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STRENGTH CRITERION OF ROCKS

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ABSTRACT

To investigate the strength characteristics of soft rocks, consolidated-drained triaxial compression tests were performed with mudstones, siltstones, tuff, weathered granite and so on. Maximum stress of applied confining pressure was about 30 MPa. Radial compression tests under confining pressure were also performed to study the mechanical properties under tensile stresses.

From these series of experiments, it may be concluded that the relationships between confining presssure and strength (maximum and residual) are generally non-linear. These strength relations can be expressed as the following power function;

$$(\tau_m/\tau_{m0})=\alpha(\sigma'_m/\sigma'_{m0})^\beta$$

where $\tau_m=(\sigma_1-\sigma_3)/2$, $\sigma'_m=(\sigma'_1+\sigma'_2+\sigma'_3)/3$, τ_{m0} and σ'_{m0} are at the case of $\sigma'_2=\sigma'_3=0$. α and β are material constants for rocks and $\sigma'_m>0$. The value of α is generally about unit and in the range of 0.96−1.23 and β is in the range of 0.44−0.85.

The normalization by $\tau_{m0}(=q_u/2)$ and σ'_{m0} $(=q_u/3)$ makes possible to represent with the same values of parameters α and β, the strength reduction due to scale effect and strength relation of sedimentary rocks which have the same geological history and of granites of various degrees of weathering which are distributed in the same petrographic province.

The applicability of proposed power function to the hard rocks and rock masses were investigated. Consequently, it is clarified that the proposed equation can be applied to the hard rocks which has the unconfined compressive strength of 20-200 MPa and also applied to the closely jointed rocks which can be regarded as the model of rock masses.

Key words: brittle failure, compressive strength, drained shear, overconsolidation, sedimentary rock, shear strength, stress-strain curve, triaxial compression test, weathering

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INTRODUCTION

Many investigations have been performed, concerning with the failure criterion of soils and/or rocks, up to now. However, it is not clarified that what failure criterion is suitable for soft rocks, which have the intermediate properties between soil and rock.

As typified by Mohr-Coulomb criterion, the failure criterion of soils is linear relation between failure stress and confining pressure. On the other hand, there are many non-linear relations for failure criterion of rocks. The failure criterion proposed by Griffith, McClintock and Walsh (modified Griffith, 1962), Murrell (extended Griffith, 1963),

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Fairhurst (1964), Hobbs (1966), Hoek (1968) and Franklin (1971) are some of these examples.

It is very difficult to evaluate the stress condition on failure plane developed in rock specimen due to its complex fracture path. In order to overcome this difficulty, Hoek (1968) has proposed the following equation expressed with directly obtained principal stresses from conventional triaxial compression tests:

$$\tau_{max}=\tau_{max,0}+A(\sigma_m)^b$$

where A and b are material constants, $\tau_{max}=(\sigma_1-\sigma_3)/2$, $\sigma_m=(\sigma_1+\sigma_3)/2$ and $\tau_{max,0}$ is the intercept of τ_{max} versus σ_m plot when $\sigma_m=0$. Hoek discussed the applicability of his equation to sandstone, but the difficulty to determine $\tau_{max,0}$ was remained. The strength criterion for soft rocks in this paper will be discussed basically along with Hoek's proposal.

The definition of soft rocks is not clear, but in this study soft rocks are characterized by the following properties;

i) unconfined compressive strength is below about 20 MN/m^2.

ii) natural initial porosity is in the range of 5-50%.

iii) mechanical properties are strongly influenced by the condition of water content and the pore water pressure.

iv) slaking and swelling properties are of characteristics on muddy stones.

These soft rocks include:

a) sedimentary rocks which are not lithificated, such as mudstone, stiff clay, tuffaceous soft rocks, soft sandstone and so on.

b) weathered rocks, especially weathered granite.

In the present paper a failure criterion of soft rocks is derived from the laboratory testing using the undisturbed samples of various soft rocks distributed in Japan. And it is clarified that the proposed failure criterion can be applicable to intact hard rocks and fissured rocks.

The proposed failure criterion has been reported by the authors at the time of the 12 th Symposium on Rock Mechanics of Japan Society of Civil Engineering (1979).

The similar failure criteria have been introduced by Adachi and Ogawa (1979) from the study of a tuffaceous rock commonly called Oya-Ishi, and by Franklin (1971) from the comparative and/or statistical studies for linear and curved criteria, based on conventional triaxial test data for a large number of rock specimens, also concrete.

MATERIALS AND METHODS

All of test specimens of rock materials used in this laboratory testing were undisturbed and saturated. Almost all of rocks used except for highly weathered granite were prepared by the block sampling method. The triaxial compression test was performed mainly under the consolidated-drained condition and partly under the consolidated undrained condition to investigate the pore-pressure behaviours.

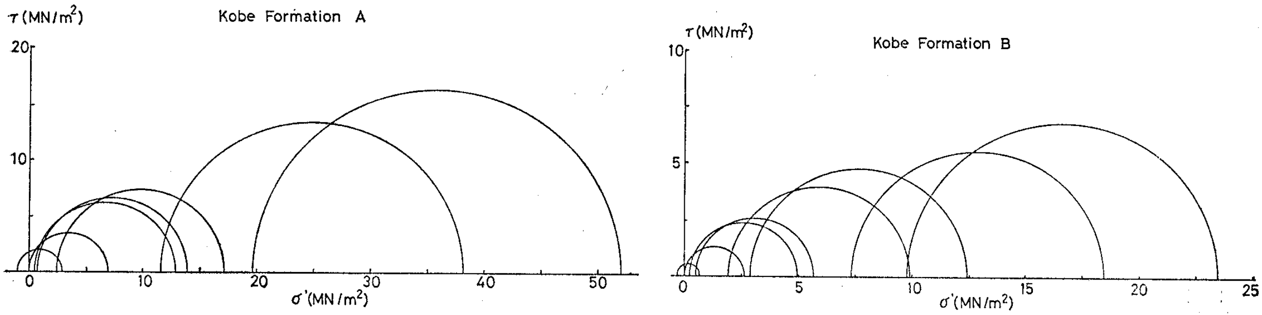
Standard size of specimens was 50 mm diameter and 100 mm length, and to confirm the size effect, 100 mm diameter and 200 mm length samples were used. The rate of loading for CD test was from 0.01%/min for porous rocks to 0.005%/min for muddy stones and that for $\overline{CU}$ test was about 0.05%/min.

Sample size for radial compression test was 50 mm diameter and 50 mm length.

Some geological and physical properties of soft rocks used in the laboratory testing are summarized in Table 1.

Table 1. Geological and physical properties

Materials	Locality of Sampling	Geological Formation	Period of Sedimentation	Maximum Thickness of Formation	Physical Properties			
					w_{sat} (%)	e_0	G_S	q_u (MPa)
Kōbe mudstone A	Myodani-cho, Kōbe city	Kōbe F. (fresh-water)	Miocene (middle-upper)	about 500 m	12.9	0.34	2.66	7.06
Kōbe mudstone B	Shimohata-cho, Kōbe city				18.4	0.49	2.67	2.63
Sano muddy stone	Sano-cho, Yokohama city	Miura F. (marine)	Miocene		45.0	0.93	2.62	9.02
Yokohama siltstone	Yokohama city	Miura F. (marine)	Miocene		32.6	0.94	2.67	2.06
Tuffaceous sandstone (Ohya-Ishi)					17.3	0.44	2.53	13.5
London clay[3)]	Ashford Common Shaft, England	London clay	Eocene	360~400 m	24.2	0.64	2.73	0.206
Keuper marl[4)] (silty mudstone)	Middle England	Keuper marl	Triassic	1 800 m	19 ~26	0.52 ~0.72	2.76	0.409



(a) **Kobe Formation A** (b) **Kobe Formation B**

Fig. 1. Mohr envelopes for drained triaxial compression tests

STRENGTH CRITERION OF SOFT AND HARD ROCKS

The envelopes of Mohr stress circle at failure of soft rocks show generally the non-linearity. In this chapter the experimental equation to represent this non-linear envelopes is considered and then the applicability of the proposed equation to intact hard rocks and fissured rocks is examined.

Strength Criterion for CD Triaxial Compressive Stresses

The strength relation of soft rocks obtained from CD triaxial compression tests are discussed in the following.

Fig. 1(a) and (b) are the Mohr envelopes at failure stresses of tertiary mudstones from Kobe Formation which has the maximum thickness about 700 m. Both envelopes show the typical non-linear relation. Sampling locations of two mudstones are at a distance of horizontally 3 km apart each other. These mudstones are a series of sediments belonging to the same sedimentary basin. But the difference of strength is remarkable, that is, the unconfined compressive strength of rocks shown in Fig. 1(a) is 7.06 MN/m^2 and that of Fig. 1(b) is 2.63 MN/m^2.

In order to compare the failure envelopes of these mudstones the mean effective stress $\sigma'_m=(\sigma'_1+2\sigma'_3)/3$ at failure and maximum shear stress $\tau_m=(\sigma_1-\sigma_3)/2$ are divided by each reference strength σ'_{m0} and τ_{m0}, which are the case of $\sigma'_3=0$ and can be obtained by the unconfined compression tests.

Strength relation at failure of these mudstones, which have a different unconfined compressive strength, can be expressed by the same line using the normalized strength as shown in Fig. 2. This may be originated in the identity of geological history, and suggests that unconfined compressive strength can be utilized as a good parameter to represent the strength relation.

To express the non-linearity, normalized strength is plotted by a logarithmic representation in Fig. 3(a). It is clear that each points are arranged on a unique line and the plots on a log-scale can be recognized as a proper representation. However, the point corresponding to unconfined compressive strength is isolated from the line. This phenomenon is considered due to the brittle nature of the rock, that is, which unconfined compressive strength is so sensitive to induced tensile stress around micro-flaws in rocks that it is not connected in the case of sensitive mudstone to the strength obtained by the triaxial compression stress state.

It is able to make correction for the line in Fig. 3(a) to pass the coordinate (1, 1) by multipling a coefficient 'A' to unconfined compressive strength. Fig. 3(b) shows the failure line obtained by the above correction. The line in Fig. 3(b) gives the strength relation of Kobe mudstones and can be expressed by the following equation:

$$\frac{\tau_m}{A\cdot\left(\frac{q_u}{2}\right)}=\left\{\frac{\sigma_m'}{A\cdot\left(\frac{q_u}{3}\right)}\right\}^{\beta'} \qquad (1)$$

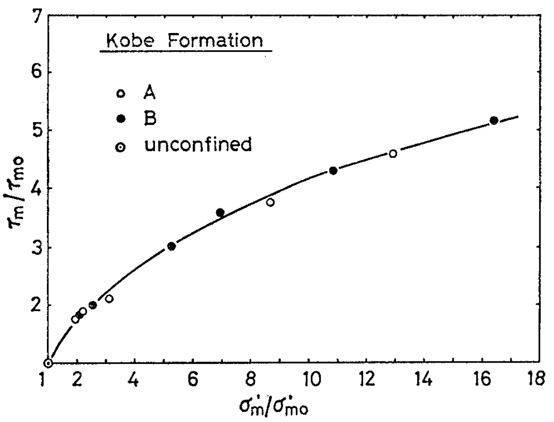


Fig. 2. Normalized strength relation of Kobe Formation A and B

Rewriting the Eq. (1) as generalized form, the next equation is obtained. (A curvilinear failure criterion expressed by octahedral stresses is considered in an Appendix.)

$$\frac{\tau_m}{\tau_{m0}}=\alpha\left(\frac{\sigma_m'}{\sigma_{m0}'}\right)^{\beta} \qquad (2)$$

where

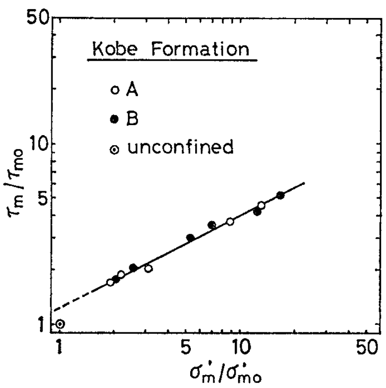
$\alpha=A^{(1-\beta')}$

$\beta=\beta'$

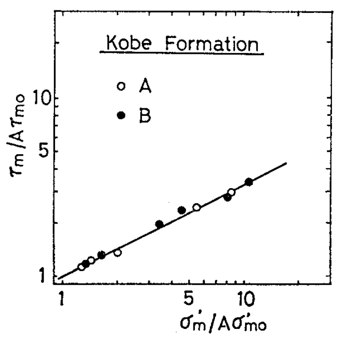
σ'_{m0}; the reference mean effective stress

τ_{m0}; the reference maximum shear stress

Let us examine the applicability of the



(a) **Kobe Formation A and B**



(b) **transformed axes by A-value**

Fig. 3. Logarithmic representation of normalized strength

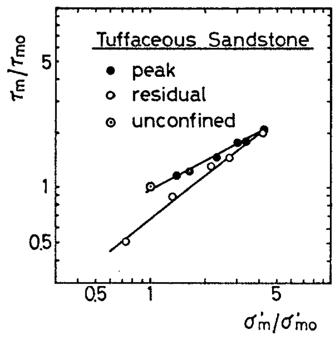


Fig. 4. Strength relation of tuffaceous sandstone

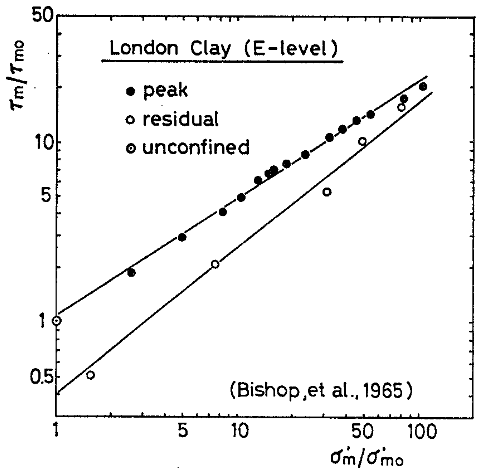


Fig. 5. Strength relation of London clay[3)]

Eq. (2) to other types of soft rocks. Figs. 4～6 show the strength relation of sandy tuff, London clay (Bishop et al., 1965), and Keuper marl (Chandler, 1967), and Fig. 7 (a) shows the results of Yokohama siltstone. From above figures, it seems that in all cases the Eq. (2) is applicable for the wide range of strength. Figs. 4～6 include the relation of the residual stress expressed by normalized strength. The residual stress shown in those Figures is defined by the fully dropped and ultimate stress after peak in stress-strain curves. (see Fig. 16). It is recognized from Figs. 4～6 that the Eq. (2) can be applied to the residual strength, too. However, in case of residual strength, it is also adequate that σ'_{m0} and τ_{m0} are replaced by unit.

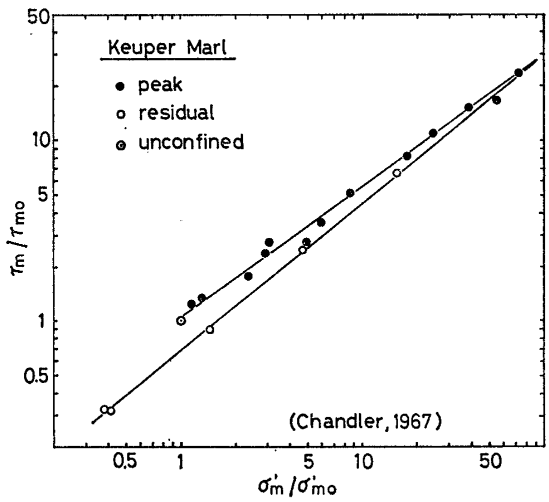
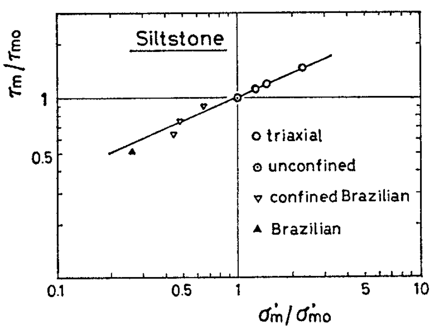
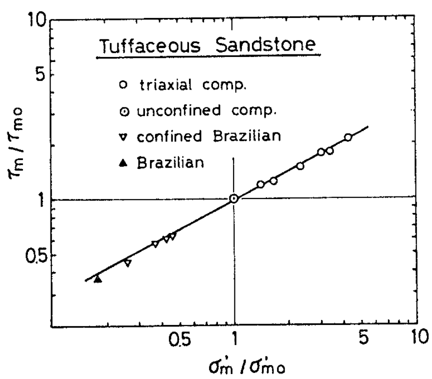


Fig. 6. Strength relation of Keuper marl[4)]



(a) siltstone (Yokohama)



(b) tuffaceous sandstone

Fig. 7. Applicability of Eq. (2) to tensile stresses

Table 2. Unconfined compressive strength and material constants

Materials	q_u (MPa)	Peak				Residual		
		α	β	A	r***	α	β	r***
London clay[3]	0.206	1.096	0.647	1.296	0.996	0.413	0.798	0.997
Keuper marl[4]	0.409	1.034	0.722	1.128	0.996	0.685	0.817	0.998
Kōbe Formation A	7.06	1.230	0.518	1.536	0.994	—	—	—
Kōbe Formation B	2.63							
Yokohama siltstone	2.06	1.011	0.440	1.020	0.995	0.676	0.793	0.996
Tuffaceous sandstone	13.5	0.959	0.527	0.916	0.994	0.681	0.774	0.997
Tennessee marble[13]	131	0.976	0.651	0.934	0.998	0.762	0.864	0.998
Dunham dolomite[11]	271	1.021	0.625	1.057	0.998	0.508	1.000	0.998
Ormond siltstone[6]	54.7	1.070	0.634	1.203	0.995	—	—	—
Panguna andesite[8]	1.23*	1.049	0.852	1.382	0.998	—	—	—
Weathered granite[15]	(9.6×10^{-3})**	0.999	0.908	0.989	0.998	—	—	—

* closely jointed rock ** estimated value *** correlative coefficient

The unconfined compressive strength q_u, the coefficients α, β in Eq. (2) and the correlative coefficient r by least squares method are summarized in Table 2. The correlative coefficient r is about unit. From the above considerations, it may be concluded that the Eq. (2) is the suitable form for strength criterion of soft sedimentary rocks.

Strength Criterion under Tensile Stress

It is well known that bacause of brittle nature of rock materials, it is not easy to perform the experiments under the pure uniaxial tensile stress. Consequently the radial compression test (or Brazilian test) is usually adopted as an alternative means of determining the tensile strength. The state of stress of Brazilian test is a plane stress state and the stresses acting along the diameter at failure are given as follows;

$$\sigma_1=3\,\sigma_t,\quad \sigma_2=0,\quad \sigma_3=-\sigma_t<0$$

where σ_1, σ_2, σ_3 are the principal stresses and σ_t is the tensile strength (tensile stress is negative).

The more general stress state can be obtained by Brazilian test under confining pressure ($\sigma_2>0$) as performed by Jaeger and Hoskins (1966). Fig. 7(a) and (b) show the Brazilian test results of siltstone and sandy tuff. These figures also show the strength obtained from the triaxial compression and Brazilian tests expressed by Eq. (2).

For both of soft rocks, value 'α' in Eq. (2) is nearly unit. According to Fig. 7, it seems that Eq. (2) can be applied to the rocks with $\alpha\fallingdotseq1$, and satisfied by the above mensioned state of stress.

Strength of Weathered Granite

Granite is a characteristic material which continuously produces decomposed materials from fresh rock to soil by weathering. The porosity of granite increases according to weathering from 0.5% in fresh rock to 50—60% in highly decomposed granite soil, and following this its strength decreases remarkably.

These properties of granite have already been reported by one of authors (1974, 1977). On the basis of these experimental data, the relationship between Eq. (2) and the strength of weathered granite is considered in this section.

Fig. 8 shows the strength relation of undisturbed weathered granites obtained from three points of sampling locations. The initial porosities of samples used are in the range of

27—38%. The correlative coefficient of the data in Fig. 8 is 0.99.

It is difficult to evaluate the unconfined compressive strength of weathered granite because of breakable nature of materials. Therefore, unconfined compressive strength has been estimated by the reverse calculation for the line shown in Fig. 8 to pass the coordinate (1, 1) and then the value is about 9.6 KN/m^2.

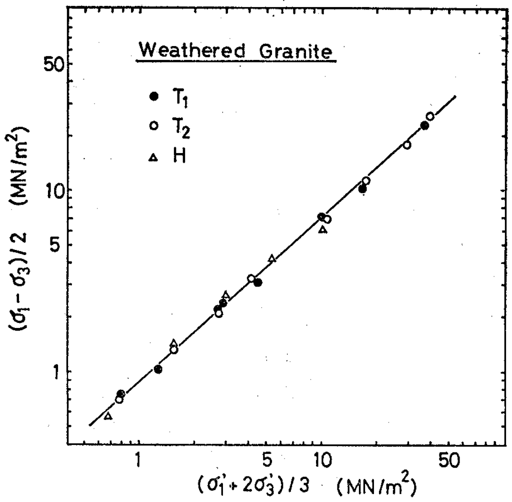


Fig. 8. Strength of weathered granite

Fig. 9 shows the strength relation of granite with porosities varied from 0.5—7%. This figure includes the triaxial compression test data of artificially granulated granite (Yoshinaka & Onodera, 1976), which was prepared by a technique of heat treatment. The procedures of heat treatment are as follows. After trimming specimens of fresh granite (q_u=165 MN/m^2) into cylindrical shape, the specimens are heated up to a temperature of 300~1 000 degrees centigrade, and then cooled down gradually to room temperature. Their porosities can be varied according to the maximum heating temperature.

The distribution of strength of these granite on τ_m/τ_{m0}–σ'_m/σ'_{m0} relation are given by two parallel lines as shown in Fig. 9. Two lines are bounded by the coordinate (1, 1).

The details of the reason why two parallel lines are presented in Fig. 9 are not clear. But it seems that these come from the same reason mentioned in 3.1. The strength reduction of heattreated granite is mainly due to the microcracks generated in the rock samples, therefore this treated granite can be recognized as a model of closely jointed rock.

Fig. 10 shows the strength relation of granite under compressive stress, and that the

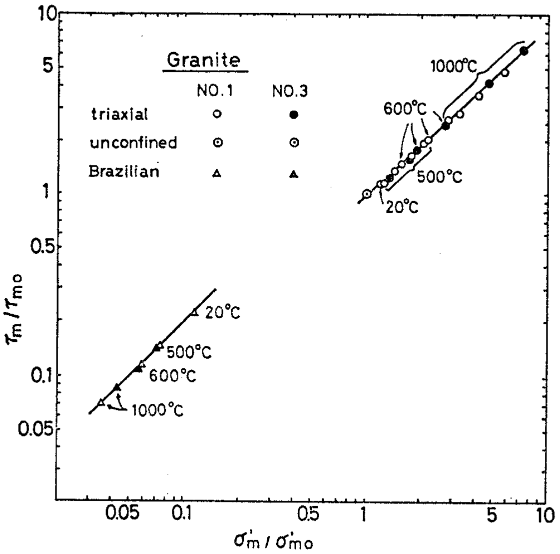


Fig. 9. Strength relation of heat-treated granite

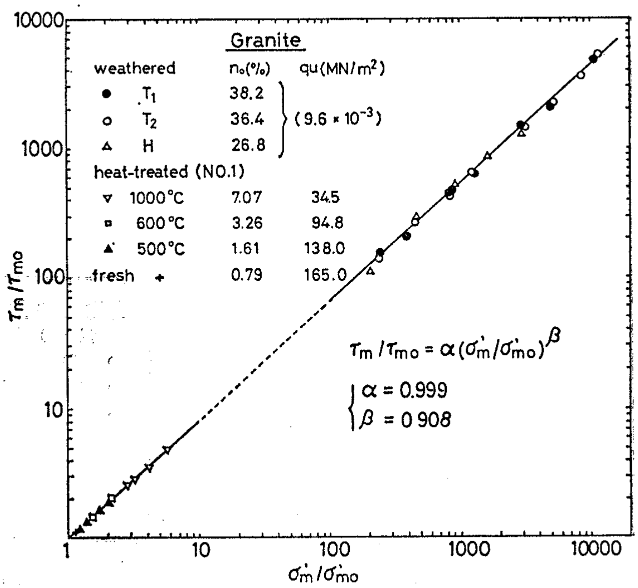


Fig. 10. Strength relation of weathered, heat-treated and fresh granite

value of β in Eq. (2) seems to be constant for very wide range of stress.

According to the above experimental evidences, it is ascertained that Eq. (2) can be applied to the strength relation of granite, which has the properties varied from fresh to highly weathered rock, with introducing the unconfined compressive strength into the reference strength σ'_{m0} and τ_{m0}.

Applicability of Proposed Equation to the Strength of Soft Rocks Reduced by Scale Effect

It is well known that strengths of rock materials decrease with increase of the volume of test specimen. These property, so-called scale effect, can be also observed on soft rocks. Therefore the relationship between Eq. (2) and strength reduction by scale effect is investigated by the triaxial compression test under CD condition. The sizes of specimens used in the test are 50 mm diameter, 100 mm length as standard specimens and 100 mm diameter, 200 mm length as large specimens.

Fig. 11(a) and (b) show the test results both of standard and large specimens of siltstone and sandy tuff. Although the strength of large samples is lower than that of standard one in the stress range considered, the coefficient values in Eq. (2) are the same in spite of the difference of sample size.

According to the experimental results, it seems that scale effect of soft rocks can be determined by unconfined compressive strength and Eq. (2).

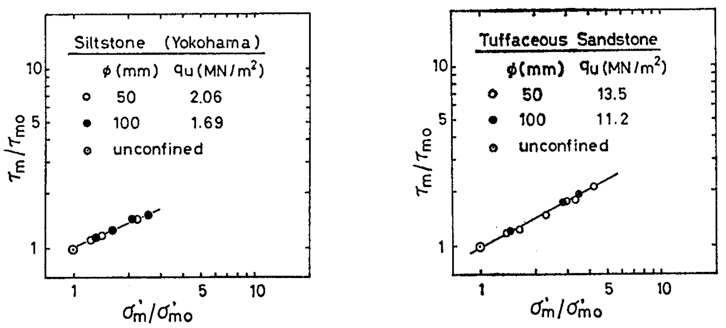
Applicability of Proposed Equation to Intact Hard Rock and Fissured Rock

Figs. 12~14 show the results of triaxial compression tests on Tennessee marble (q_u= 131 MN/m^2) reported by Wawersik and Fairhurst (1970), Dunham dolomite (q_u=271 MN/m^2) by Mogi (1971) and Ormond siltstone (q_u=54.7 MN/m^2) by Hobbs (1970).

It is clarified from Figs. 12~14 that representation of Eq. (2) is a suitable equation to express the strength of these intact hard rocks. Each values in Eq. (2) are summarized in Table 2. Values of 'α' are nearly unit and values of 'β' for peak strength are in the range of 0.63—0.65.

Applicability of Eq. (2) to fissured rock is investigated by using the results of triaxial compression tests on Panguna andesite reported by Jaeger (1970). It is so difficult to determine the mechanical properties of fissured but undisturbed rock with laboratory testing that the data published until now are very limited. In this sense, experimental result of Panguna andesite is very precious.

The sample size of Panguna andesite is 15 cm diameter, 30 cm height. The sample is divided up by a network of open joints and of veins with a rather weak fillings, the



(a) **siltstone (Yokohama)** (b) **tuffaceous sandstone**

Fig. 11. Applicability of Eq. (2) to scale effect

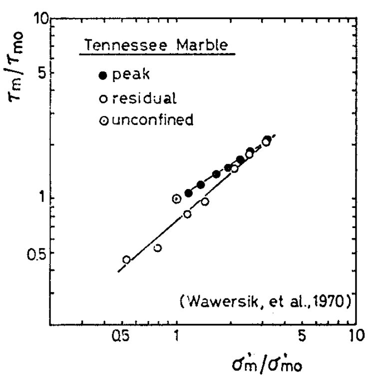


Fig. 12. Strength relation of Tennessee marble[13)]

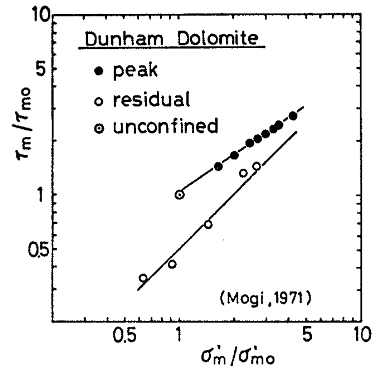


Fig. 13. Strength relation of Dunham dolomite[11)]

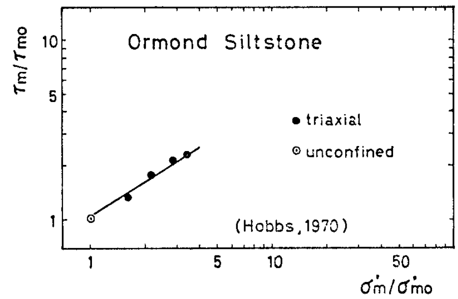


Fig. 14. Strength relation of Ormond siltstone[6)]

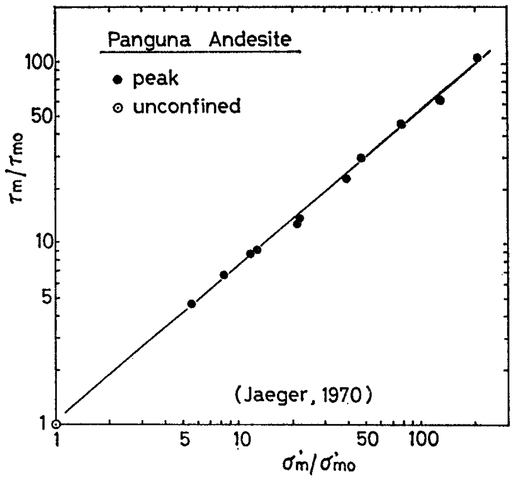


Fig. 15. Strength relation of Panguna andesite[8)]

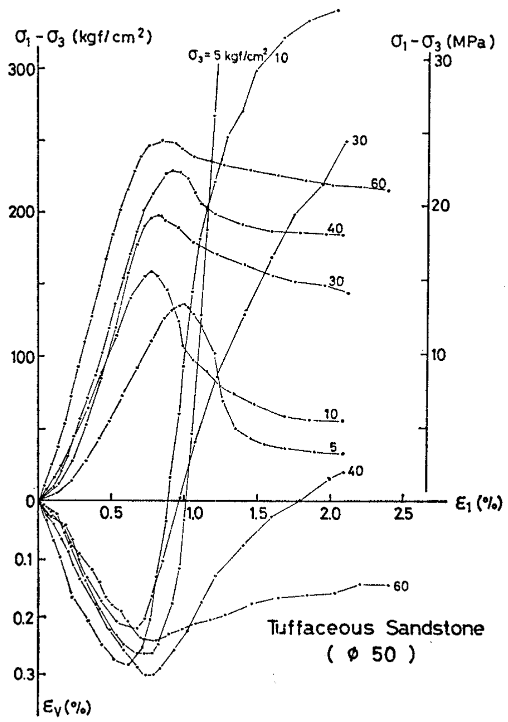


Fig. 16. Stress-strain curves of tuffaceous sandstone

spacing of these being so close that a cross section of the core would usually contain 50 to 100 individual areas separated by planes of weakness. The unconfined compressive strength of intact rock (268 MN/m^2) is quite a strong one. Fig. 15 shows the experimental results and the regression line by Eq. (2).

It seems that in the case of fissured rock the Eq. (2) is suitable for expression of strength relation, and the coefficients in Eq. (2) are $\alpha=1.049$ and $\beta=0.852$.

Strain-Softening and Yielding with Increase of Confining Pressure

Generally the rock meterials have strong strain-softening properties under moderate confining pressure. Soft rocks also have a characteristic of strain-softening as well as hard rock materials. But compared to hard rocks, the stress-strain curves of soft rocks show an easy conversion from strain softening to strain hardening with increase of

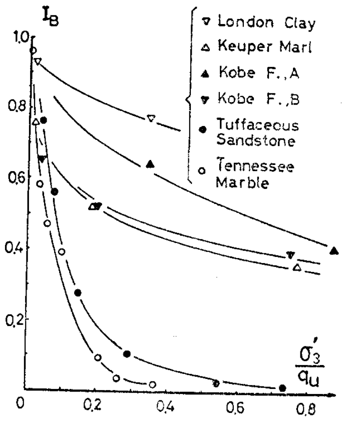


Fig. 17. Relation between Brittleness Index (I_B) and normalized confining pressure

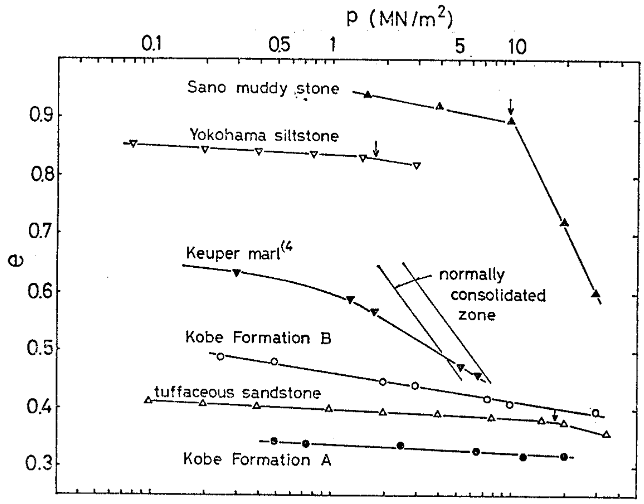


Fig. 18. e-log p curves under equal all round pressure (arrow denotes the yielding point)

confining pressure. Fig. 16 is a typical example of this characteristics. ($n_0=29.7\%$, $q_u=13.5\,\mathrm{MN/m^2}$). In this section the relationships between strain softening and confining pressure are considered from a point of ultimate and residual strengths given by Eq. (2).

Bishop[16] defined the Brittleness Index I_B to represent the strain softening properties of stiff clay as follows;

$$I_B=1-\left(\frac{\tau_r}{\tau_f}\right) \qquad (3)$$

where τ_f; peak shear stress

τ_r; residual shear stress

I_B is an important index to consider the progressive failure. Fig. 17 shows the relation between I_B and confining pressure of some of soft rocks and hard rocks. The abscissa of Fig. 17 is the confining pressure normalized by each unconfined compressive strength.

Although I_B decreases almost exponentially for all rocks with increase of confining pressure, the intensity of strength reduction depends on the nature of rock materials. The effect of confining pressure on I_B can be determined by substituting Eq. (2) to Eq. (3).

If strain softening occurs when mean effective stress is constant, the Brittleness Index I_B is expressed as follows;

$$I_B=1-\left(\frac{\alpha_r}{\alpha_f}\right)\left(\frac{\sigma_m'}{\sigma_{m0}'}\right)^{\beta_r-\beta_f} \qquad (4)$$

The mean effective stress at which strain softening occurs no more, can be obtained by setting $I_B=0$;

$$(\sigma_m')_{I_B=0}=\sigma_{m0}'\left(\frac{\alpha_f}{\alpha_r}\right)^{1/(\beta_r-\beta_f)}=\frac{q_u}{3}\left(\frac{\alpha_f}{\alpha_r}\right)^{1/(\beta_r-\beta_f)} \qquad (5)$$

Each coefficients in Eq. (5) are the material constants.

Next, we consider the relationship between the mean effective stress at $I_B=0$, yielding and unconfined compressive strength. Fig. 18 shows $e-\log p$ curves under equal all round pressure. The allows in Fig. 18 represent the yielding stress p_0, however, the very points at p_0 on curves are not distinguishable owing to the drawing $e-\log p$ relations of various rocks with wide range of initial void ratio from 0.3 to 1.0 in the same scale.

Table 3. Mean effective stresses at $(\tau_f=\tau_r)$, $(\sigma_m')_{I_B=0}$, and yielding stresses under equal all round pressure, p_0

Materials	$(\sigma_m')_{I_B=0}$ (MPa)	$(\sigma_m')_{I_B=0}/q_u$	p_0 (MPa)	$(\sigma_m)_{I_B=0}/p_0$
Tuffaceous sandstone	18.0	1.37	17.6	1.0
Yokohama siltstone	2.16	1.05	1.76	1.2
Sano muddy stone*	7.02	0.78	9.80	0.7
Hoshikawa siltstone**	4.67	1.36	3.92	1.2
London clay[3]	11.9～	58～	3.92	～3.1
Keuper marl[4]	10.4	25.2	3.92～4.90	2.1～2.7
Kobe Formation A	(67.4)	(9.6)	—	—

* $q_u=9.02$ MPa ** $q_u=3.45$ MPa

Table 3 shows the relation of yielding properties of soft rocks between consolidation yielding pressure p_0 and the mean effective stress under shearing deformation at $I_B=0$, and the relation of unconfined compressive strength q_u as a strength index of soft rocks to those yielding pressures.

The value of $(\sigma'_m)_{I_B=0}$ is nearly equal to the unconfined compressive strength for rocks such as siltstone, sandstone, hard limestone, hard dolomite. And the value $(\sigma'_m)_{I_B=0}$ for siltstone and sandstone are nearly equal to the yielding strength by consolidation with equal all round pressure. On the other hand, for mudstones the value $(\sigma'_m)_{I_B=0}$ is higher than the unconfined compressive strength and yielding strength by consolidation pressure, that is, from Table 3, $(\sigma'_m)_{I_B=0}$ may be equal to (20–50)q_u and (2–3)p_0.

CONCLUSIONS

1) A proposed equation expressed by power law is suitable for strength criterion of failure and residual stresses of soft rocks, and the equation is possible to be applied to the strength criterion of intact hard rocks and fissured rocks. The equation is as follows;

$$\frac{\tau_m}{\tau_{m0}}=\alpha\left(\frac{\sigma'_m}{\sigma'_{m0}}\right)^{\beta}$$

where $\tau_m=(\sigma_1-\sigma_3)/2$, $\sigma'_m=(\sigma'_1+\sigma'_2+\sigma'_3)/3$ and $\sigma'_m>0$. α and β are the material constants of rocks as summarized in Table 2. σ'_{m0} and τ_{m0} are the reference strength. As the reference strength, it is convenient to adopt the unconfined compressive strength. When $\sigma'_{m0}=q_u/3$ and $\tau_{m0}=q_u/2$, there are many cases of $\alpha=1$. (Refer to an Appendix)

2) Strain softening occurs when the mean effective stress is below the following value:

$$(\sigma'_m)_{I_B=0}=\left(\frac{q_u}{3}\right)\left(\frac{\alpha_f}{\alpha_r}\right)^{1/(\beta_r-\beta_f)}$$

The value $(\sigma'_m)_{I_B=0}$ is nearly equal to the unconfined compressive strength for rocks such as siltstone, sandstone, hard limestone, hard dolomite. And the value $(\sigma'_m)_{I_B=0}$ for siltstone and sandstone are also nearly equal to the yielding strength by consolidation with equal all round pressure. On the other hand, for mudstones the value $(\sigma'_m)_{I_B=0}$ is higher than the unconfined compressive strength and yielding strength by consolidation pressure, that is, from Table 3, $(\sigma'_m)_{I_B=0}$ may be equal to (20–50) q_u and (2–3)p_0.

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NOTATION

A=correction factor, $\alpha^{1/(1-\beta')}$
e=void ratio
G_s=specific gravity
I_B=Brittleness Index
p=consolidation pressure
p_0=consolidation yielding pressure
q_u=unconfined compressive strength
r=correlative coefficient
w_{sat}=saturated water content
α=material constant in Eq. (2)
α_f, α_r=α-value for failure and residual stresses
β=material constant in Eq. (2)
β_f, β_r=β-value for failure and residual stresses
ε_1=axial strain
ε_v=volumetric strain
$\sigma'_1, \sigma'_2, \sigma'_3$=principal effective stresses $(\sigma'_1>\sigma'_2>\sigma'_3)$
σ'_m=mean effective stress at failure, $(\sigma'_1+\sigma'_2+\sigma'_3)/3$
σ'_{m0}=reference mean effective stress, $(=q_u/3)$
σ'_{oct}=octahedral normal stress, $(\sigma'_1+\sigma'_2+\sigma'_3)/3$
$(\sigma'_{oct})_0$=reference octahedral normal stress, $(=q_u/3)$
σ_t=tensile strength (negative)
$(\sigma'_m)_{I_B=0}$=mean effective stress at $(\tau_f=\tau_r)$
τ_m=maximum shear stress, $(\sigma_1-\sigma_3)/2$
τ_{m0}=reference maximum shear stress $(=q_u/2)$
τ_{oct}=octahedral shear stress, $1/3[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2]^{1/2}$
$(\tau_{oct})_0$=reference octahedral shear stress, $(\sqrt{2}\,q_u/3)$
τ_f, τ_r=shear stresses for failure and residual state

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APPENDIX

A Curvilinear Failure Criterion Expressed by Octahedral Stresses

The following failure criterion (6) is obtained by introducing the octahedral shear stress τ_{oct} in stead of maximum shear stress τ_m into the Eq. (2) :

$$\frac{\tau_{oct}}{(\tau_{oct})_0}=\alpha\left\{\frac{\sigma_{oct}'}{(\sigma_{oct}')_0}\right\}^{\beta} \qquad (6)$$

where $\tau_{oct}=1/3\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\}^{1/2}$, $\sigma_{oct}'=1/3(\sigma_1'+\sigma_2'+\sigma_3')$, $(\tau_{oct})_0$ and $(\sigma_{oct}')_0$ are the reference strengths.

Eq. (6) corresponds to a curvilinear representation of following Eq. (7), what is called extended von Mises* and also a generalized form of Eq. (8) which is so-called extended Griffith (Murrell, 1963) and has the theoretical back-ground :

$$\tau_{oct}=\frac{\sqrt{2}}{3}\alpha\,\sigma_{oct} \qquad (7)$$

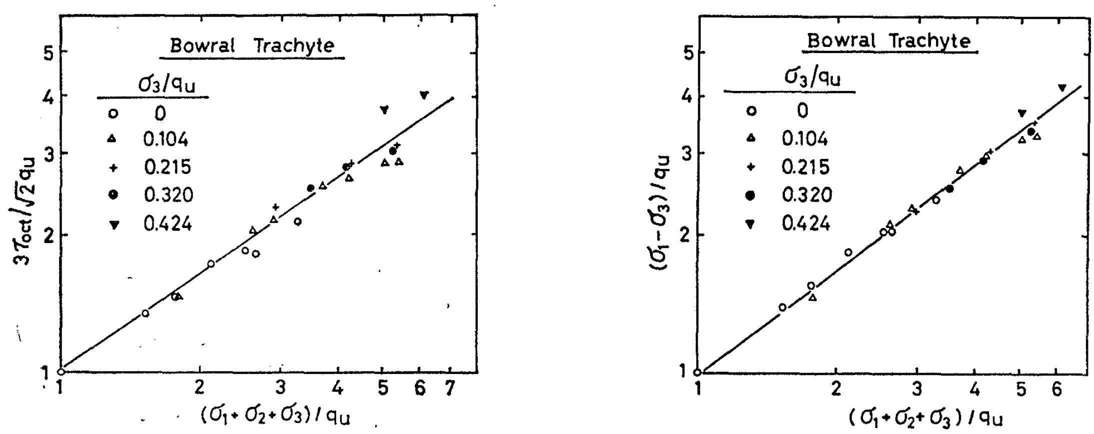
$$\tau_{oct}=2\sqrt{2}\ \sigma_t^{1/2}\ \sigma_{oct}^{1/2} \qquad (8)$$

On the other hand, Eq. (2) corresponds to a generalized form of Eq. (9) what is called extended Tresca*

$$(\sigma_1-\sigma_3)=\alpha\left(\frac{\sigma_1'+\sigma_2'+\sigma_3'}{3}\right) \qquad (9)$$

It seems to be very interesting to examine what degree of approximation by using Eq. (6) may be possible for explanation of experimental data of true triaxial tests, and to compare Eq. (2) with Eq. (6). However, these data are so limited that the investigation may only introduce the problems.

When unconfined compressive strength is adopted as a reference strength, next relations are



(a) expressed by Eq. (10) (b) expressed by Eq. (11)

Fig. 19. True triaxial data of Bowral Trachyte

* discussed by Bishop in the sixth Rankine Lecture, Geotechnique, Vol. XII, No. 2, 1966.

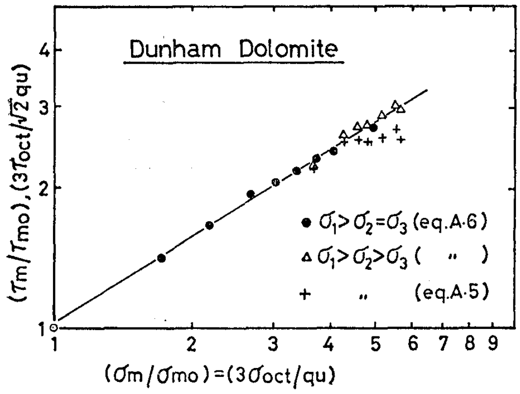


Fig. 20. conventional and true triaxial compression data of Dunham dolomite[11], expressed by Eq. 10 (eq. A 5) & 11 (eq. A 6)

obtained from Eqs. (6) and (2) respectively ;

$$\frac{\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\}^{1/2}}{\sqrt{2}\ q_u}$$

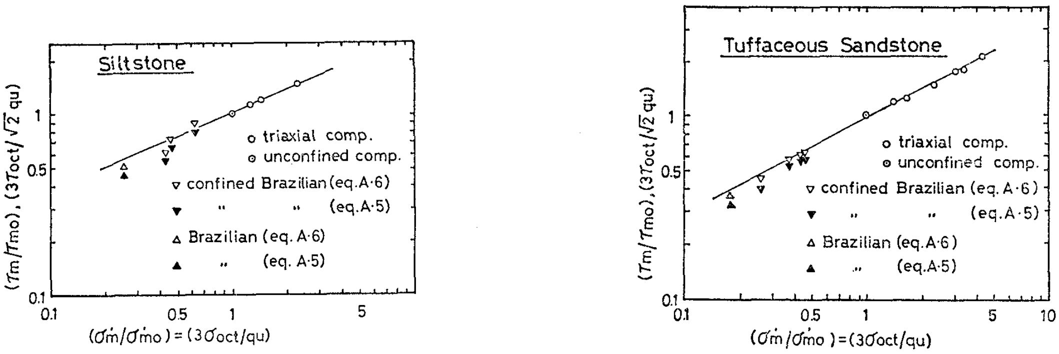
$$=\alpha\left(\frac{\sigma_1'+\sigma_2'+\sigma_3'}{q_u}\right)^{\beta} \tag{10}$$

$$\left(\frac{\sigma_1-\sigma_3}{q_u}\right)=\alpha\left(\frac{\sigma_1'+\sigma_2'+\sigma_3'}{q_u}\right)^{\beta} \tag{11}$$

The results may be summarized as follows :

1) Figs. 19 (a) and (b) are representations of the true triaxial test data of Bowral Trachyte* (q_u=155 MPa) expressed by Eq. (10) and (11). In this case, it seems that Eq. (10) expressed by τ_{oct} gives the data more scattering representation compared with Eq. (11).

2) Fig. 20 shows the results of conventional triaxial ($\sigma_1>\sigma_2=\sigma_3$) and true triaxial ($\sigma_1>\sigma_2>\sigma_3$) compression tests of Dunham Dolomite (q_u=267 MPa ; Mogi, 1970). From this, it seems that the discrepancy of strength lines obtained from two series of tests under different stress conditions is magnified when using the octahedral shear stress and Eq. (10).



(a) **siltstone** (b) **tuffaceous sandstone**

Fig. 21. Comparison of Eq. 10 (eq. A 5) & Eq. 11 (eq. A 6) by triaxial and confined Brazilian tests

3) Figs. 21 (a) and (b) show the results of the strength relation of two kinds of soft rocks, obtained from conventional triaxial compression and confined Brazilian tests. From these figures, it seems that the distances between each plotted points obtained from confined Brazilian test and the strength line obtained from triaxial compression test are enlarged when using τ_{oct} and Eq. (10) compared with Eq. (11) Considering from above mentioned results, we may conclude at present, that Eq. (2) is better than Eq. (10) judging from the scattering of data and continuity of strength relation under different test conditions. However, more detail discussion concerning these equations should wait for accumulation of true triaxial compression data and theoretical developments.

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