Vibration Isolation System Using Negative Stiffness*

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A new vibration isolation system using negative stiffness realized by active control technique is proposed in this paper. The serial connection of a normal spring and a suspension system with negative stiffness enables the isolation system to have low stiffness for vibration from the ground and high (*theoretically infinite*) stiffness against direct disturbance acting on the isolation table. A control method of realizing negative stiffness with a linear actuator is presented in an analytical form. The validity of this method is confirmed experimentally with an apparatus equipped with a voice coil motor. It is also confirmed experimentally that high stiffness against direct disturbance on the isolation table can be achieved in the proposed vibration isolation system.

***Key Words*:** Vibration Isolation, Vibration Control, Actuator, Mechatronics, Servo Mechanism, Precision Instrument, Negative Stiffness, Compliance Control

1. Introduction

Demands for high-performance vibration isolation systems have been increasing in various scientific and industrial fields such as high-precision measuring and semiconductor manufacturing. There are two categories of disturbance whose effects should be attenuated by them. One is vibration transmitted from ground through suspension (spring). The other is disturbance caused by changes in weight and motions of instrument on the isolation table (direct disturbance). Lower stiffness of suspension is better for reducing the former while higher stiffness is better for suppressing the effects of the latter. A trade-off between them is inevitable in conventional passive-type vibration isolation systems so that their performances are limited inherently.

They can be made compatible by applying active control technology[1]. However, they must be equipped with high-performance sensors such as servo-type accelerometers so that active vibration isolation systems tend to be costly, roughly speaking, nearly ten times as expensive as passive ones. This restricts the application fields of active vibration isolation systems.

To solve such a problem, we have proposed a vibration isolation system using a zero-power magnetic suspension system[2)-(4]. Since a zero-power magnetic suspension system behaves as if it has a negative stiffness, infinite stiffness against disturbance on the isolation table can be achieved by combing it with a mechanical spring. The principles of the proposed vibration isolation are explained as follow. It is assumed that there is a serial spring consisting of a normal spring and a spring whose stiffness is negative (negative spring). If the absolute value of the stiffness of the normal spring is equal to that of the negative spring, the total stiffness of the serial spring becomes infinite. Therefore, even if direct disturbance acts on the isolation table, the table has no steady-state displacement. It means that the suspension system has *infinite stiffness*, or *zero compliance* to direct disturbance. Moreover, if the absolute value of the stiffness of each spring is low enough, the isolation table is isolated from ground vibration well. It enables the system to have good characteristics in both the performances of isolation from ground vibration and suppression of the effects of direct distur-

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bance. It is to be noted that zero-power magnetic suspension systems can be realized with relative-displacement sensors that are usually less expensive than servo-type accelerometer.

In the zero-power magnetic suspension system, however, the magnitude of negative stiffness is a function of the gap between the electromagnet and the suspended object. When the mass on the isolation table changes, therefore, the negative stiffness varies from the nominal value so that the stiffness against disturbances acting on the isolation table becomes lower[3],[4].

In this paper, we propose to use a linear actuator instead of an electromagnet for generating a suspension system with negative stiffness. It enables the vibration isolation system to keep high stiffness for a wider range of operation than the original system. This paper is organized as follows. First, the concept of the proposed vibration isolation system is briefly described. Second, a control method of realizing negative stiffness with a linear actuator is clarified by a transfer function approach. Third, the basic characteristics of the proposed vibration isolation system are studied. Fourth, experimental results are presented to demonstrate the effectiveness of the proposed method of generating infinite stiffness.

2. Concept of Vibration Isolation System

First, it will be shown that infinite stiffness can be generated by connecting a normal spring with a negative spring. When two springs with spring constants of k_1 and k_2 are connected in series as shown in Fig. 1, the total stiffness k_c is given by

$$k_c=\frac{k_1k_2}{k_1+k_2}. \qquad (1)$$

This equation shows that the total stiffness becomes lower than that of each spring when normal springs are connected. However, if one of the springs has negative stiffness that satisfies

$$k_1=-k_2, \qquad (2)$$

the resultant stiffness becomes infinite, that is

$$|k_c|=+\infty. \qquad (3)$$

This research applies this principle to generating high stiffness for direct disturbances acting on the isolation table.

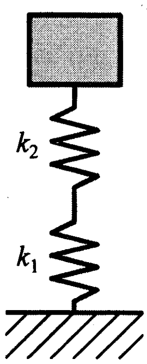

Fig. 1 Series spring

Figure 2 shows the configuration of one of the proposed vibration isolation systems. A middle mass m_1 is connected to the base through a spring k_1 and a damper c_1 that work as a conventional vibration isolator. A linear actuator is attached to the middle mass. The actuator suspends and drives an isolation table m_2.

The linear actuator is controlled for the suspension system to have negative stiffness. In the initial steady states shown in Fig. 3(a), the distance between the table m and the base is kept to be L. When the mass of the table increases by Δm, the distance increases by ΔL as shown in Fig. 3(b). The displacement of the table is in the direction opposite to the added force Δmg so that the static stiffness of the suspension is given by

$$-\frac{\Delta mg}{\Delta L}(\equiv -k_n). \qquad (4)$$

When the actuator is controlled in this way, the vibration isolation system shown by Fig. 2 behaves as follows. When the isolation table m_2 is subject to downward force f_0, the table would move upwards by f_0/k_n if the middle mass were fixed. Meanwhile, the

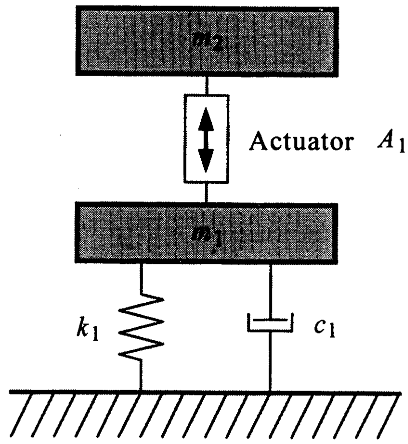

Fig. 2 Structure of vibration isolation system

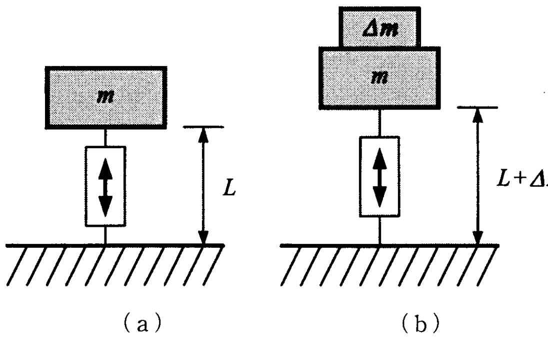

Fig. 3 Negative stiffness for direct disturbances

middle mass moves downwards by f_0/k_1 because of the increase of the suspension force. If $k_1=k_n$, the increase of the length of the actuator is cancelled by the downward displacement of the middle mass. Thus the isolation table is maintained at the same position as before so that the system has infinite stiffness or zero compliance to direct static disturbance.

3. Realization of Negative Stiffness

In this section, the design of control system for realizing negative stiffness with a linear actuator is described. A voice coil motor (VCM) is adopted as an actuator.

3.1 Basic equation

Figure 4 shows a single-degree-of-freedom model used in analysis. The equation of motion is

$$m_2\ddot{x}_2+c_2\dot{x}_2+k_2x_2=f_a+f_d, \tag{5}$$

where

m_2 : mass of the table,
x_2 : displacement from the equilibrium point,
k_2 : stiffness of supporting spring,
c_2 : damping coefficient of support,
f_a : force generated by the actuator,
f_d : direct disturbance acting on the table.

Since the force generated by the actuator is proportional to the coil current i, f_a is given by

$$f_a=k_i i, \tag{6}$$

where

k_i : current-force coefficient.

From Eqs.(5) and (6), we get a transfer representation of the dynamics:

$$X(s)=\frac{1}{s^2+a_1s+a_0}(b_0I(s)+d_0F_d(s)), \tag{7}$$

where

$$a_0=\frac{k_2}{m_2},\ a_1=\frac{c_2}{m_2},\ b_0=\frac{k_i}{m_2},\ d_0=\frac{1}{m_2}.$$

3.2 Design of control system

The displacement of the table is treated as an output signal here. When linear control law is applied, therefore, the control input is generally represented as

$$I(s)=-\frac{h(s)}{g(s)}X(s). \tag{8}$$

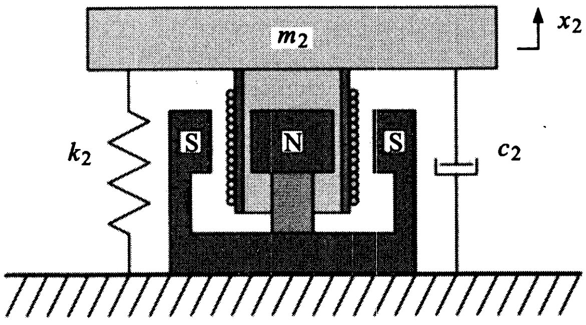

Fig. 4 Basic model for control system design

When the transfer function of the controller is proper, the polynomials are represented as

$$g(s)=s^n+\sum_{k=0}^{n-1}g_ks^k, \tag{9}$$

$$h(s)=\sum_{k=0}^{n}h_ks^k. \tag{10}$$

Substituting Eq.(8) into Eq.(7) leads to

$$X(s)=\frac{g(s)}{(s^2+a_1s+a_0)g(s)+b_0h(s)}d_0F_d(s). \tag{11}$$

The disturbance is assumed to be stepwise so that it can be modeled as

$$F_d(s)=\frac{F_0}{s}\quad (F_0\text{: const}). \tag{12}$$

It is assumed that the controller is selected to stabilize the closed loop system. The steady-state displacement $x(\infty)$ is given by

$$\frac{x(\infty)}{F_0}=\lim_{s\to 0}\frac{g(s)}{(s^2+a_1s+a_0)g(s)+b_0h(s)}d_0$$
$$=\frac{d_0g_0}{a_0g_0+b_0h_0}. \tag{13}$$

For the system to have negative stiffness with a magnitude of k_n, the following equation must be satisfied

$$\frac{d_0g_0}{a_0g_0+b_0h_0}=-\frac{1}{k_n}. \tag{14}$$

For assigning the closed-loop poles arbitrarily, second- or higher-order compensators are necessary. When a second-order compensator is used, the characteristic polynomial of the closed-loop system becomes

$$t_c(s)=s^4+(a_1+g_1)s^3+(a_0+g_0+a_1g_1+b_0h_2)s^2$$
$$+(a_0g_1+a_1g_0+b_0h_1)s+(a_0g_0+b_0h_0). \tag{15}$$

To obtain a system with a characteristic equation of the form

$$t_d(s)=s^4+e_3s^3+e_2s^2+e_1s+e_0, \tag{16}$$

we can match coefficients to obtain

$$g_0=-\frac{e_0}{d_0k_n}=-e_0\frac{m}{k_n}, \tag{17}$$

$$g_1=e_3-a_1, \tag{18}$$

$$h_0=\frac{1}{b_0}(e_0-a_0g_0), \tag{19}$$

$$h_1=\frac{1}{b_0}(e_1-a_0g_1-a_1g_0), \tag{20}$$

$$h_2=\frac{1}{b_0}(e_2-a_0-g_0-a_1g_1). \tag{21}$$

When the transfer function of the compensator is restricted to be strictly proper, third- or higher-order compensators are necessary to assign the poles arbitrary in addition to satisfying Eq.(14). When a third-order compensator is used, the characteristic polynomial of the closed-loop system becomes

$$t_c(s)=s^5+(a+g_2)s^4+(a_0+a_1g_2+g_1)s^3$$
$$+(a_0g_2+a_1g_1+g_0+b_0h_2)s^2$$
$$+(a_0g_1+a_1g_0+b_0h_1)s+(a_0g_0+b_0h_0). \tag{22}$$

To obtain a system with a characteristic equation of the form

$$t_d(s)=s^5+e_4s^4+e_3s^3+e_2s^2+e_1s+e_0, \tag{23}$$

we can match coefficients to obtain

$$g_0 = -\frac{e_0}{d_0 k_n} = -e_0 \frac{m}{k_n}, \tag{24}$$

$$g_2 = e_4 - a_1, \tag{25}$$

$$g_1 = e_3 - a_0 - a_1 g_2, \tag{26}$$

$$h_0 = \frac{e_0}{b_0}\left(\frac{a_0}{d_0 k_n} + 1\right) = \frac{e_0}{b_0}\left(\frac{k}{k_n} + 1\right), \tag{27}$$

$$h_1 = \frac{1}{b_0}(e_1 - a_0 g_1 - a_1 g_0), \tag{28}$$

$$h_2 = \frac{1}{b_0}(e_2 - g_0 - a_0 g_2 - a_1 g_1). \tag{29}$$

4. Analysis of Basic Characteristics

4.1 Basic equation

A basic model of the proposed vibration isolation system with a VCM shown in Fig. 5. The translation motions in the vertical direction will be treated here. The equations of motion are

$$m_1 \ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_0) + k_1(x_1 - x_0) + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) = -k_i i, \tag{30}$$

$$m_2 \ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = k_i i + f_d. \tag{31}$$

where

x_0, x_1, x_2: displacements of the floor, the middle mass and the isolation table,

f_d: direct disturbance acting on the isolation table.

It assumed for simplicity that the initial values are zero. From Eqs. (30) and (31), we get

$$(m_1 s^2 + c_1 s + k_1) X_1(s) - (c_1 s + k_1) X_0(s) = (c_2 s + k_2)(X_2(s) - X_1(s)) - k_i I(s), \tag{32}$$

$$m_2 s^2 X_2(s) = k_i I(s) - (c_2 s + k_2)(X_2(s) - X_1(s)) + F_d(s). \tag{33}$$

The control current for realizing negative stiffness can be represented as

$$I(s) = -\frac{h(s)}{g(s)}(X_2(s) - X_1(s)), \tag{34}$$

where $g(s)$ and $h(s)$ are selected to satisfy Eq. (14) and to stabilize the closed-loop system.

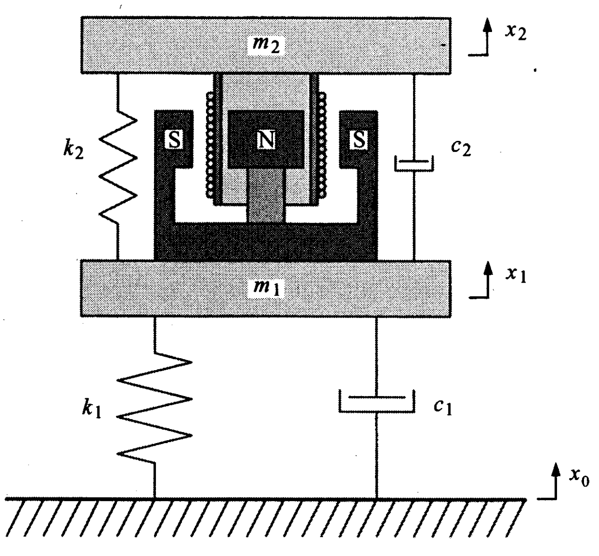

Fig. 5 Basic model of a zero-compliance system

From Eqs. (32) to (34), we get

$$X_1(s) = \frac{(c_1 s + k_1) t_2(s)}{t_c(s)} X_0(s) + \frac{\tilde{k}_2(s)}{t_c(s)} F_d(s), \tag{35}$$

$$X_2(s) = \frac{(c_1 s + k_1) \tilde{k}_2(s)}{t_c(s)} X_0(s) + \frac{t_1(s) + \tilde{k}_2(s)}{t_c(s)} F_d(s), \tag{36}$$

where

$$t_1(s) = m_1 s^2 + c_1 s + k_1, \tag{37}$$

$$\tilde{k}_2(s) = k_2 + c_2 s + k_i \frac{h(s)}{g(s)}, \tag{38}$$

$$t_2(s) = m_2 s^2 + \tilde{k}_2(s), \tag{39}$$

$$t_c(s) = t_1(s) t_2(s) + m_2 s^2 \tilde{k}_2(s). \tag{40}$$

It is to be noted that $\tilde{k}_2(s)$ satisfies

$$\begin{aligned} \lim_{s \to 0} \tilde{k}_2(s) &= k_2 + k_i \frac{h_0}{g_0} \\ &= m\left(a_0 + b_0 \frac{h_0}{g_0}\right) \\ &= -k_n, \end{aligned} \tag{41}$$

4.2 Response to direct disturbance

To estimate the stiffness for direct disturbance, the direct disturbance f_d is assumed to be stepwise, that is,

$$F_d = \frac{F_0}{s} \quad (F_0 : \text{const}). \tag{42}$$

When the vibration of the floor is neglected, the steady-state displacement of the table is obtained as

$$\begin{aligned} \frac{x_2(\infty)}{F_0} &= \lim_{s \to 0} \frac{t_1(s) + \tilde{k}_2(s)}{t_c(s)} \\ &= \frac{k_1 - k_n}{k_1(-k_n)} = \frac{1}{k_1} - \frac{1}{k_n}. \end{aligned} \tag{43}$$

Therefore, when

$$k_1 = k_n, \tag{44}$$

we get

$$\frac{x_2(\infty)}{F_0} = 0. \tag{45}$$

Equation (45) shows that the suspension system between the isolation table and the floor has infinite stiffness statically because there is no steady-state deflection even in the presence of stepwise disturbances acting on the table.

The steady-state displacement of the middle mass and the variation of gap are obtained as

$$\frac{x_1(\infty)}{F_0} = \frac{1}{k_1}, \tag{46}$$

$$\frac{x_2(\infty) - x_1(\infty)}{F_0} = -\frac{1}{k_n}. \tag{47}$$

Equations (46) and (47) indicates that the middle mass moves downward and the length of the actuator increases when a downward force acts on the isolation table ($F_0 < 0$). It supports well the predictions on the behavior of the proposed vibration isolation system described in Section 2.

5. Experiment

5.1 Experiment setup

Figure 6 is a schematic diagram of the developed apparatus for experimental study. The stator of the VCM, which is corresponding to the middle mass m_1, is suspended by a pair of plate springs to be in translation in the vertical direction. The isolation table is attached to the moving shaft of the VCM. The displacements of the middle mass and the isolation table to the base are detected by eddy-current gap sensors with a resolution of 1 μm. Subtracting the former from latter gives the relative displacement of the isolation table to the middle mass. Designed control algorithms are implemented with a DSP-based digital controller. A second-order compensator is used for realizing zero-power control in the following experiments. The control period is 100 μs.

5.2 Experimental results

Two kinds of experiments are carried out. The first is for estimating negative stiffness generated by the VCM, and the other is for measuring the displacement of the isolation table for direct disturbance. In the following graphs, load and displacement in the *downward* direction are represented to be *positive*, and *vice versa*.

Figure 7 shows the results of the first experiment. In this experiment, the stator of the VCM is clamped. The amplitude of negative stiffness is set to be

(a) $k_n=15$ [kN/m], (b) $k_n=20$ [kN/m],
(c) $k_n=25$ [kN/m],

in designing the controller (see Eq. (14)). The results show that the displacement of the table is in the direction opposite to the load and their relation is quite linear. The amplitude of negative stiffness is also estimated from the results as

(a) $k_n=14.9$ [kN/m], (b) $k_n=19.7$ [kN/m],
(c) $k_n=25.1$ [kN/m].

The amplitudes of negative stiffness set in designing the controller is almost equal to the actual values.

In the second experiment, the clamp of the stator of the VCM is released and then load is added to the isolation table. Since the total stiffness of the plate springs that is corresponding to k_1 is 16.4 [kN/m], the amplitudes of negative stiffness is set to equal this value. Figure 8 shows the displacement of the stator of the VCM to the base and that of the isolation table to the base. It is observed that the position of the table is maintained almost in the same, while the position of the stator changes proportion to the load. This result demonstrates well that the developed

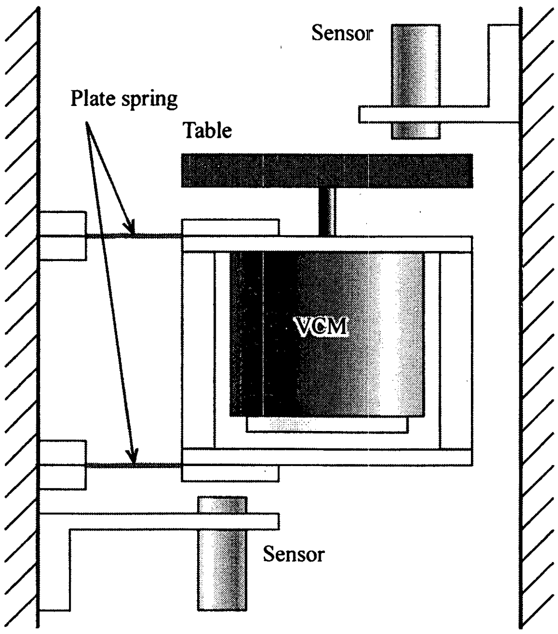

Fig. 6 Experimental apparatus

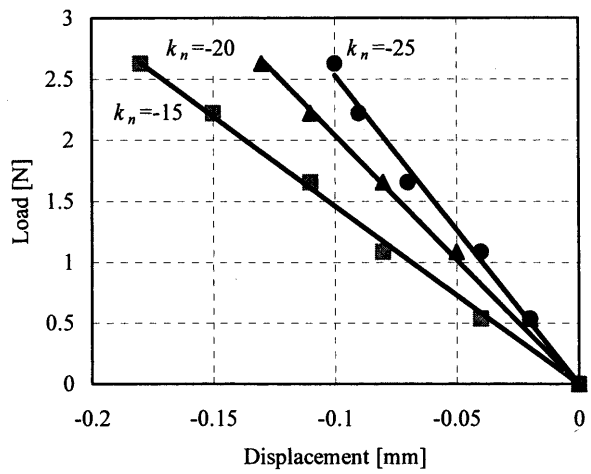

Fig. 7 Relation between the displacement of the table and load acting on the table

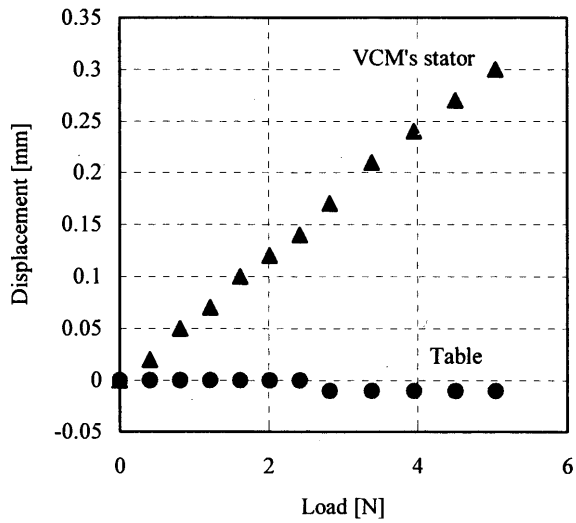

Fig. 8 Displacements of the VCM's stator and the table when load force acts on the table

mechanism with a VCM can realize high stiffness against static direct disturbance acting on the isolation table.

6. Conclusions

A new vibration isolation system using negative stiffness realized by active control technique was proposed in this paper. The serial connection of a normal spring and a suspension system with negative stiffness enables the isolation system to have low stiffness for vibration from the ground and high (*theoretically infinite*) stiffness against direct disturbance acting on the isolation table. A control method of realizing negative stiffness with a linear actuator was presented in an analytical form. The validity of this method and the effectiveness of the proposed method of realizing high stiffness against static direct disturbance were confirmed experimentally.

Acknowledgments

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