Active Dynamic Vibration Absorber with Automatic Frequency-Tracking Performance*

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A disturbance cancellation controller for the active dynamic vibration absorber is modified to follow the frequency of disturbance without any complicated adaptive algorithm. The original compensator has an internal model of a sinusoidal disturbance. When the frequency of disturbance varies, the dynamics of the model must also be altered to be identical to those of disturbance for perfect regulation. In the modified compensator, control input is generated by calculating a convolution integral instead of solving the corresponding state-space equation; exogenous signals synchronized with the actual disturbance are used in the convolution. The performance of the designed controller is experimentally studied.

Key Words: Active Vibration Control, Dynamic Vibration Absorber, Frequency Tracking, Internal Model Principle, Disturbance Cancellation

1. Introduction

The active type dynamic vibration absorber can realize higher-performance vibration control than the passive one. Its industrial application has been recently activated in the fields of mechanical and civil engineering. In designing the control system, various control theories are selectively applied according to the purpose and target of vibration control[(1)-(4)]. The authors applied the internal model principles, and constructed a control system, which can reduce the vibration to a low level over a wide range of frequencies and to zero at a specified frequency[(5)]; this frequency and the property of reducing vibration to zero are refereed to as absorption frequency and output regulation in the following. This controller incorporates a disturbance model into the feedback loop. Since this model supplies transmission zeroes to cancel the unstable poles of the disturbance signal, it preserves the output regulation property even if the parameters in both primary and auxiliary systems are perturbed. This property of robustness was confirmed experimentally using an apparatus with an electromagnetic servomechanism. Moreover, the control law has been generalized for the absorber to have two or more absorption frequencies[(6)]. It was shown that the vibration is actually removed at two to four frequencies by using this controller[(7)].

For changes in the frequency of disturbance, however, the regulation property is lost because the unstable poles move from their nominal values and the pole-zero cancellation becomes imperfect. Such a problem can be overcome by altering the internal model according to change in the frequency of disturbance; this performance is refereed to as frequency tracking.

There are several methods of achieving the frequency-tracking performance. An usual way is to detect the frequency and change the corresponding parameters in the controller based on the detected value. However, on-line frequency measurement with high accuracy is sometimes difficult or costly.

Proposed here is to use exogenous signals synchronized with disturbance to construct an internal model in the controller. Such synchronous signals are easily obtained in some applications, for example, where vibration sources come from rotational machinery. The frequency-tracking performance of the

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modified controller is experimentally studied using an active dynamic vibration absorber system with an electromagnetic servomechanism.

2. Basic Model

A schematic diagram of the vibration absorber system considered in this paper is illustrated in Fig. 1. A primary mass m_1, which is disturbed by an external force $p(t)$, is mounted on a base through a spring k_1 and a damping element c_1. An active dynamic vibration absorber is attached to the mass. It consists of a lever hanging from a fixed point O with a ball-bearing pivot, and a pair of electromagnets acting on the lever. The equations of motion for small values of θ are[(4)]:

$$(m_1+m_2)\ddot{x}_1+c_1\dot{x}+k_1x_1+m_2\ddot{x}_2=p(t) \quad (1)$$

$$m_2\ddot{x}_1+(1+r_2)m_2\ddot{x}_2+c_2\dot{x}_2+k_2x_2=-\hat{k}_i i \quad (2)$$

where

$$x_2=l_g\theta,\ r_2=\frac{I_G}{m_2l_g^2},\ c_2=\frac{c_\theta}{l_g^2},\ \hat{k}_i=2k_i\frac{l_e}{l_g},$$

$$k_2=\frac{m_2g}{l_g}-2k_s\left(\frac{l_e}{l_g}\right)^2,$$

x_1 : displacement of the primary mass,
m_2, I : mass and moment of inertia of the lever,
c_θ : damping constant of the absorber,
l_g : distance of the mass center of the lever from the fixed point of the lever,
l_e : distance of the electromagnets from the fixed point of the lever,
k_i, k_s : coefficients of the linearized model of the electromagnets,
i : control current,
g : acceleration of gravity.

The dynamics described by Eqs. (1) and (2) can be represented in the state-vector form,

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{b}\boldsymbol{u}(t)+\boldsymbol{d}\boldsymbol{p}(t), \quad (3)$$

where

$$\boldsymbol{x}=\begin{bmatrix}x_1\\ \dot{x}_1\\ x_2\\ \dot{x}_2\end{bmatrix},\ \boldsymbol{u}=i,\ \boldsymbol{b}=\begin{bmatrix}0\\ b_0\\ 0\\ -(1+r_1)b_0\end{bmatrix},\ \boldsymbol{d}=\begin{bmatrix}0\\ (1+r_2)d_0\\ 0\\ -d_0\end{bmatrix}$$

$$\boldsymbol{A}=\begin{bmatrix}0 & 1 & 0 & 0\\ -(1+r_2)a_1 & -(1+r_2)a_3 & a_3 & a_4\\ 0 & 0 & 0 & 1\\ a_1 & a_2 & -(1+r_1)a_3 & -(1+r_1)a_4\end{bmatrix},$$

$$a_1=\frac{k_1}{m_a},\ a_2=\frac{c_1}{m_a},\ a_3=\frac{k_2}{m_a},\ a_4=\frac{c_2}{m_a},$$

$$r_1=\frac{m_1}{m_2},\ b_0=\frac{\hat{k}_i}{m_a},\ d_0=\frac{1}{m_a},$$

$$m_a=(1+r_2)m_1+m_2.$$

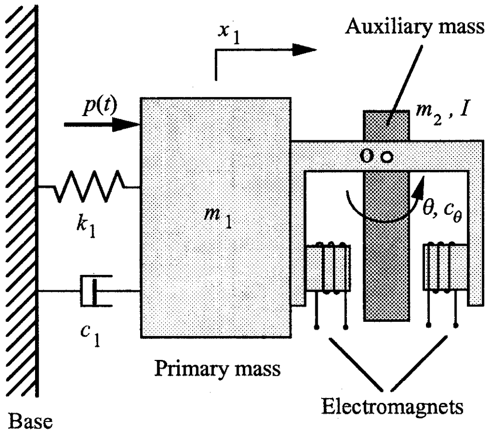


Fig. 1 Model of an active dynamic vibration absorber system

3. Controller with an Internal Model

3.1 Structure of controller

The force acting on the primary mass can be treated as a disturbance to the controlled object. In designing the controller, $p(t)$ is assumed to be sinusoidal

$$p(t)=P_0\cos\omega t \quad (4)$$

Modeled in this way, the disturbance signal is described by a state space equation:

$$\dot{\boldsymbol{w}}(t)=\boldsymbol{E}\boldsymbol{w}(t), \quad (5)$$

$$p(t)=\boldsymbol{C}\boldsymbol{w}(t), \quad (6)$$

where

$$\boldsymbol{w}(t)=\begin{bmatrix}w_1\\ w_2\end{bmatrix},\ \boldsymbol{E}=\begin{bmatrix}0 & -\omega\\ \omega & 0\end{bmatrix},\ \boldsymbol{C}=[1\ 0],$$

$$w_1=P_0\cos\omega t,\ w_2=P_0\sin\omega t.$$

The main object of the control system design is to reduce the vibration of the primary mass m_1 essentially to zero at the specified frequency ω_0 and to non-resonant levels at the other frequencies. In order to achieve this purpose, a disturbance model is incorporated in the feedback path; this model supplies transmission zeroes to cancel the unstable poles of the disturbance signal. It leads to a dynamic compensator described as[(5)]

$$\dot{\boldsymbol{z}}(t)=\boldsymbol{E}_0\boldsymbol{z}(t)+\boldsymbol{H}\boldsymbol{x}(t), \quad (7)$$

$$u(t)=\boldsymbol{F}\boldsymbol{x}(t)+\boldsymbol{G}\boldsymbol{z}(t), \quad (8)$$

where

$$\boldsymbol{z}(t)=\begin{bmatrix}z_1\\ z_2\end{bmatrix},\ \boldsymbol{E}_0=\begin{bmatrix}0 & -\omega_0\\ \omega_0 & 0\end{bmatrix},\ H=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix},$$

$$\boldsymbol{F}=[f_1\ f_2\ f_3\ f_4],\ \boldsymbol{G}=[g_1\ g_2].$$

The block diagram of this dynamic compensator is shown in Fig. 2.

3.2 Pole assignment

The matrices $\boldsymbol{F}$ and $\boldsymbol{G}$ should be chosen to stabilize the closed-loop system. Since the characteristics of vibration are closely related to the zeroes and poles of the system, a pole assignment method will be used

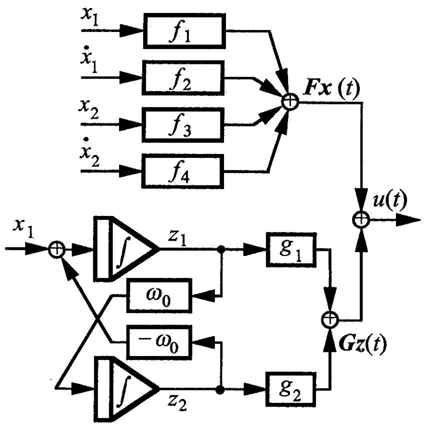


Fig. 2 Block diagram of the controller with an internal model of disturbance

here to select $\boldsymbol{F}$ and $\boldsymbol{G}$; note that the closed-loop system has the transmission zeroes canceling the unstable poles of disturbance.

Let a desired characteristic polynomial denoted by

$$t_d(s)=s^6+\beta_5 s^5+\beta_4 s^4+\beta_3 s^3+\beta_2 s^2+\beta_1 s+\beta_0 \quad (9)$$

Then, we shall find $\boldsymbol{F}$ and $\boldsymbol{G}$ such that

$$\begin{vmatrix} s\boldsymbol{I}-(\boldsymbol{A}+\boldsymbol{bF}) & -\boldsymbol{bG} \\ -\boldsymbol{H} & s\boldsymbol{I}-\boldsymbol{E}_0 \end{vmatrix}=t_d(s) \quad (10)$$

In order to obtain the elements of $\boldsymbol{F}$ and $\boldsymbol{G}$, a transfer function approach will be used here. From Eqs.(7) and (8), the control input is represented as

$$U(s)=(f_1+f_2 s)X_1(s)+(f_3+f_4 s)X_2(s) + \frac{g_1 s+g_2\omega_0}{s^2+\omega_0^2}X_1(s) \quad (11)$$

where the Laplace transformed variables are denoted by their capitals, and the initial condition $\boldsymbol{x}(0)$ is assumed to be zero for simplicity. From Eqs.(1), (2), and (11), we get

$$\frac{X_1(s)}{P(s)}=\frac{(s^2+\lambda a_{24}s+\lambda a_{23})(s^2+\omega_0^2)(1+r_2)d_0}{t_c(s)-b_0 s^2(g_1 s+g_2\omega_0)} \quad (12)$$

where

$$\gamma=\frac{\lambda}{1+r_1},\quad \lambda=r_1 r_2+r_1+r_2,$$
$$a_{21}=-(1+r_2)a_1+b_0 f_1,\quad a_{22}=-(1+r_2)a_2+b_0 f_2,$$
$$a_{23}=a_3+b_0 f_3,\quad a_{24}=a_4+b_0 f_4,$$
$$a_{41}=a_1-(1+r_1)b_0 f_1,\quad a_{42}=a_2-(1+r_1)b_0 f_2,$$
$$a_{43}=-(1+r_1)a_{23},\quad a_{44}=-(1+r_1)a_{24},$$
$$t_c(s)=(s^2-a_{22}s-a_{21})(s^2-a_{44}s-a_{43})-(a_{24}s+a_{23})(a_{42}s+a_{41}).$$

Comparing $t_c(s)$ with $t_d(s)$ gives:

$$f_1=\frac{1}{b_0}\{\omega_0^2+(1+r_2)a_1+(1+r_1)a_{23}+\lambda a_2 a_{24}-\beta_4\} \quad (13)$$

$$f_2=\frac{1}{b_0}\{(1+r_2)a_2-a_{44}-\beta_5\} \quad (14)$$

$$f_3=\frac{1}{b_0}\left(\frac{\beta_0}{\lambda a_1\omega_0^2}-a_3\right) \quad (15)$$

$$f_4=\frac{1}{b_0}\left(\frac{\beta_1-\frac{a_2}{a_1}\beta_0}{\lambda a_0\omega_0^2}-a_4\right) \quad (16)$$

$$g_1=\frac{1}{b_0}\left(\beta_3-\frac{\beta_1}{\omega_0^2}-\beta_5\omega_0^2\right) \quad (17)$$

$$g_0=\frac{1}{b_0\omega_0}\left\{\beta_2-\frac{\beta_0}{\omega_0^2}-(\beta_5-\omega_0^2)\omega_0^2\right\} \quad (18)$$

4. Controller Using Exogenous Signals

Equation (12) shows that the closed-loop system with the designed controller has transmission zeroes at $\pm j\omega_0$. Thereby the output regulation is achieved when the excitation force $p(t)$ has an angular frequency ω_0 because the transmission zeroes cancel the unstable poles of disturbance. The internal model principle guarantees that the output regulation is preserved in the presence of any small perturbations in the controlled object. For a change of the frequency of disturbance, however, the regulation property is lost because the unstable poles move from the nominal values $\pm j\omega_0$ and the pole-zero cancellation becomes imperfect. Proposed here is to use exogenous signals synchronized with disturbance for the transmission zeroes to follow the frequency of disturbance automatically.

A controller using exogenous signals will be obtained by modifying the controller with an internal model. For output regulation, the parameter ω_0 of the internal model in the controller (11) must coincide with the actual angular frequency ω of disturbance.

$$U(s)=(f_1+f_2 s)X_1(s)+(f_3+f_4 s)X_2(s) + \frac{g_1 s+g_2\omega}{s^2+\omega^2}X_1(s) \quad (19)$$

The modification uses the following formula of Laplace transformation.

$$L^{-1}[F_1(s)F_2(s)]=\int_0^\infty f_1(t-\tau)f_2(\tau)d\tau \quad (20)$$

where

$$F_1(s)=L[f_1(s)],\quad F_2(s)=L[f_2(t)].$$

The third term in the right-side of (19),

$$V(s)=\frac{g_1 s+g_2\omega}{s^2+\omega^2}X_1(s), \quad (21)$$

corresponds to an internal model of disturbance, which is incorporated into the feedback loop. The Laplace transforms of sine- and cosine-functions are

$$L[\cos\omega t]=\frac{s}{s^2+\omega^2} \quad (22)$$

$$L[\sin\omega t]=\frac{\omega}{s^2+\omega^2} \quad (23)$$

Applying the formula of convolution to Eq.(19) gives

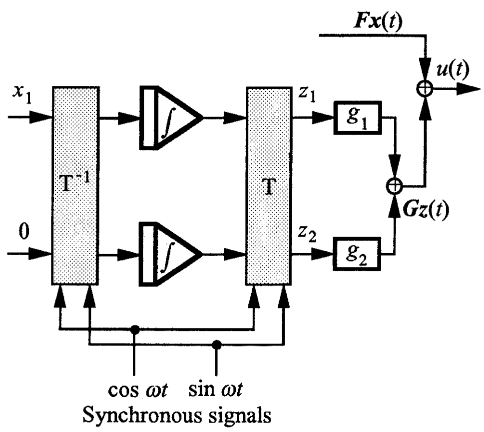
$$v(t)(=L^{-1}[V(s)])$$
$$=\int_0^\infty\{g_1\cos\omega(t-\tau)+g_2\sin\omega(t-\tau)\}x_1(\tau)d\tau$$
$$=[g_1\ \ g_2]\begin{bmatrix}\cos(\omega t+\alpha) & -\sin(\omega t+\alpha)\\ \sin(\omega t+\alpha) & \cos(\omega t+\alpha)\end{bmatrix}$$

$$\times \int_0^\infty \begin{bmatrix} \cos(\omega\tau+\alpha) \\ -\sin(\omega\tau+\alpha) \end{bmatrix} x_1(\tau) d\tau \qquad (24)$$

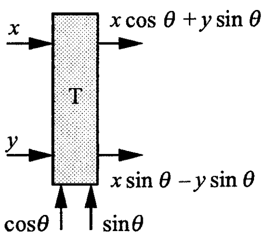
It is to be noted that Eq. (24) holds for any value of the phase angle α. The control input can be calculated according to

$$u(t)=f_1x_1+f_2\dot{x}_1+f_3x_2+f_4\dot{x}_2$$
$$+[g_1 \ g_2]\begin{bmatrix} \cos(\omega t+\alpha) & -\sin(\omega t+\alpha) \\ \sin(\omega t+\alpha) & \cos(\omega t+\alpha) \end{bmatrix}$$
$$\times \int_0^\infty \begin{bmatrix} \cos(\omega\tau+\alpha) \\ -\sin(\omega\tau+\alpha) \end{bmatrix} x_1(\tau) d\tau \qquad (25)$$

The block diagram of the modified controller is shown in Fig. 3 where α is set zero for simplicity. In this figure, the element T convert coordinates between the fixed reference system and a reference system rotating at an angular velocity ω; the element T^{-1} performs the inverse conversion. If the parameter ω is fixed to be ω_0 in these conversions, this controller has the same dynamics as the controller described by Eqs. (7) and (8) so that the property of output regulation is lost for a change of the frequency of disturbance. When sinusoidal signals synchronized with the actual disturbance are used in these conversions, the critical parameter ω equal the actual value automatically. The property of output regulation is, therefore, preserved even if the frequency of disturbance varies from the nominal value.



(a) Modified controller



(b) Transformation by the component T

Fig. 3 Block diagram of the controller using exogenous synchronous signals

5. Experiment

The schematic drawing of an experimental setup is shown in Fig. 4, which was manufactured for mass measurement[9]. The active dynamic vibration absorber is attached to the frame mounted to the base by a pair of plate springs. An auxiliary system is attached to the mass. It consists of a lever hanging from a fixed point O at one end with a ball-bearing pivot, and a pair of electromagnets acting on the lever. A motor fixed to the frame drives an unbalanced rotor which generates a sinusoidal force in the horizontal direction. The magnitude of this force is readily derived,

$$p(t)=m_u r\omega^2 \cos \omega t \qquad (26)$$

where

$m_u r$: magnitude of rotor unbalance,

ω : rotational speed of motor.

The parameters of the setup are listed in Table 1.

The outline of the experimental system is shown in Fig. 5. The displacement of the frame is measured by a capacitive sensor with a resolution of 0. 1 mm (catalogue value). The displacement of the lever is measured by an eddy-current sensor with a resolution

Table 1 System parameters

parameter	value
m_1	5.42 kg
m_2	1.09 kg
I	1.94×10^{-3} kgm^2
k_1	1.17×10^5 N/m
c_1	29.0 Ns/m
c_θ	0.0
l_e	65.0 mm
l_g	28.7 mm
a_3	2.02×10^3 1/(ms^2)
b_0	6.26 1/(As^2)
$m_u r$	26.0 gmm

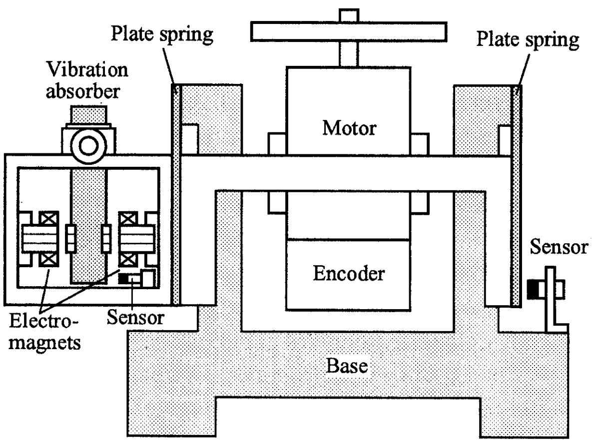


Fig. 4 Schematic drawing of the experimental setup

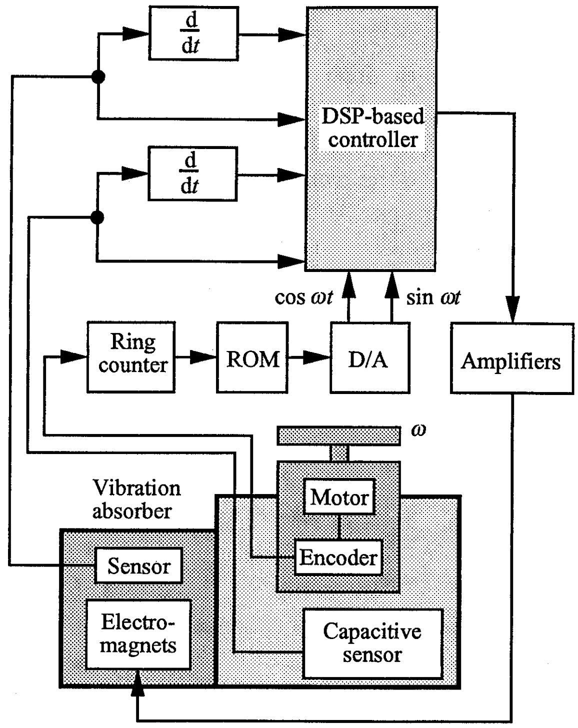


Fig. 5 Outline of the control system

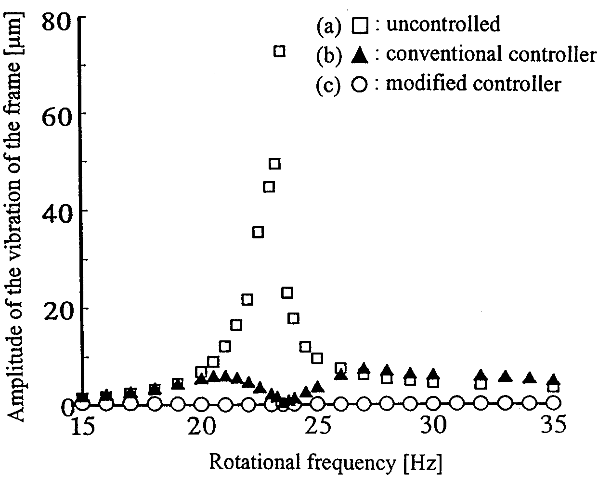


Fig. 6 Response of the system at each rotational speed

1 mm (catalogue value). These detected signals are fed into a DSP-based controller with eight 12-bit A/D converters and eight 12-bit D/A converters ; the DSP is TMS320C30 from Texas Instruments. Two-phase synchronous signals, which are used in calculating the convolution integral in Eq. (25), are generated based on pulse signals produced by a rotary encoder attached to the motor axis. These signals are also fed into the controller through A/D converters. The DSP calculates control input based on Eqs. (7) and (8) or Eq. (18), and produces command signals to the amplifiers of a pair of electromagnets through D/A converters. The control period is 100 μsec.

Figure 6 shows the movement of the frame at various rotational speeds in the cases where

(a) the absorber is not activated,

(b) the original servocompensator, given by Eqs. (7) and (8), is activated,

(c) the modified servocompensator, given by (25), is activated.

In designing the controller of (b), the characteristic polynomial of the closed-loop system is given by

$$t_d(s)=\prod_{n=1}^{3}(s^2+2\zeta_n\omega_n+\omega_n^2)$$

$$\omega_n=2\pi\times 23.5,\ \zeta_n=0.2 \quad (n=1, \cdots, 3)$$

In the case of (c), all the parameters except ω of the internal model are made equal to those of (b).

When the absorber is not activated ((a)), the maximum amplitude of the vibration of the frame is 75 mm at $\omega=2\pi\times 22.5$ [1/s]. When the original controller is used ((b)), the vibration is removed completely at the specified frequency $\omega_0=2\pi\times 23.5$ [1/s], but remains to some extent at the other rotational speeds. The modified compensator succeeds in removing the vibration at any rotational frequency ((c)). This result shows that the controller using synchronous signals has automatic frequency-tracking performance.

6. Conclusions

A disturbance cancellation controller designed based on the internal model principle was modified to have automatic frequency-tracking performance by using exogenous signals synchronized with disturbance. The obtained results are summarized as follows.

(1) The modified controller generates its control input by calculating the convolution integral instead of solving the state-space equation.

(2) When exogenous signals synchronized with the disturbance are used in calculating the convolution integral, the transmission zeroes of the closed-loop system are made at the same locations as the unstable poles of the disturbance signal automatically.

(3) In the experiments carried out by using an apparatus with an electromagnetic servomechanism, the designed controller succeeded in removing the vibration for sinusoidal excitation of any frequency between 15 Hz and 35 Hz.

The developed controller can achieve output regulation without fail because the constructed internal model has no error. It will be practical and very useful especially in measurement systems using dynamic vibration absorbers because such synchronous signals are easy to obtain and the elimination of vibration is necessary in these systems[10],[11].

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