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EXPERIMENTAL STUDY OF ANISOTROPIC SHEAR STRENGTH OF SAND BY PLANE STRAIN TEST

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ABSTRACT

Anisotropic parallel alignment of particles is universally observed not only in river, beach and coastal dune sands but also in artificially deposited sands. Anisotropic shear strength caused by the anisotropic parallel alignment of particles can be observed more clearly in the plane strain condition $\varepsilon_2=0$ than in the symmetrical stress condition $\sigma_2=\sigma_3$. This must be chiefly due to the fact that re-arrangement of particles during shear deformation can be more easily performed in the latter condition than in the former one. Shear strength of sand in a plane strain test has been generally believed to be 10% to 20% greater than that obtained in a triaxial compression test. When the specimen is compressed at a small angle to a bedding plane, however, it is possible that the shear strength in the plane strain test is smaller than that in the triaxial compression test. The effect of anisotropic shear strength cannot be ignored when stability problems and earth pressure problems in plane strain condition are analyzed.

Key words: anisotropy, angle of internal friction, bearing capacity, model test, plane strain, sand, soil structure
IGC: D6

INTRODUCTION

Oda (1972), Oda and Koishikawa (1977) and Arthur and Menzies (1972) have mentioned that sand deposited under the action of gravity force shows the anisotropic shear strength in a triaxial compression test. This is due to the preferred alignment of long axes of particles nearly parallel to a bedding plane. We now discuss in this paper the shear strength of a sand in the plane strain condition by taking account of its fabric anisotropy.

Some of failures of foundation occur under conditions of plane strain $\varepsilon_2=0$. Many authors (i.e., Bjerrum and Kummeneje, 1961; Cornforth, 1964; Leussink and Wittke, 1963; Tong, 1970; Lee, 1970) have discussed the characteristics of shear strength and of dilatancy observed in the plane strain test of sands with the following conclusions:

(1) The friction angle in the plane strain test ϕ_p is usually 10% to 20% greater than that measured by the triaxial compression test ϕ_t, when a dense sand is tested under a low confining pressure. No significant difference between ϕ_p and ϕ_t can be observed when a sand is loose or when a dense sand is tested at a sufficiently high confining pressure.

(2) A sand with a given void ratio is failed at a smaller axial strain in the plane strain test than in the triaxial compression test.

(3) A sand dilates more extensively in the triaxial compression test than in the plane

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strain test.

The effect of the fabric anisotropy on the mechanical response of sand was not considered by these authors. Some experimental evidences we have obtained clearly indicate that some of the above noted conclusions must be rectified and that the fabric anisotropy is significantly influential on the shear strength when a sand is failed under the plane strain condition.

PARALLEL ALIGNMENT OF PARTICLES IN SANDS

The particle alignment is an important factor when we discuss the mechanical characteristics of anisotropic sand (Oda, 1972; Arthur and Menzies, 1972).

In our experimental study, eleven samples of naturally deposited sands were taken by the use of thin wall samplers (7.5cm inner diameter and 20cm high) carefully pushed into typical sand beds deposited recently under different sedimentary conditions; i.e., river sands (6 specimens), beach sands (2 specimens) and coastal dune sands (2 specimens). Their physical properties are summarized in Table 1. The particle arrangement of these sands was fixed by infiltrating polyester resin binder into voids after oven-dried at 60°C. Maximum volume reduction through the oven-dried treatment was only 2.5%. Two thin sections which were made to be parallel to a vertical section (*V*-section) and a horizontal section (*H*-section) were prepared from each of thus fixed sands (Oda, 1972). Photo. 1 (a) is a typical example of particle arrangement observed in *V*-section of a river sand showing the remarkable parallel alignment of particles.

Fifteen disturbed sands were collected from various localities. Each of these sands was poured into vacant mold (5 cm in inner diameter and 10 cm high) by the following two ways:

(1) A sand was poured into the mold by a spoon as loosely as possible. The relative density of the sand ranges from 10% to 30%. This method of pouring is tentatively called *L*-method.

(2) A sand was poured into the mold by *L*-method and was compacted by repeated up and down vibrations by hands to make the relative densities ranging from 60% to 80% (*V*-method).

Table 1. Physical properties of naturally deposited sands

	Name of Sample	Locality	Specific gravity	Mean size (mm)	Uniformity coefficient	Void ratio	Relative density (%)
River sand	No. 6	Fuji river	2.73	0.65	1.11	0.92	14
	No. 11	Yōro river	2.61	0.22	1.50	0.76	84
	No. 12	Obitsu river	2.59	0.25	1.69	0.88	35
	No. 31	Sinano river	2.66	0.29	2.00	0.96	31
	No. 32		2.67	0.21	2.09	1.04	0
	No. 33		2.64	0.35	2.00	0.97	14
Beach sand	No. 20	Shirako beach	2.63	0.25	1.75	0.71	78
	No. 52	Kugenuma beach	2.82	—	—	0.87	60
Dune sand	No. 16	Ohara, Chiba	3.23	0.25	1.50	0.65	67
	No. 19	Shirako, Chiba	2.67	0.23	1.47	0.69	73

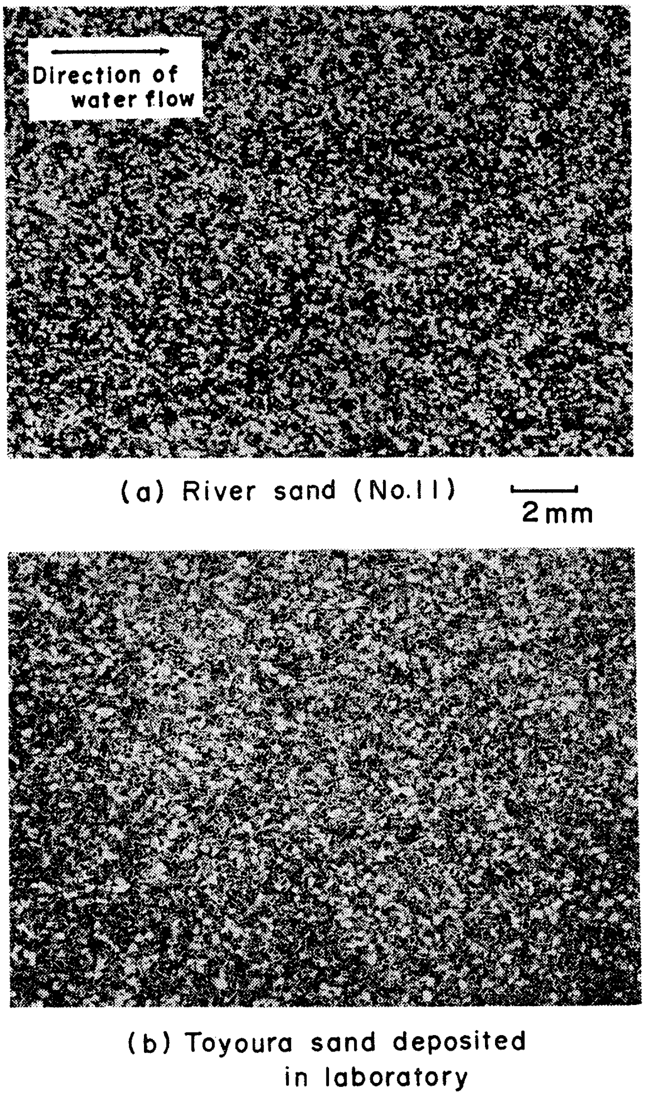

Photo. 1. Photographs showing the parallel alignment of particles in two *V*-sections prepared from (a) a naturally deposited river sand and (b) Toyoura sand deposited in air in a laboratory

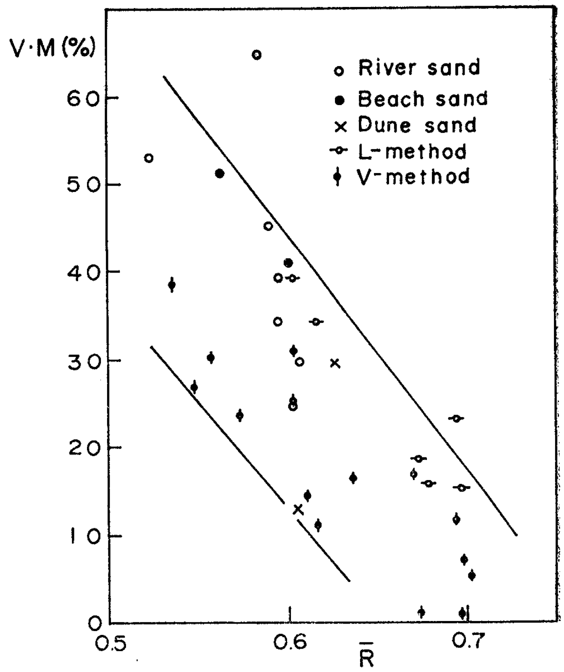

Fig. 1. Relation between the vector magnitude *V·M* showing the intensity of *particle* alignment in *V*-section and the axial ratio *R* showing the particle shape

These poured sands were fixed by the polyester resin binder and *V*- and *H*-sections were also prepared. Parallel alignments of particles in the above noted specimens were determined by measuring the angles θ_i between the horizontal direction and apparent long axes of about 250 particles observed in these thin sections.

Oda (1972) and Oda and Koishikawa (1977) have already shown the following experimental evidences: The preferred orientation of long axes of particles can be found to be parallel to the horizontal in sands artificially compacted by *L*-and *V*-methods. The intensity of such preferred orientation of particles is closely related to the shape characteristic of particles (axial ratio), the method of compaction, gravity force and subsequent disturbance after deposition such as penetration of hand plunger. *V*-section of naturally deposited sands such as river and beach sands show a similar pattern of particle alignment to that of the disturbed sands, although they show an imbrication due to the effect of water flow at the time of deposition. The above mentioned facts are also supported by the present additional study (Fig. 1).

Vector magnitude $V{\cdot}M$ used as an ordinate in Fig. 1 is an index to represent the intensity of parallel alignment of apparent long axes of non-spherical particles observed in *V*-section. The clear definition of vector magnitude was given by Curray (1956) and Oda (1972). According to it, the vector magnitude $V{\cdot}M$ varies from 0% which means a complete random arrangement of apparent long axes without any preferred direction to 100% which means their complete alignment with a clear preferred direction. The increase of vector magnitude indicates the increase of parallel alignment of particles. Axial ratio $\bar{R}$ in Fig. 1 is an index for characterizing the particle shape of non-spherical particles. The axial ratio 1 means completely spherical shape of particles. With decreasing the axial ratio, the shape of particles becomes flat or rod-like (Oda, 1972).

From Fig. 1, it can conservatively be said as follows: In the sands packed by *L*- and *V*-methods in a laboratory, the intensity of particle alignment in *V*-section increases with decreasing the axial ratio. The sands packed by *L*-method, however, have slightly higher values of vector magnitude than those compacted by *V*-method. The parallel alignments of particles in *V*-sections of river and beach sands are similar to those of the sand packed by *L*-method. A dune sand shows no significant parallel orientation even in its *V*-section as compared with other river and beach sands. Although the anisotropic orientation of particles in naturally deposited sands may be simulated by adopting a suitable method in a laboratory, naturally deposited sands must be distinguished from artificially deposited ones in the following two points as pointed out by Oda and Koishikawa (1977):

(1) The preferred direction of particle alignment in naturally deposited sands inclines at an acute angle less than 30° to the horizontal while the preferred orientation of particles in artificially deposited sands is practically parallel to the horizontal.

(2) While there is no significant parallel alignment of particles in *H*-section of any artificially deposited sands, a marked alignment can be seen in *H*-section of naturally deposited sands.

PLANE STRAIN TEST AND TRIAXIAL COMPRESSION TEST

Toyoura Sand

Toyoura sand tested is a well sorted fine sand (mean grain size=0.18 mm: uniformity coefficient=1.5) which in composed of 75% quartz, 22% feldspar and 3% magnetite. Its constituent particles were so strong that particle crushing was negligible when tested at the condition of $\sigma_3=4\,kg/cm^2$. Maximum and minimum void ratios were 0.99 and 0.63 respectively.

Plane Strain Test

Twenty eight plane strain tests were performed on specimens having various tilting angles δ of an initial horizontal plane (bedding plane in Figs. 2 and 3) to the maximum principal stress axis by the following ways:

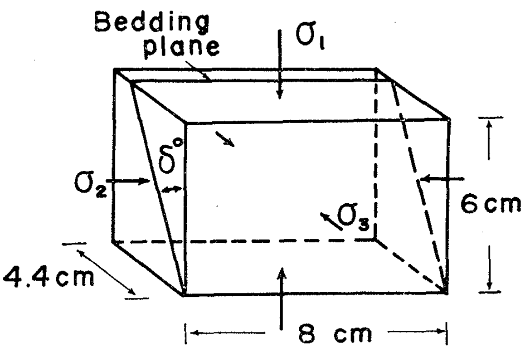

Fig. 2. Tilting angle δ of the bedding plane to the maximum principal stress axis in the plane strain test

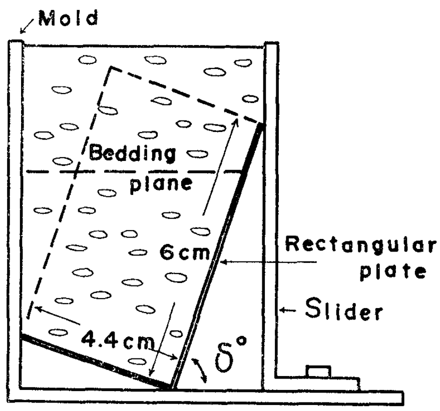

Fig. 3. Method to make the specimen with the tilting angle δ in the plane strain test

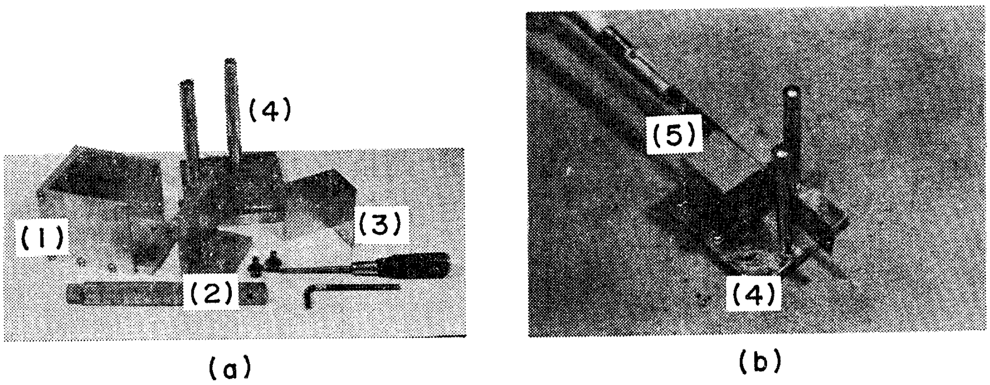

Photo. 2. Instruments to make specimens with various angles (plane strain test). (1) Mold; (2) Slider; (3) Rectangular plate; (4) Sample trimmer; (5) Specimen

(1) A rectangular plate having a dimension 8×4.4×6 cm was set in a sample former to have various tilting angles δ to the bedding plane by adjusting a slider of the sample former (Photo. 2(a) and Fig. 3).

(2) Oven-dried Toyoura sand was poured into the sample former through a funnel from 90 cm height. This pouring method was convenient to get a uniform sand bed whose void ratio was within 0.66 to 0.67 (relative density ≒ 90%). Its particle arrangement shown by Photo. 1(b) is very similar to that of a river sand shown by Photo. 1(a).

(3) The sample former filled with dried Toyoura sand was soaked very slowly into water and was frozen. The frozen specimen was taken away from the sample former and was fixed to a sample trimmer (Photo. 2(b)). The specimen was quickly trimmed by a saw and a straight edge to get a cuboidal specimen the dimension of which was 8 cm long, 4.4 cm wide and 6 cm high. In the plane strain test, the tilting angle δ was selected as 0°, 15°, 24°, 30°, 45°, 60° or 90°.

(4) Trimmed specimen was covered with a cuboidal rubber membrane (0.2 mm thick) and set quickly in a plane strain apparatus. Cell pressure σ_3 and vertical deviator stress $(\sigma_1-\sigma_3)$ were applied in the identical manner to a conventional triaxial test, and intermediate principal stress σ_2 was controlled by a null technique similar to that used in the measurement of pore water pressure. All four rigid polished platens were lubricated by two greased rubber membranes in order to provide frictionless sliding surfaces.

(5) A cell pressure of about 0.3 kg/cm² was applied to stiffen the specimen untill the ice in it completely thawed. After that, the cell pressure was increased up to a certain value (0.5, 1, 2 or 4 kg/cm²) with the simultaneous increase of intermediate stress σ_2 which was about 3% greater than the cell pressure to give tight contact of the specimen against two side platens. K_0-consolidation pressure has been usually applied in the previous study of plane strain test. It is worthy of note, however, that the shear strength of sand is not affected by the ways of consolidation (Bishop and Eldin, 1953; Bjerrum, Kringstad and Kummeneje, 1961).

(6) Axial stress σ_1 was increased up to the complete failure under the drained condition by the continuous increase of σ_2 to maintain the plane strain condition.

Triaxial Compression Test

In order to examine the difference of behaviour in a plane strain test and in a triaxial compression test, Toyoura sand was compressed triaxially by the following procedures:

(1) A mold of 7.5 cm inner diameter and 20 cm long was sunk into a bucket filled with water and was inclined at a tilting angle δ (Fig. 4). Toyoura sand was poured slowly into the mold through an upper opening and the side wall of the mold was tapped

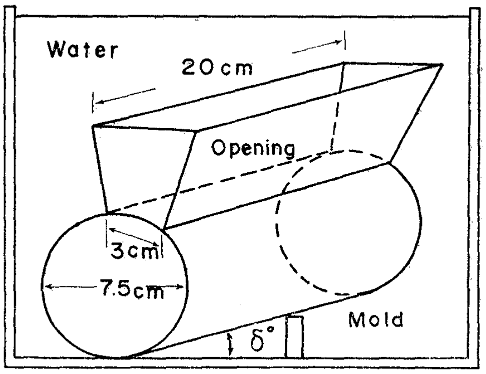

Fig. 4. Method to make the specimen with the tilting angle δ in the triaxial compression test

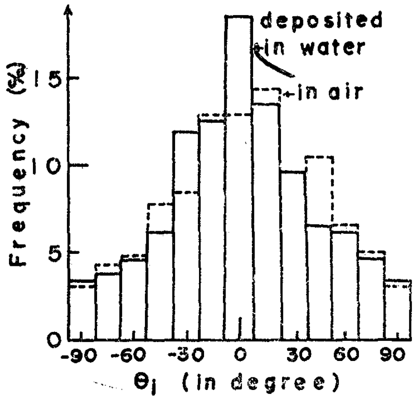

Fig. 5. Frequency histograms of θ_i in two *V*-sections of Toyoura sand artificially deposited in a laboratory

sufficiently to give a specimen having a void ratio within 0.67 to 0.68. Tilting angle δ was selected as 0°, 30°, 60° or 90° in the triaxial compression test. The water bucket with the inclined mold filled with the saturated sand was frozen in the same way as in the case of the plane strain test.

(2) The frozen sand was taken out from the mold and was trimmed by a saw and a straight edge to get a columnar specimen of 7.5 cm diameter and 8 cm long.

(3) The frozen specimen covered with a rubber membrane was placed in a triaxial chamber and a drained triaxial compression test was performed in the similar way to the plane strain test. Friction between the sand specimen and two end platens was minimized by inserting two greased rubber membranes between them. Cell pressure was selected as 0.5, 1 or 2 kg/cm^2.

According to the microscopic examination of *V*-sections, the apparent long axes were well aligned parallel to the bedding plane as shown by the solid line in Fig. 5. The broken line in Fig. 5 shows also the similar histogram obtained from the specimen of plane strain test.

EXPERIMENTAL RESULTS OF SHEAR TESTS

Shear Plane

No apparent shear plane could be observed at the peak deviator stress $(\sigma_1-\sigma_3)_f$. When an appreciable strain was added after the peak, however, thin films of shear planes began to appear. In the previous study, Oda (1972) has reported that any systematic preferred orientation of shear plane with respect to the bedding plane cannot be observed in the triaxial compression test with frictional end platens. In the present study by using frictionless end platens, however, there was a marked tendency of shear planes to coincide with the bedding plane when the specimen with $\delta=30°$ was sheared. When the specimen with $\delta=90°$ was sheared, there were some examples to show many shear planes in a conical fashion. These observations seem to support the usefulness of the frictionless end platens in the triaxial compression test (Rowe and Barden, 1964).

Two distinct shear plane (main and secondary) were generally developed at the end of plane strain tests. The main shear plane have a tendency to coincide with the bedding plane when the specimen with $\delta=24°$ or 30° were sheared.

Stress-Strain Relation

Stress-strain relation obtained from the shear tests at the cell pressures of 0.5 and 2 kg/cm^2

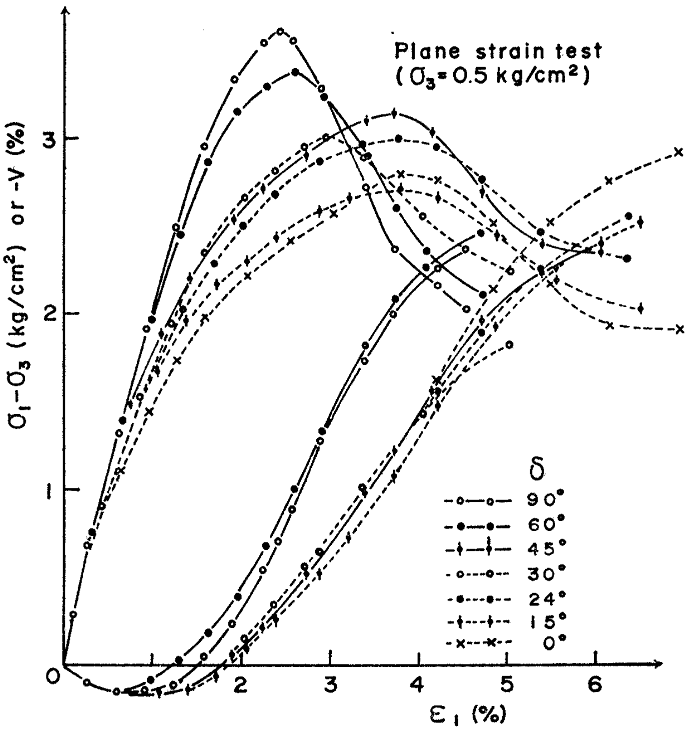

Fig. 6. Stress-strain relations in the plane strain tests of $\sigma_3=0.5\,kg/cm^2$

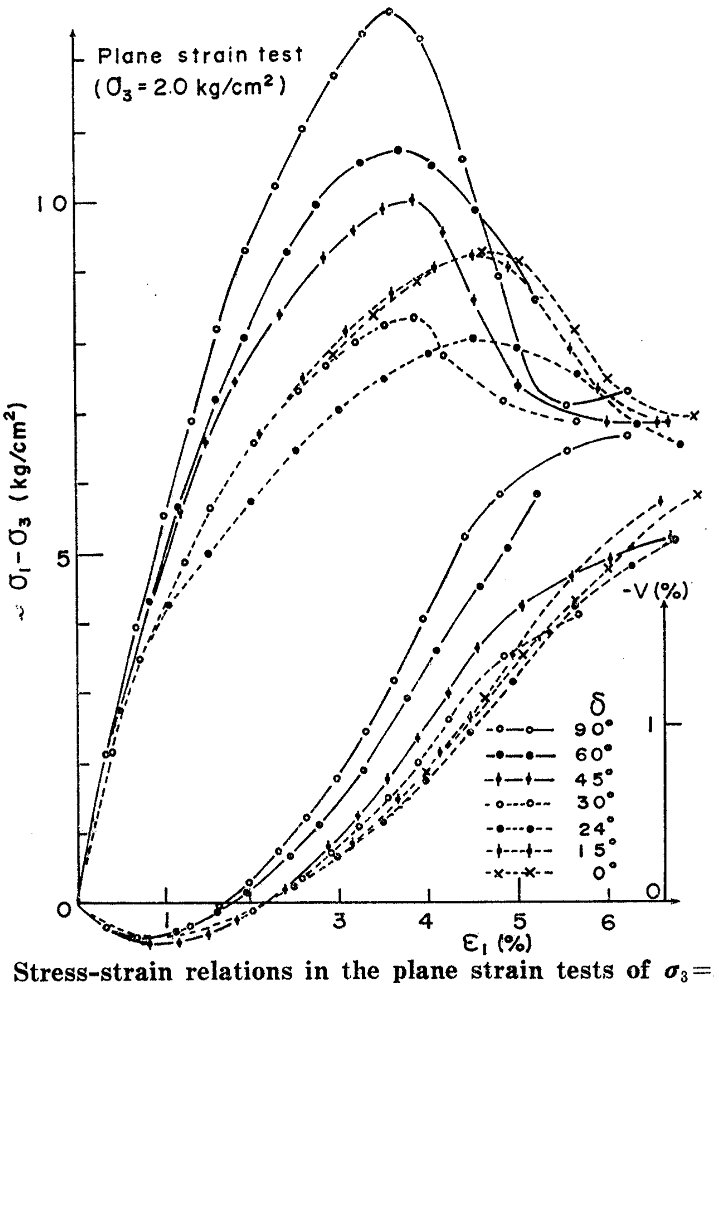

Fig. 7. Stress-strain relations in the plane strain tests of $\sigma_3=2\,kg/cm^2$

are shown in Figs. 6, 7, 8 and 9. These figures demonstrate an important effect of tilting angle δ to the stress-strain relation as follows:

(1) Peak deviator stress $(\sigma_1-\sigma_2)_f$ and dilatancy rate $(dv/d\varepsilon_1)_f$ at failure increase with the increase of δ. The increase is not so conspicuous in the triaxial compression test as compared with the plane strain test. This fact means that the anisotropic behaviour of sand is more emphasized in the plane strain condition $\varepsilon_2=0$ as compared with the case of symmetrical stress condition $\sigma_2=\sigma_3$. It is worthy of note that, in the plane strain test of $\sigma_3=2$ kg/cm^2, the specimens with $\delta=24°$ and 30° have lower values of $(\sigma_1-\sigma_3)_f$ than the other ones.

(2) The specimens in the plane strain test are failed in a brittle manner with small failure strains ranging from 2.5% to 5%, irrespective of δ. With the increase of cell pressure from 0.5 to 4 kg/cm^2, the failure strain slightly increases. The specimens of triaxial compression test, however, are failed in a ductile manner with relatively large failure strains over 5%, except for the test on the specimen of $\delta=90°$ and of $\sigma_3=0.5$ kg/cm^2. The failure strain also increases with increasing the cell pressure in the triaxial compression test.

The experimental observation that the effect of δ on the peak deviator stress is emphasized in the plane strain test can well be explained by imperfect re-arrangement of particles during the small shearing process up to failure in this strain condition. Mahmood and Mitchell (1974) reported a significant effect of shear direction on the shear strength of anisotropic sand specimen in the direct shear test failed at the plane strain condition. On the

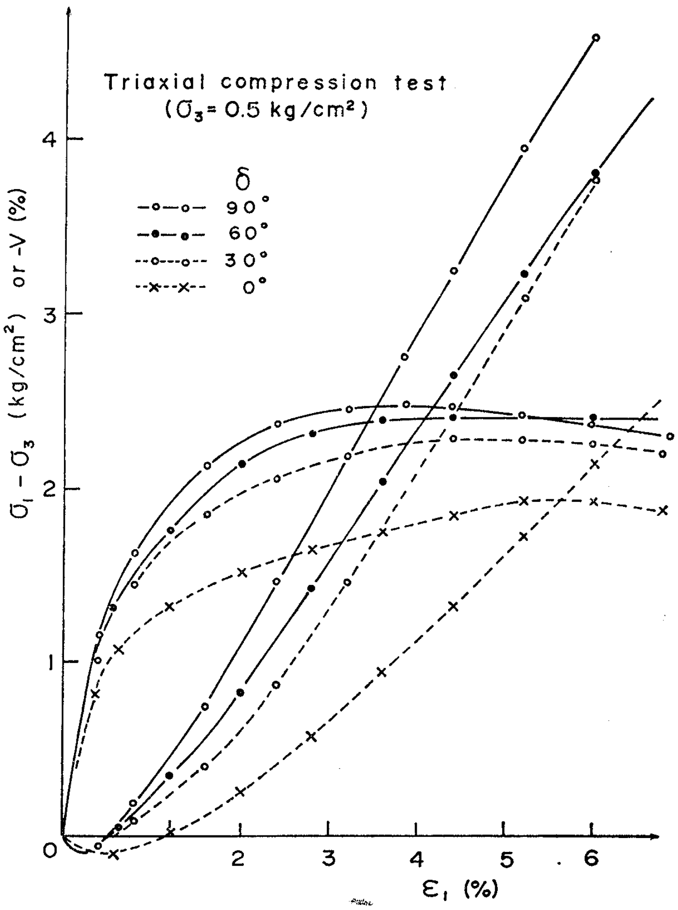

Fig. 8. Stress-strain relations in the triaxial compression tests of $\sigma_3=0.5$ kg/cm^2

other hand, sand particles can move more easily without strain confinement in the triaxial compression test. Therefore, the ultimate fabric of sand at failure and, at the same time, its shear strength become similar to each other in the triaxial compression test especially at a high cell pressure, regardless of their tilting angle δ.

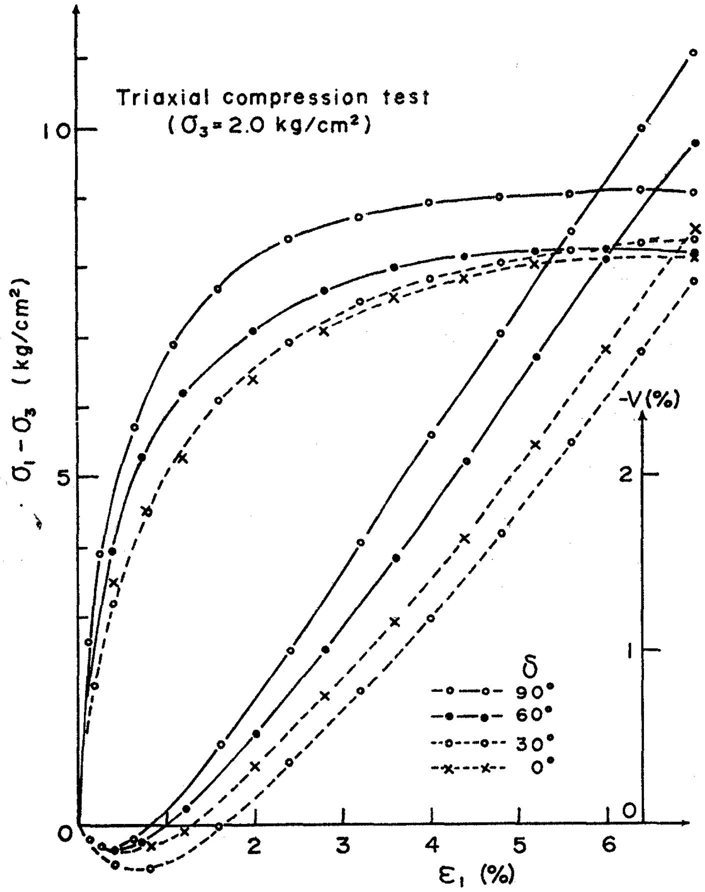

Fig. 9. Stress-strain relations in the triaxial compression tests of $\sigma_3=2\,\mathrm{kg/cm^2}$

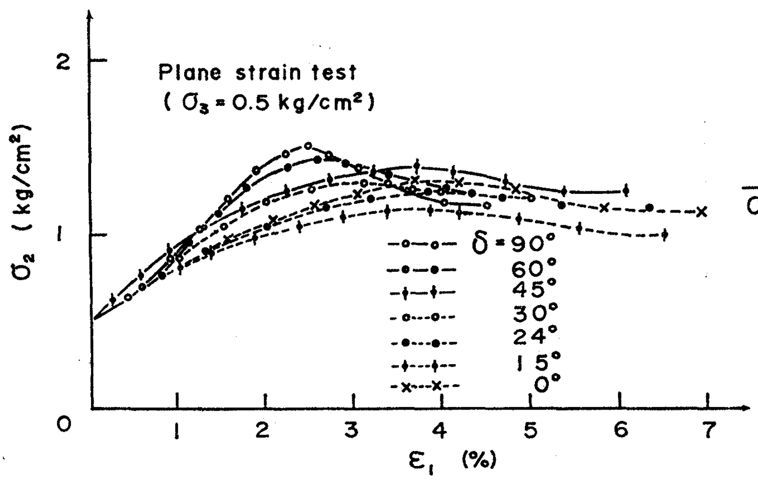

Fig. 10. A typical example showing the gradual increase of σ_2 up to failure in the plane strain tests of $\sigma_3=0.5\,\mathrm{kg/cm^2}$

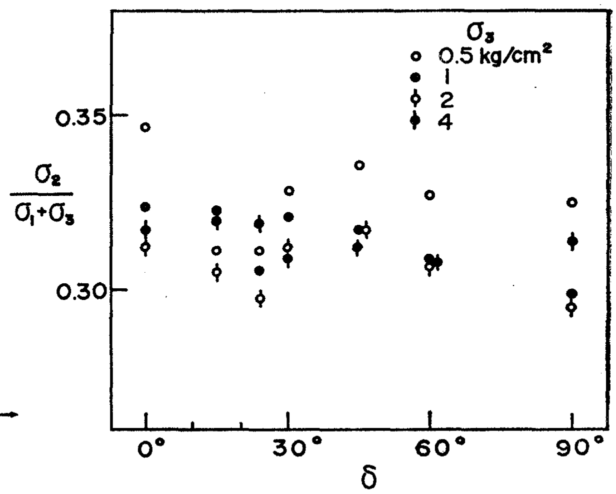

Fig. 11 Relation between stress ratio $\sigma_2/\sigma_1+\sigma_3$ and tilting angle δ

(3) The intermediate stress σ_2 increases gradually during the process of shearing up to the failure (Fig. 10). This characteristic is also facilitated by increasing the tilting angle δ. However, the stress ratio of σ_2 to $(\sigma_1+\sigma_3)$ at failure seems to be independent of the tilting angle δ (Fig. 11).

Shear Strength

Principal stress ratios at failure $(\sigma_1/\sigma_3)_f$ are shown by plotting them against the tilting angle δ in Fig. 12 for the plane strain test and in Fig. 13 for the triaxial compression test. On the basis of the comparison between Figs. 12 and 13, the following characteristics can be pointed out:

(1) The stress ratio $(\sigma_1/\sigma_3)_f$ in the plane strain test is generally higher than that in the triaxial compression test, as has been said by Cornforth (1964) and other authors.

(2) When the cell pressure is 2 or 4 kg/cm^2 in the plane strain test, the ratio $(\sigma_1/\sigma_3)_f$ becomes minimum at $\delta=24°$. It has been said in this paper that the main shear plane observed is nearly parallel to the bedding plane when the specimens with $\delta=24°$ and 30° are sheared. In the triaxial compression test, on the other hand, the gradual decrease of $(\sigma_1/\sigma_3)_f$ with the decrease of δ is apparent without a clear minimum point although the shear plane accords with the bedding plane in the specimen of $\delta=30°$. Such gradual decrease of $(\sigma_1/\sigma_3)_f$ in the triaxial compression test has already been reported by Oda (1972), Arthur and Menzies (1972) and Arthur and Phillips (1975) (Fig. 13).

(3) With the increase of cell pressure from 0.5 to 4 kg/cm^2, there is a marked reduction of $(\sigma_1/\sigma_3)_f$, especially in the plane strain test as has been reported by Tong (1970). This indicates a not-straight line of the Mohr's failure envelope of sand.

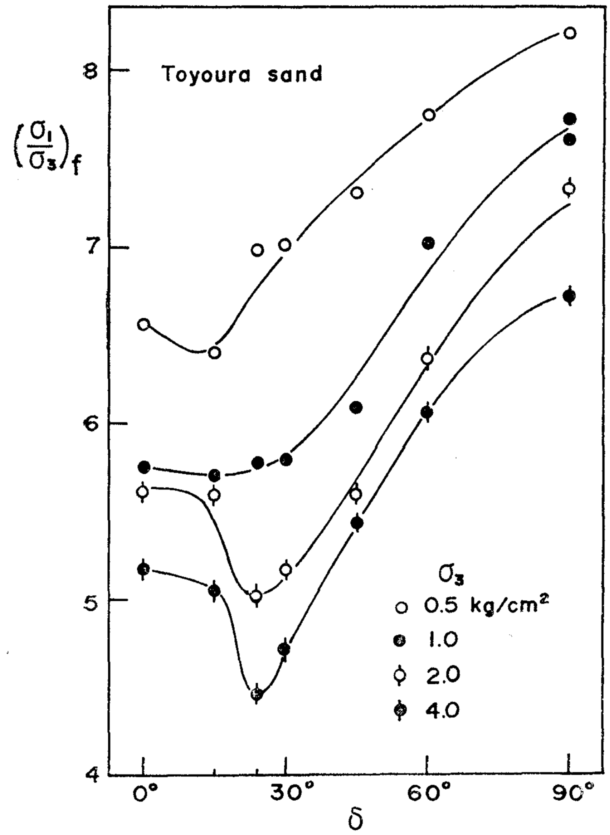

Fig. 12. Severe effect of tilting angle δ on stress ratio at failure $(\sigma_1/\sigma_3)_f$ in the plane strain test (e=0.66−0.67)

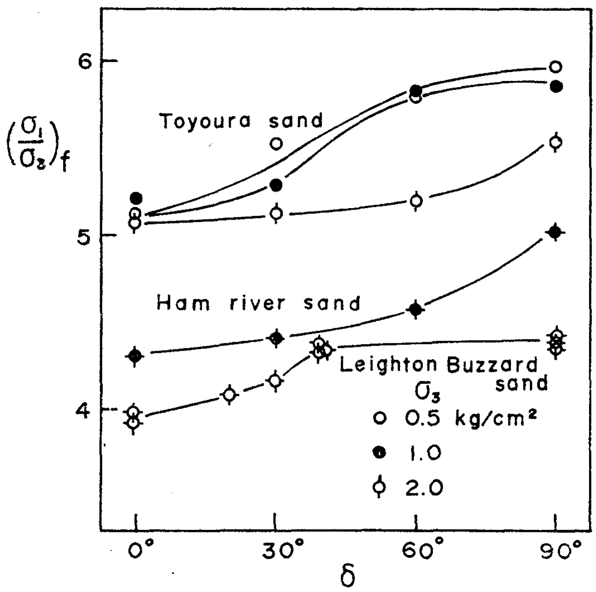

Fig. 13. Slight effect of tilting angle δ on stress ratio at failure $(\sigma_1/\sigma_3)_f$ in the triaxial compression test (e=0.67−0.68). The symbols ϕ and ◆ show the results of triaxial compression tests on Leighton Buzzard sand and Ham river sand quoted from Arthur and Menzies (1972) and Arthur Phillipse (1975), respectively.

This is usually believed to be due to the particle crushing during its shear deformation. According to the sieve analysis before and after the shear test, however, no significant particle crushing could be observed in the present study. Further study is necessary to make the reason clear.

(4) The Mohr's failure envelope lines in the plane strain and triaxial compression tests are summalized in Fig. 14. The hatched area is determined by the following two extreme failure envelope lines; i.e., the upper failure line for the specimens with $\delta=90°$ and the lower failure line for the specimens with $\delta=24°$ in the plane strain test. In the same figure, the solid area is also determined by the upper failure line for the specimens with $\delta=90°$ and the lower failure line for the specimens with $\delta=0°$ in the triaxial compression test. The shear strength in the plane strain test is higher than that in the triaxial compression test if the tilting angle is large enough. When the tilting angle is within 0° to 30°, the reverse result is possible.

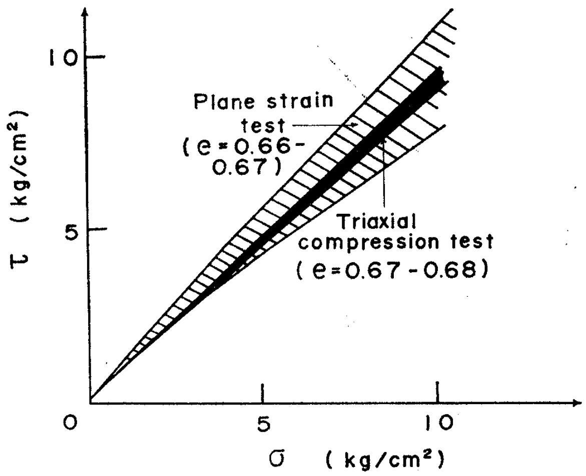

Fig. 14. Compiled figure of the Mohr's failure envelopes in the plane strain and triaxial compression tests

APPLICATION OF ANISOTROPIC SHEAR STRENGTH OF SAND IN ANALYSIS OF SOIL ENGINEERING PROBLEMS

Earth Pressure of Sand

Rankin's active earth pressure σ_a and passive earth pressure σ_p in a cohesionless sand bounded by a horizontal plane can generally be calculated by using the same friction angle in the following equations:

$$\sigma_a=\gamma z\tan^2\left(45°-\frac{\phi}{2}\right) \quad (1)$$

$$\sigma_p=\gamma z\tan^2\left(45°+\frac{\phi}{2}\right) \quad (2)$$

where γ is a unit weight of sand and z is a depth. Since a cohesionless sand is generally anisotropic as discussed in the previous sections, the friction angle which can be used in Eqs. (1) and (2) must be changed. That is, a reasonable friction angle ϕ_a for Eq. (1) is determined by the plane strain test on the specimens with $\delta=90°$ and a friction angle ϕ_p for Eq. (2) must be derived from the plane strain test on the specimens with $\delta=0°$. If not so, over-estimations of σ_p and σ_a may be calculated.

For example, the angles ϕ_a and ϕ_p are obtained as 47° and 41.5° respectively, by neglecting apparent cohesion (Fig. 15). If we use $\phi=47°$ uniformly as a friction angle of isotropic sand, the calculated value of σ_p results in an overestimation of about 30%.

Bearing Capacity of Sand

When we consider a bearing capacity of sand beneath a strip footing, the theoretical estimation of the effect of anisotropic shear strength of sand on the safty analysis of foundation is very difficult because slip lines generally intersect the bedding plane at various angles. Comparison of two types of bearing capacity tests (V- and H-bearing capacity tests) may be useful for the experimental estimation of the effect of anisotropic shear strength.

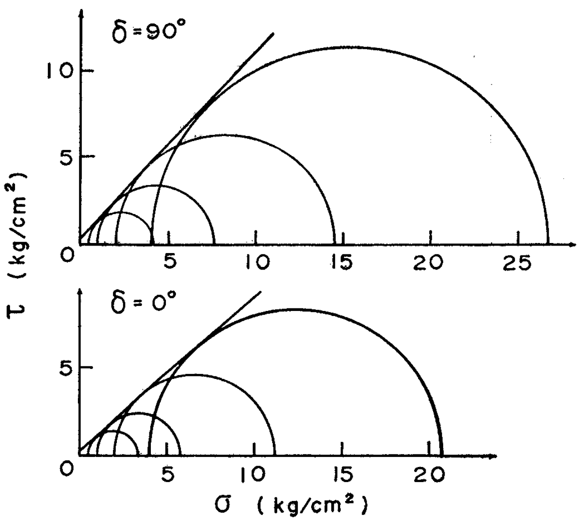

Fig. 15. Mohr's failure envelopes in the specimens with $\delta=90°$ and $\delta=0°$

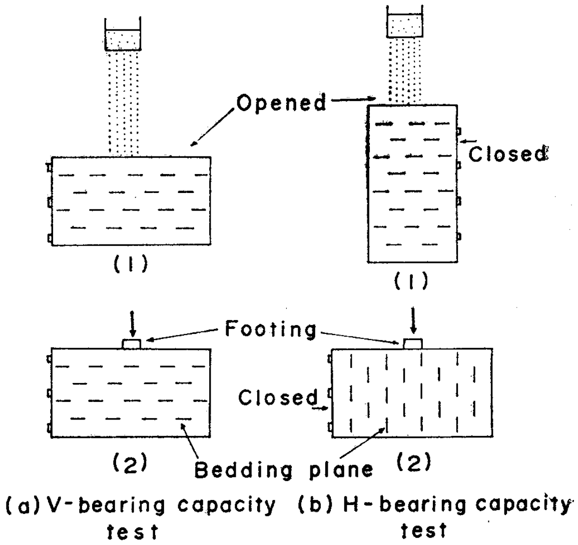

Fig. 16. Two types of bearing capacity test

Fig. 16 shows experimental procedures for *V*- and *H*-bearing capacity tests. In both types of tests, Toyoura sand was poured from various heights into a vacant steel sided box to get uniform sand beds with void ratios ranging from 0.66 to 0.77. The dimension of the box was 30 cm long, 7 cm wide and 20 cm deep. By stiffening the box, its lateral deflection was restricted less than 0.01 mm over the range of strip footing penetration up to failure and the plane strain condition of the test was nearly satisfied. A smooth footing lubricated by two membranes and silicon grease the size of which was 3.5 cm wide and 6.7 cm long was penetrated perpendicular (*V*-bearing capacity test) to or parallel (*H*-bearing capacity test) to the bedding plane.

Ultimate bearing capacity and modulus of subgrade reaction are shown in Figs. 17 and 18. The ultimate bearing capacity and the modulus of subgrade reaction in the *V*-bearing capacity test are much higher than those in the *H*-bearing capacity test. It goes without saying that if we are going to discuss the bearing capacity and subgrade reaction of sandy bed, we have to consider the appreciable effect of the anisotropic shear strength due to the anisotropic arrangement of particles.

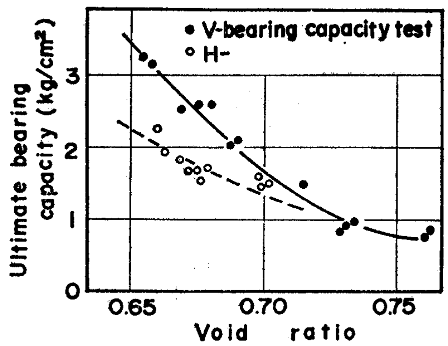

Fig. 17. Important effect of anisotropic strength of sand on its ultimate bearing capacity

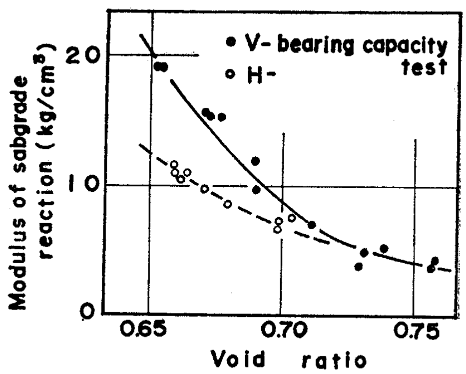

Fig. 18. Important effect of anisotropic behaviour of sand on its modulus of subgrade reaction

CONCLUSIONS

From the experimental study on the anisotropic behaviour of sand, the following conclusions were obtained:

(1) Anisotropic parallel alignment of particles is an important fabric characteristic not only in natural sand beds recently deposited at river, beach and dune but also in artificially deposited sand beds.

(2) Anisotropic shear strength of sand is especially important when sand is deformed at the condition with strain confinement. This is probably due to the fact that re-arrangement of particles is more difficult in this condition than in the condition without any strain confinement.

(3) Shear strength in the plane strain test is not always 10% to 20% greater than that in the triaxial compression test. Reverse result is possible when specimens with the tilting angles $\delta=24°$ and 30° are sheared.

(4) When the cell pressure in the plane strain test is greater than 2 kg/cm^2, the mobilized strength is minimized at $\delta=24°$.

(5) The effect of anisotropic shear strength of sandy materials can never be neglected in considering stability problems and earth pressure problems in plane strain condition.

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NOTATION

e = void ratio
$\bar{R}$ = axial ratio showing particle shape
v = volumetric strain
$V\cdot M$ = vector magnitude showing intensity of particle alignment
δ = tilting angle of bedding plane
ε_1 = axial strain
σ_1 = axial stress (in effective)
σ_2 = intermediate stress (in effective)
σ_3 = cell pressure (in effective)
$(\sigma_1/\sigma_3)_f$ = stress ratio at failure
$(\sigma_1-\sigma_3)_f$ = stress difference at failure
σ_a = Rankin's active pressure
σ_p = Rankin's passive pressure
ϕ_a = internal friction angle to calculate Rankin's active pressure
ϕ_p = internal friction angle to calculate Rankin's passive pressure

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