PROCEEDINGS OF SCIENCE

Fourier Analysis of the Parametric Resonance of the Neutrino Oscillation in the Presence of Inhomogeneous Matter

Joe Sato*
Physics Department, Saitama University, 255 Shimo-Okubo, Sakura-ku, Saitama 338-8570, Japan
E-mail: joe@phy.saitama-u.ac.jp

Masafumi Koike
Physics Department, Saitama University, 255 Shimo-Okubo, Sakura-ku, Saitama338-8570, Japan
E-mail: koike@krishna.th.phy.saitama-u.ac.jp

Toshihiko Ota
Institüt für Theoretische Physik und Astrophysik, Universität Würzburg, 97074 Würzburg, Germany
E-mail: Toshihiko.Ota@physik.uni-wuerzburg.de

Masako Saito
Physics Department, Saitama University, 255 Shimo-Okubo, Sakura-ku, Saitama 338-8570, Japan
E-mail: msaito@krishna.th.phy.saitama-u.ac.jp

We study the parametric resonance of the neutrino oscillation through the matter whose density varies spatially. The Fourier analysis of the matter effect enables us to clarify the parametric resonance condition, which is summarized in a frequency matching between the neutrino oscillation and the spatial variation of the matter density. As a result, the n-th Fourier mode of a matter density profile modifies the energy spectrum of the $\nu_\mu \to \nu_e$ appearance probability at around the n-th dip.

10th International Workshop on Neutrino Factories, Super beams and Beta beams
June 30 - July 5 2008
Valencia, Spain

*Speaker.

PoS(Nufact08)140

The oscillation probability of neutrinos passing through the interior of the Earth is strongly affected by the matter on the baseline. In such cases, the spatial variation of the matter density can make the probability quite different from that under the constant matter density [1, 2, 3]. The possibility of the parametric resonance of the neutrino oscillation with the matter density has been discussed [1]. We exploit the Fourier analysis of the matter density profile to study the effect of parametric resonance [3, 4]. Generally, the parametric resonance of an oscillation emerges when an oscillation parameter changes with a resonant frequency, which is typically twice the natural frequency of the system. Our formulation based on the Fourier analysis gives a clear view of this frequency matching, which is essential to the parametric resonance.

We carry out our analysis in a two-flavor framework. This simplification does not spoil our point as can be seen in Fig. 1, in which we compare the $\nu_\mu \to \nu_e$ appearance probability in the three- and two-flavor calculation, taking the baseline length $L = 12000\,\mathrm{km}$. The three-flavor result assumes the density profile of the Preliminary Reference Earth Model (solid curve), while two-flavor result uses the constant density case (dotted curve) and the cosine variation around the average density (broken curve). Renormalized by the factor of $\sin^2 2\theta_{23}$, the two-flavor result with the cosine profile shows the similar feature as the three-flavor one.

The evolution equation of two flavors of neutrinos ν_e and ν_μ in the matter density $\rho(x)$ is

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix} = \frac{1}{2E}\left[\frac{\delta m^2}{2}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix} + \begin{pmatrix}a(x) & 0\\ 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e(x)\\ \nu_\mu(x)\end{pmatrix}, \tag{1}$$

where $\delta m^2, \theta$, and E are the mass-square difference, the mixing angle, and the energy of neutrinos, respectively. The matter effect $a(x)$ is given by $a(x) = 2\sqrt{2}EG_F N_A Y_e \rho(x)$, where G_F is the Fermi constant, N_A is the Avogadro number, and Y_e is the proton-to-nucleon ratio. Defining $\Delta \equiv \delta m^2 L/2E$, $\Delta_m(x) \equiv a(x)L/2E$, and $z(x) = \nu_e(x)\exp[\int_0^x \mathrm{d}s\,\mathrm{i}\Delta_m(s)/2]$, and introducing an Fourier expansion $\Delta_m(\xi) = \sum_{n=0}^{\infty}\Delta_{mn}\cos 2n\pi\xi$ with $\xi = x/L$, we obtain from Eq. (1)

$$z''(\xi) + \frac{1}{4}\Bigg[(\Delta_{m0} - \Delta\cos 2\theta)^2 + \Delta^2\sin^2 2\theta + 2(\Delta_{m0} - \Delta\cos 2\theta)\sum_{n=1}^{\infty}\Delta_{mn}\cos 2n\pi\xi$$
$$+\left(\sum_{n=1}^{\infty}\Delta_{mn}\cos 2n\pi\xi\right)^2 + 4n\pi\mathrm{i}\sum_{n=1}^{\infty}\Delta_{mn}\sin 2n\pi\xi\Bigg]z(\xi) = 0. \tag{2}$$

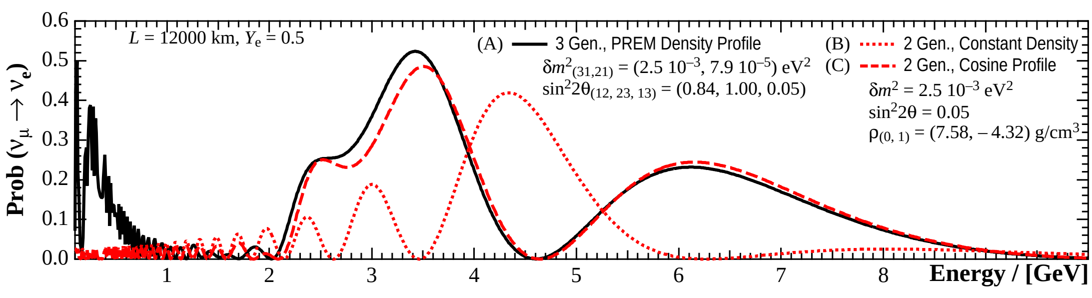

Figure 1: Oscillation probabilty from ν_μ to ν_e for the baseline length of 12000 km. The solid curve (A) is evaluated by three-flavor analysis with the matter density profile taken from the PREM. The dotted (B) and broken (C) curves are results of the two flavor calculation with (B) constant density with $\rho_0 = 7.58\,\mathrm{g/cm^3}$, and (C) cosine profile $\rho(x) = \rho_0 + \rho_1\cos 2\pi x/L$ with $\rho_1 = -4.32\,\mathrm{g/cm^3}$. Values of other parameters taken are described in the figure.

PoS(NuFact08)140

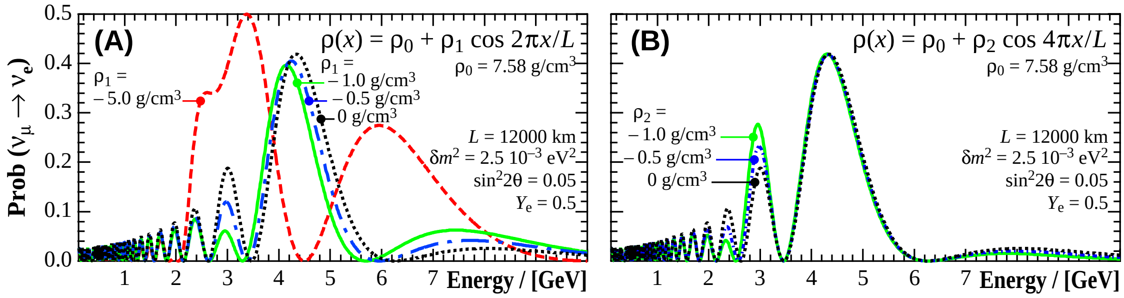

Figure 2: The $\nu_\mu \to \nu_e$ oscillation probabilties with $\rho_n = 0, -0.5$, and $-1.0\,\mathrm{g/cm^2}$, where (A) $n = 1$ and (B) $n = 2$. The former includes $\rho_1 = -5.0\,\mathrm{g/cm^2}$ case also. Values of other parameters are shown in the graphs.

The parametric resonance can arise under the presence of the density profile. Suppose that only the n-th Fourier mode of the density profile is present so that $\Delta_\mathrm{m}(\xi) = \Delta_{\mathrm{m}0} + \Delta_{\mathrm{m}n} \cos 2n\pi\xi$. The parametric resonance is expected when the frequency of the density profile $2n\pi$ is twice the natural frequency of the system $\omega_0 \equiv \sqrt{(\Delta_{\mathrm{m}0} - \Delta \cos 2\theta)^2 + \Delta^2 \sin^2 2\theta - \frac{1}{2}\sum_{n=1}^{\infty} \Delta_{\mathrm{m}n}^2}/2$, *i.e.* $\omega_0 = 2n\pi/2 = n\pi$. This frequency matching condition amounts to

$$E = E_n^{(\mathrm{res})} \equiv \frac{\delta m^2 L}{2} \frac{1}{\Delta_{\mathrm{m}0} \cos 2\theta \pm \sqrt{4n^2\pi^2 - \Delta_{\mathrm{m}0}^2 \sin^2 2\theta - \frac{1}{2}\sum_{n=1}^{\infty} \Delta_{\mathrm{m}n}^2}}. \tag{3}$$

We show in Fig. 2 the $\nu_\mu \to \nu_e$ oscillation probability for various values (A) of ρ_1 and (B) of ρ_2. Figure 2 indicates that the n-th Fourier mode of the matter density modifies the oscillation probability around the n-th dip: $\omega_0 = n\pi$, which corresponds to the parametric resonance energy Eq. (3). This modification is understood by the parametric resonance as shown elsewhere [5].

In summary, we studied the parametric resonance of the neutrino oscillation through the matter whose density varies spatially, analyzing the matter effect by the Fourier expansion. We have shown that the n-th Fourier mode of the density profile modifies the oscillation probability around the parametric resonance energy $E_n^{(\mathrm{res})}$ defined in Eq. (3). This condition is understood in terms of the frequency matching between the natural frequency of the system and that of the density profile.

References

[1] V. K. Ermilova, *et al., Kr. Soob. Fiz. [Short. Notices of the Lebedev Institute]* **5** *(1986) 26; E. K. Akhmedov, Sov. J. Nucl. Phys.* **47** *(1988) 301 [Yad. Fiz.* **47** *(1988) 475]; P. I. Krastev and A. Y. Smirnov, Phys. Lett. B* **226** *(1989) 341; Q. Y. Liu and A. Y. Smirnov, Nucl. Phys. B* **524** *(1998) 505; Q. Y. Liu, S. P. Mikheyev and A. Y. Smirnov, Phys. Lett. B* **440** *(1998) 319; E. K. Akhmedov, Nucl. Phys. B* **538** *(1999) 25; E. K. Akhmedov, A. Dighe, P. Lipari and A. Y. Smirnov, Nucl. Phys. B* **542** *(1999) 3; E. K. Akhmedov, M. Maltoni and A. Y. Smirnov, Phys. Rev. Lett.* **95** *(2005) 211801.*

[2] S. T. Petcov, Phys. Lett. B **434** *(1998) 321; M. V. Chizhov and S. T. Petcov, Phys. Rev. Lett.* **83** *(1999) 1096; M. V. Chizhov and S. T. Petcov, Phys. Rev. D* **63** *(2001) 073003.*

[3] T. Ota and J. Sato, Phys. Rev. D **63** *(2001) 093004; T. Ota and J. Sato, Phys. Rev. D* **67** *(2003) 053003.*

[4] M. Koike and J. Sato, Mod. Phys. Lett. A **14** *(1999) 1297.*

[5] M. Koike et al., in preparation.