter is necessary for stabilization. For a second-order loop filter, $F(s)$ is represented as

$$F(s)=\frac{d(s)}{c(s)}=\frac{d_2s^2+d_1s+d_0}{s^2+c_1s+c_0}. \tag{27}$$

The closed-loop transfer function becomes

$$T(s)=K_tb(d_2s^2+d_1s+d_0)$$
$$/\{s^5+c_1s^4+(c_0-a)s^3+(K_tbd_2-ac_1)s^2$$
$$+(K_tbd_1-ac_0)s+K_tbd_0\} \tag{28}$$

where

$$K_t=K_aK_dK_\omega.$$

Therefore, we can assign the closed-loop poles arbitrarily by adjusting the coefficients of the filter. Here, we designate the desired poles by a characteristic polynomial $g(s)$:

$$g(s)=s^5+g_4s^4+g_3s^3+g_2s^2+g_1s+g_0. \tag{29}$$

The coefficients are determined as follows:

$$c_1=g_4,\ c_0=g_3+a,\ d_2=\frac{g_2+ac_1}{K_tb},\ d_1=\frac{g_1+ac_0}{K_tb},$$
$$d_0=\frac{g_0}{K_tb} \tag{30}$$

The controller itself is stable because $c_1>0$ and $c_0>0$.

If we restrict $F(s)$ to strictly proper real-rational functions, a filter of third or higher order is necessary for arbitrary pole assignment. When $F(s)$ is of the third order, it is represented by

$$F(s)=\frac{d_2s^2+d_1s+d_0}{s^3+c_2s^2+c_1s+c_0}. \tag{31}$$

Then the closed-loop transfer function becomes

$$T(s)=K_tb(d_2s^2+d_1s+d_0)$$
$$/\{s^6+c_2s^5+(c_1-a)s^4+(c_0-ac_2)s^3$$
$$+(K_tbd_2-ac_1)s^2+(K_tbd_1-ac_0)s+K_tbd_0\}. \tag{32}$$

When the desired characteristic polynomial of the closed-loop system is given by

$$g(s)=s^6+g_5s^5+g_4s^4+g_3s^3+g_2s^2+g_1s+g_0, \tag{33}$$

the coefficients of the filter must satisfy

$$c_2=g_5,\ c_1=g_4+a,\ c_0=g_3+ac_2$$
$$d_2=\frac{g_2+ac_1}{K_tb},\ d_1=\frac{g_1+ac_0}{K_tb},\ d_0=\frac{g_0}{K_tb}. \tag{34}$$

## 4. Static Accuracy

The open-loop transfer function of PLMS, $G(s)$, automatically contains an integration. Thus we expect that the steady-state position error is zero for step disturbances in the designed closed-loop systems. It will be shown in the following.

When a disturbance force $p$ is acting on the suspended object, the equation corresponding to Eq. (3) or (18) becomes

$$x(s)=\frac{1}{s^2-a}(bi(s)+dp(s)). \tag{35}$$

From Eqs. (5) to (9) or Eqs. (19) to (24), we obtain

$$x(s)=\frac{K_tbF(s)}{s(s^2-a)+K_tbF(s)}x_r(s)$$
$$+\frac{bs}{s(s^2-a)+K_tbF(s)}p(s) \tag{36}$$

where

$$x_r(s)=\frac{s\theta_r(s)}{K_\omega}.$$

In most magnetic suspension systems, the desired position $x_r$ is constant (usually zero). Assuming that the closed-loop system is stable and $p(s)=1/s$, we find

$$\lim_{t\to\infty}x(t)=x_r. \tag{37}$$

In active-type magnetic bearings with conventional position sensors, an integral is introduced into the feedback loop artificially to realize high accurate positioning. Most analog integrators are not ideal for the technical reason that commercially available op amps have DC error. Even with a digital controller, some error remains because of quantization with a finite number of word length. In contrast to these integrators, the integration contained in a PLL is ideal because it comes from the fact that frequency is the time derivative of phase (see Eqs. (6) and (22)). It is thus expected that PLMS's will accomplish extremely accurate positioning.

## 5. PLMS of Self-Sensing Type

One of the main reasons why magnetic suspensions have been used only in special fields such as vacuum technology is the cost of the total system. Reduction of the cost, size and weight of magnetic suspension systems is important to increase their industrial applications. An effective method for this purpose is the combined use of electromagnets for force generator and position sensor.

In the pulse-restrained magnetic suspension, the windings were used on a time-sharing basis to serve alternately as position sensor and force coils[4]. In the sensorless or self-sensing magnetic bearing, an observer was used to reconstruct the displacement and velocity of the rotor from the coil current and voltage[5]~[7]. Another method of self-sensing is to make use of the PWM carrier frequency component[8]. In addition to the latter two methods of self-sensing, Vischer proposes the use of a power amplifier named hysteresis amplifier and the demodulation of its switching signal for measuring the gap[5].

The schematic diagram of the hysteresis amplifier is shown in Fig. 5[9]. Since the switching rate depends on the load impedance, it changes with the gap between the suspended object and the magnet. The switching signal is, therefore, modulated in frequency.

In a PLMS with the hysteresis amplifiers, switching signals from these amplifiers can be used instead of those produced by the frequency type of displace-

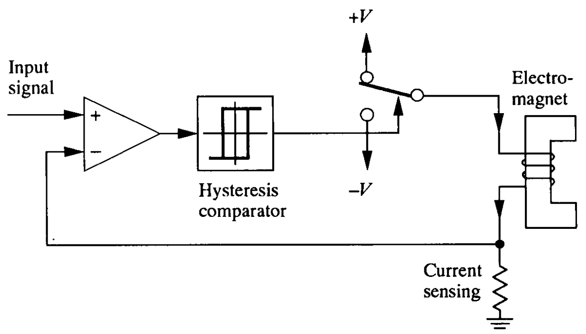


Fig. 5 Schematic diagram of hysteresis amplifier

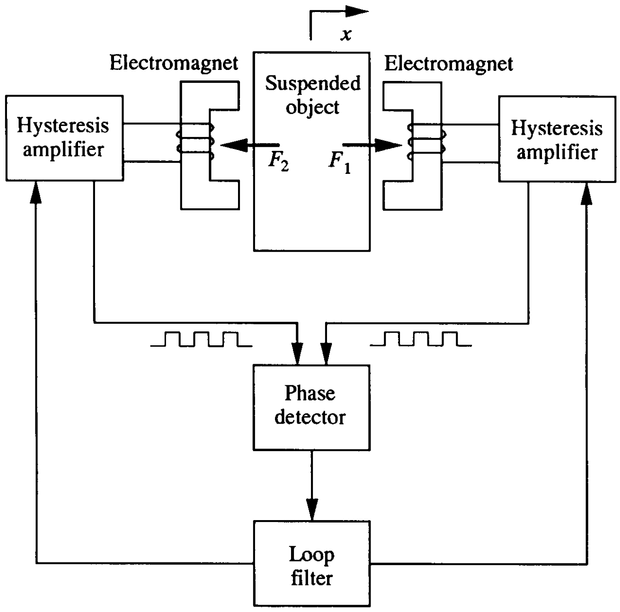


Fig. 6 Self-sensing configuration of PLMS

ment sensors. These are directly inputted to the phase detector as shown in Fig. 6. The self-sensing type of PLMS proposed here is so simple that the costs will be substantially reduced.

## 6. Conclusions

The phase-locked active magnetic suspension system (PLMS) characterized by a displacement sensor of frequency type and stabilization with PLL techniques has been proposed. It has been shown that the closed-loop poles can be assigned arbitrarily by using loop filters whose transfer functions are real-rational and (strictly) proper; the steady-state response error is zero for step disturbances because the open-loop transfer function automatically contains an ideal integration. The self-sensing type of PLMS has been proposed which uses a hysteresis amplifier whose switching frequency varies with the position of the suspended object.

An experimental study of PLMS's is under way, in addition to the investigation of different aspects of PLMS's.


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