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A METHOD TO ESTIMATE MODULUS OF HORIZONTAL SUBGRADE REACTION FOR A PILE

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ABSTRACT

This paper proposes an experimental formula to estimate the modulus of horizontal subgrade reaction k to be used in design practice for analyzing the lateral deflection of a pile at the top and the ground surface.

In this paper, (1) the relation between the values of deformation modulus measured by different soil investigation methods was obtained from the results of a comparison study, (2) the effect of loading width to the modulus of horizontal subgrade reaction was investigated by the horizontal plate loading tests carried out in test pits, and (3) the influence of deflection of a pile at the top or ground surface to the value k was investigated on the basis of some experimental data of horizontal loading tests. According to those results, the experimental formula to estimate the value k was obtained.

Key words: bearing capacity, deformation, in-situ test, modulus of horizontal subgrade reaction, pile

IGC: E2/D5

INTRODUCTION

Many studies have been conducted on estimation of the modulus of horizontal subgrade reaction, k, for a pile (or piles) on the assumption that it may be considered as a beam on an elastic or inelastic foundation and various formulae are proposed for the value of the modulus k.

At the same time efforts have been made by many investigators to obtain numerical solutions using those values of k for analyzing the lateral behavior of piles. Generally, expressions for k which have been proposed may be included in Eq. (1) or in a similar form.

$$p=kx^m y^n \qquad (1)$$

where p is the horizontal subgrade reaction, y is the lateral deflection of the pile at its depth x, and k, m and n are constants representing the subgrade condition. m (>0) shows the tendency to change with depth in soil and n ($1>n>0$) shows the stress-strain characteristics of soil, that is, $n=1$ means elastic and $n\neq 1$ non-linear elastic.

The constants, m and n, in Eq. (1) are evaluated variously: for $n=1$ by Chan (1937) (m

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$=0$), Row (1956) ($m=1$), Terzaghi (1955) ($m=1$), Reese-Matlock (1960) ($m=1$) and Palmer-Tompson (1948) (m, arbitary), and for $n \neq 1$ by Rifaat (1935) ($m=1$), Shinohara-Kubo (1964) ($n=1/2$, $m=1$), Hayashi-Miyajima (1963) ($n=1/2$, $m=0$) and so on. Using these constants, the solution was obtained by each investigator.

The studies to obtain a general expression of the modulus k such as Eq. (1) or others are very important to analyze the lateral resistance of piles. However, dealing with practical design problems, we can not help thinking of the following difficulties:

(1) Is it possible to determine each constant in Eq. (1) by means of the presently available techniques of soil invesigation methods ? Even if possible, can it be accomplished with any degree of accuracy in comparison with efforts for the complicated and troublesome computations required for the case of $n \neq /1$ and $m \geqq 0$?

(2) Its it possible to adequately evaluate the influence of loading width on the modulus of horizontal subgrade reaction ?

To deal with the problems described above, we present in this paper the following approach which have been proposed by Yoshida (1964) and Yoshinaka (1967).

On the assumption that the theoretical function F (x) to express the exact lateral deflection process of a pile is obtained, the constants required in F (x) are for instance a, b, c etc. While point A in Fig. 1 can be determined by a straight line of the secant $\overline{OA}$ in place of the function F (x), then the constant required to show the point A is only the inclination angle α.

At the present time no satisfactory soil investigation method is available to determine the constants a, b, c, etc., in order to express the function F (x) with sufficient accuracy, and therefore, the point A may be shown by rough approximation within a range of some tolerable error, using the only constant α.

When the state of actual deflection of a pile is shown in Fig. 2 (a), the same deflection y_f can be obtained from the unified modulus of subgrade reaction converted into the simplified state as illustrated in Fig. 2 (b).

If the values required in the design are the lateral deflections of the pile at some special points such as at the top or the ground surface, or the maximum bending moment as in many cases, they could be obtained by determining the ground condition corresponding to the state of Fig. 2 (b), even if the bending state over the whole length of the pile differs

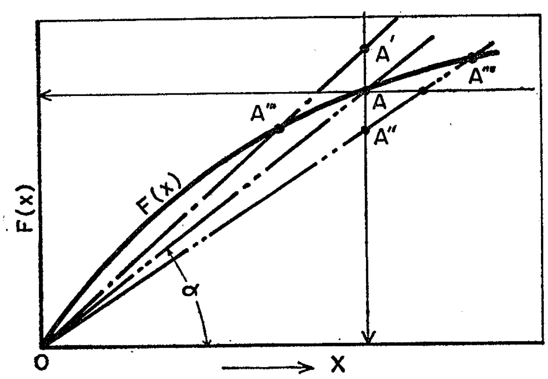

Fig. 1. Approximate expressions of function (Fx)

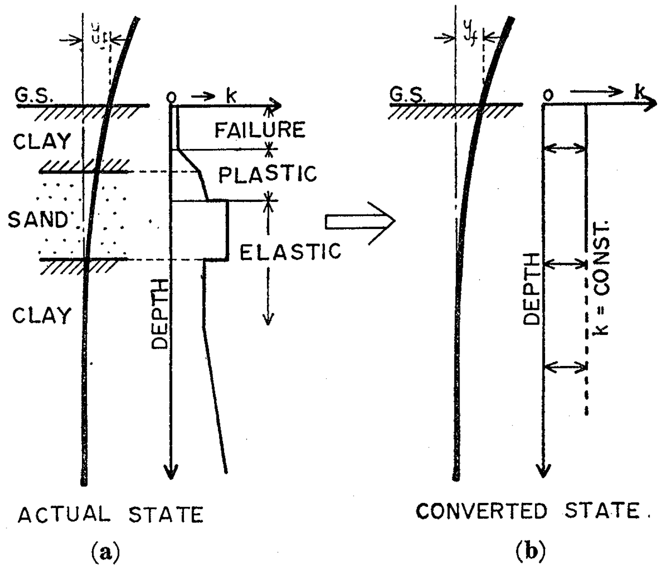

Fig. 2. Deflection of a pile and soil reaction

between Fig. 2 (a) and Fig. 2 (b).

Our treatment is justified in view of the characteristics of the lateral deflection of a pile; that is, (1) the influence of the type of expressions on the modulus k decreases in the following order: the lateral deflection, the bending moment and the shearing force, and (2) because the effective depth of ground with respect to deflection of the upper part of piles is limited within relatively shallow depths as pointed out by many investigators, it is not necessary to obtain the modulus k with the same accuracy along the whole length of piles.

Based on the consideration described above, we present a method to evaluate the modulus of k for the practical design, using the simplest expression of Eq. (1) which assumes $m=0$ and $n=1$. Also a study on the method to measure the deformation properties of subgrade is described. In addition, a study on the modulus k, fundamentally based on the proposal by Yoshida (1964), conducted by Imai et al. (1969) using LLT (lateral load tester), new concept on the specific, standard and general k-values is introduced.

RELATION BETWEEN THE MODULUS OF DEFORMATION OF SOILS AND THE TESTING METHODS

There are many methods to measure the deformation properties of soil. But the values measured do not always agree with other results due to the difference of the methods used to determine the modulus of deformation. As well known, the difference between static and dynamic methods is conspicuous, but even among the static methods the difference is considerable.

Static methods include the uniaxial and triaxial compression tests employing undisturbed samples, the plate loading test, the bore-hole deformation test, etc. In this paper an experimental study carried out by Yoshida and Yoshinaka (1967) and Yoshinaka (1967) on the relation between the values of the deformation modulus obtained by these methods is shown. The calculation of the modulus of deformation of soils or foundations is based on the elastic theory, in which the Poisson's ratio of soils is assumed to be about 0.3.

Fig. 3 shows the relation between the values of the modulus from the compression tests on samples and from the borehole deformation tests (the pressure meter tests) conducted in the same soil condition. The latter consists mainly of two types: the unicell type devised by Kögler (1933) (60—80 cm in length of probe, 50—80 mm in diameter), and the tricell type by Menard (1962). It is already confirmed by Yoshida and Yoshinaka (1967) that the difference in the mechanism of the both types does not affect the measured values of the deformation modulus. Therefore, the values determined by these methods are not distinguished in this paper.

Though the stress condition caused by each test method differs, Fig. 3 indicates that the results of the both methods agree very well, namely, $E_c=E_b$ where E_c is the modulus of deformation by compression tests on undisturbed soil samples and E_b by bore-hole deformation tests.

Fig. 4 shows the relation between the results from the horizontal loading tests with the circular rigid plate and the borehole deformation tests conducted at the same point in the ground consisting of sand and clayey soil. It is shown that there is a good correlation without the influence of soil type and the relation may be approximated as

$$E_{SO}=3E_b \qquad (2)$$

where E_{SO} is the modulus of deformation of soil by a bearing plate, 30 cm in diameter. The deformation properties of soil from the loading plate will be described in the next chapter in detail.

As the standard penetration tests are most commonly performed in the field, it is con-

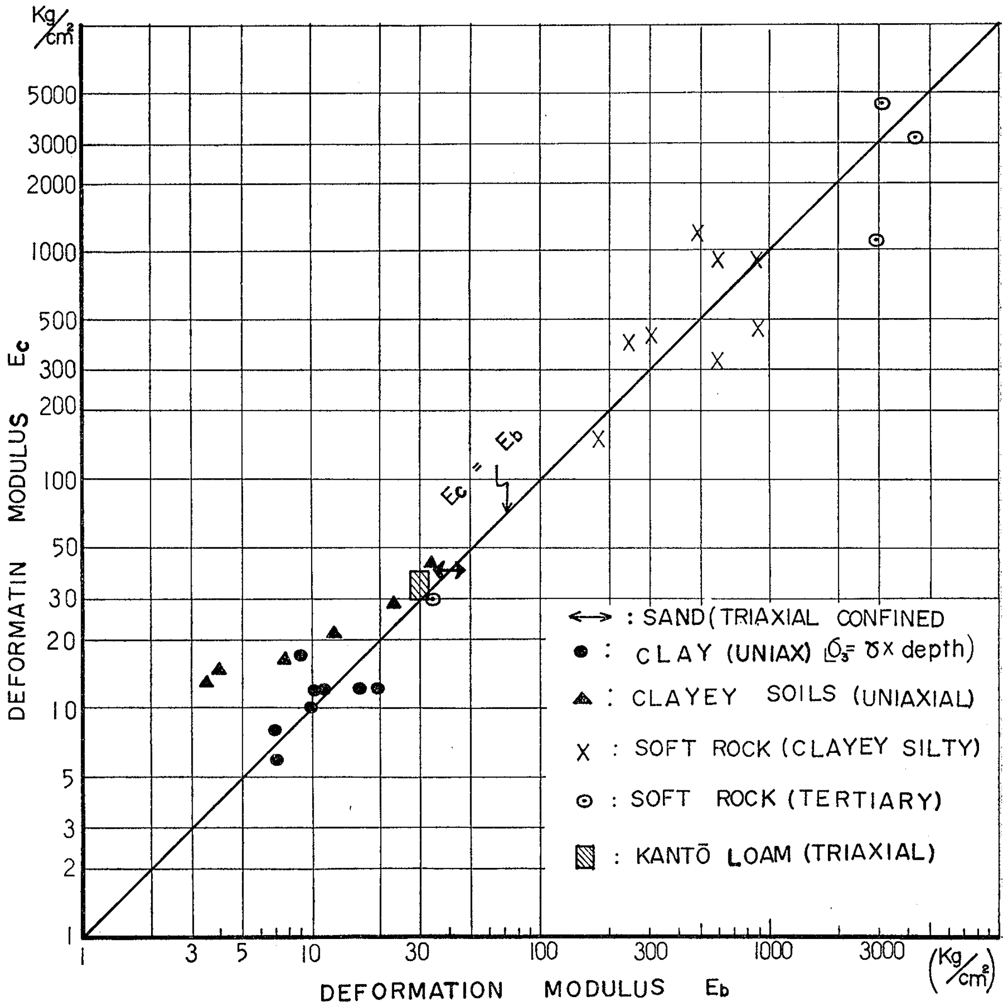

Fig. 3. Relationship between E_b and E_c

(E_b: modulus of deformation by bore-hole deformatian test, E_c: by compression test on undisturbed sample)

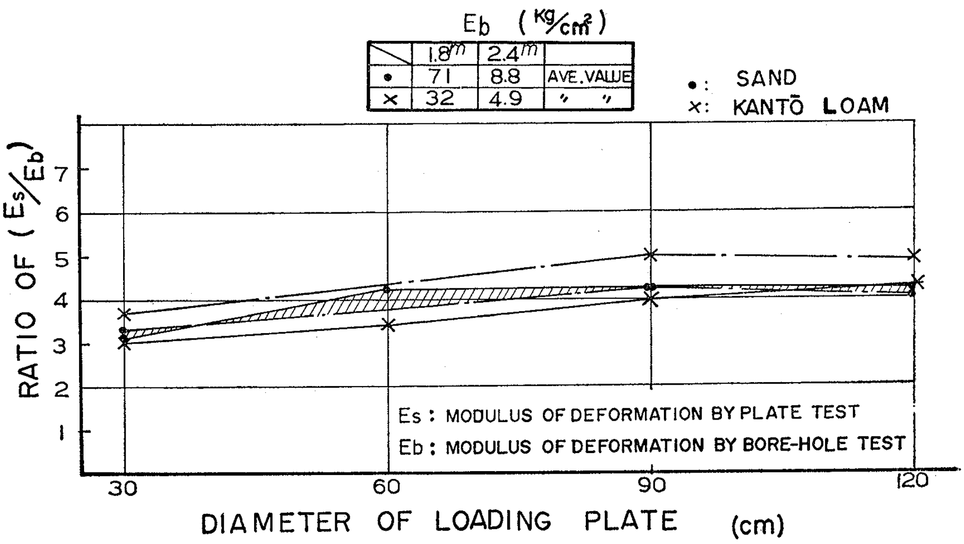

Fig. 4. Comparison of E_s and E_b

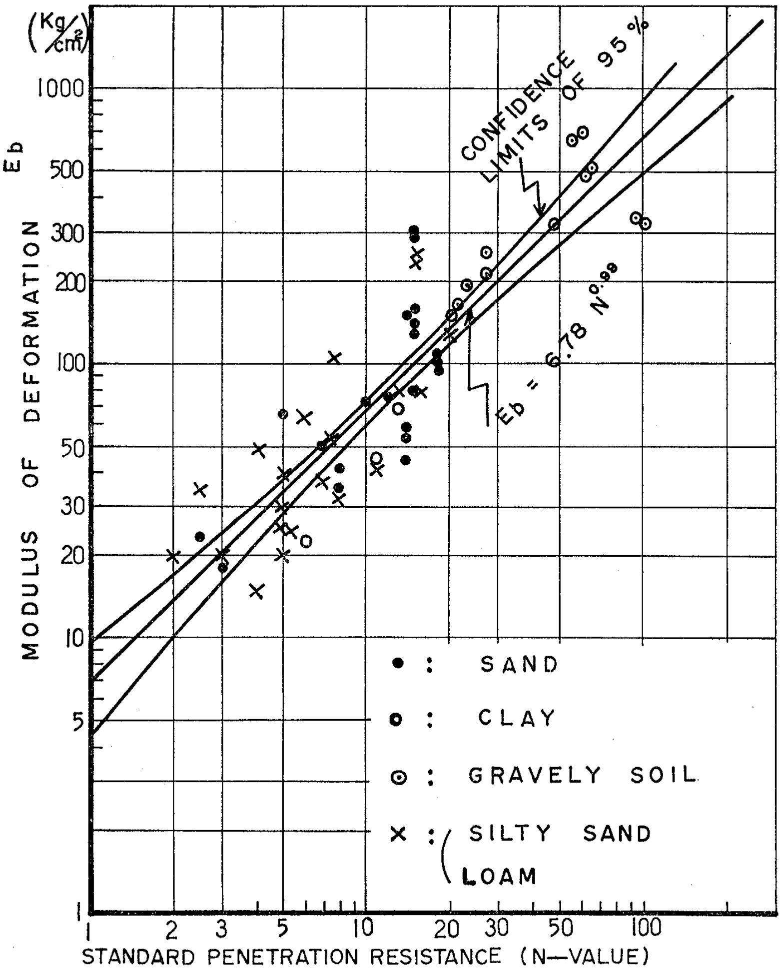

Fig. 5. Relationship between E_b and N-value

venient if the N-value can be converted into the modulus of deformation. The relation of N-value and the modulus of deformation is shown in Fig. 5. From the figure it is clear that there is some correlation between these measured values statistically and the relation may be expressed as,

$$E_b = 6.78\ N^{0.998} \doteqdot 7\ N \qquad (3)$$

where E_b is the modulus of deformation defermined by the borehole tests and N is the standard penetration resistance.

INFLUENCE OF LOADING WIDTH ON THE MODULUS OF HORIZONTAL SUBGRADE REACTION

The elastic theory indicates that the settlement of a loading plate is simply proportional to the width of loading, but it is doubtful whether the same relation holds for soil. It is well known that the settlement of a foundation on sandy ground decreases with the width of the foundation, as shown by Terzaghi (1955), that is, the modulus of vertical subgrade reaction in sand increases with the loading width, relatively.

We carried out a study on the similar relation for the horizontal case in test pits as shown in Fig. 6. The soil conditions are as follows: in case of sand, the test pits were filled with

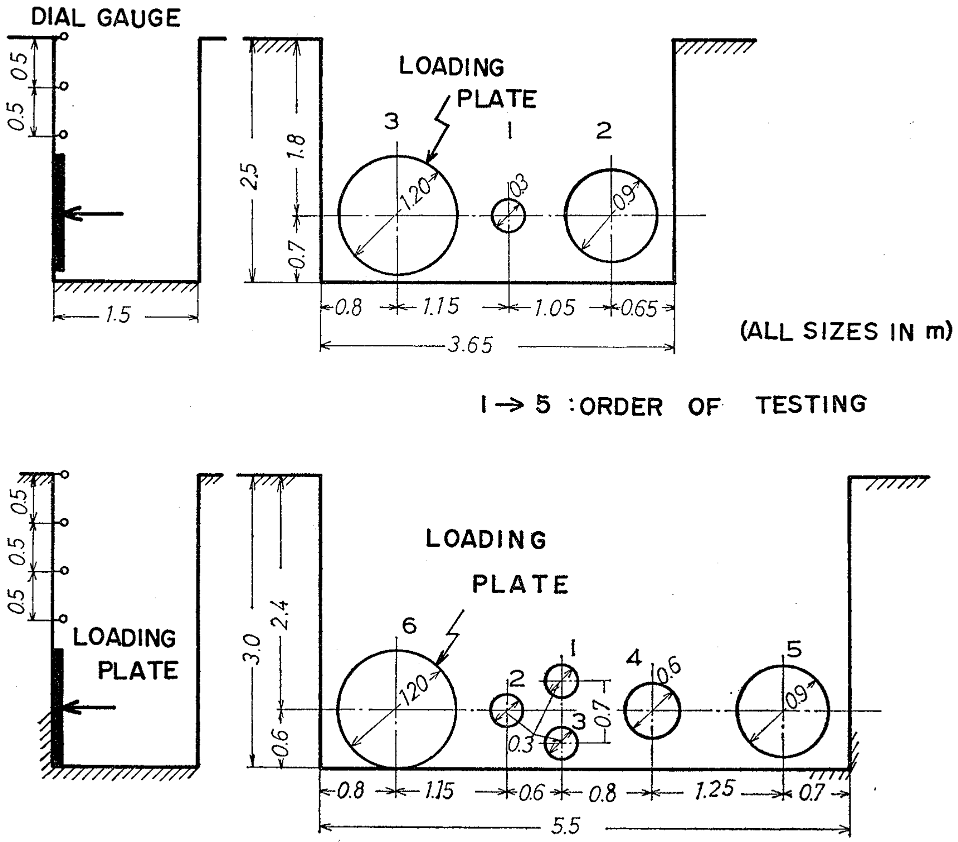

Fig. 6. Arrangement of horizontal loading tests

compacted soils, $G_S=2.71$, $\gamma_d=1.40$—1.47, and $w=8$—13%, and in case of clayey soil, the ground consisted of a natural deposit of volcanic ash, the so called Kanto-loam, $G_S=2.73$, $\gamma_d=0.52$—0.65, $w=110$—135%.

The test plates used for loading were rigid and circular, 30, 60, 90 and 120 cm in diameter. The sequence for loading is arranged from smaller to larger plates, the rate of loading was slow, and the loading pattern was cyclic.

The results of experiments are summarized in Fig. 7. The figure shows the ratio of the k value obtained by plates of various diameters to that of 30 cm in diameter versus the diameter of plates. The value of k is defined as,

$$k=\frac{\text{Load intensity (kg/cm}^2\text{)}}{\text{Settlement (cm)}} \tag{4}$$

and is calculated from the secant of the linear part below the yield load of a stress-strain curve. From Fig. 7 it is clear that the influence of loading width on the horizontal subgrade reaction k is expressed by

$$\frac{k}{k_0}=\left(\frac{B}{B_0}\right)^{-l} \tag{5}$$

where l is constant being about 3/4 and B_0 is the standard diameter of 30 cm.

Eq. (5) can be expressed by the modulus of deformation E_S given to diameter B, that is, the value E_S obtained from the result of a loading test using the relation between k and E_S based on the elastic theory;

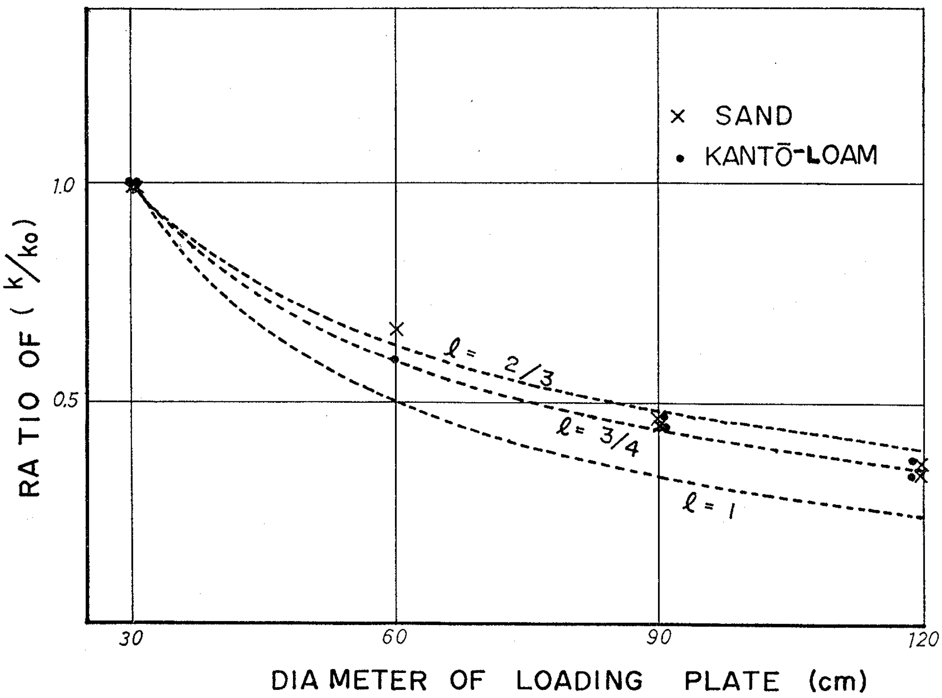

Fig. 7. Effect of loading width on k-value

$$k=\frac{E_s}{I_\rho(1-\mu_s^2)B} \tag{6}$$

where I_ρ is the shape factor and μ_s is Poisson ratio of soil.

By substituting Eq. (6) at B and B_0 in Eq. (5), then

$$E_s=E_{so}\left(\frac{B}{B_0}\right)^{1-l} \tag{7}$$

Thus, Eq. (7) becomes an experimental formula for correcting the modulus of deformation of soils for the effect of loading width.

LATERAL DEFLECTION AT PILE TOP

A pile subjected to a horizontal force at its top resists with the lateral subgrade reaction and the rigidity of the pile itself. As mentioned in the preceding section the subgrade reaction on piles is so complicated it is difficult at the present time to express the modulus k accurately.

However, considering the deflection from the practical point of view, the following two points are to be noted:

(1) The effective depth of soil relating lateral deflection of piles is limited only to the upper part of piles, even if the pile is driven in the foundation consisting of several strata with different engineering properties.

(2) When piles are used for the foundation, the deflection which may actually occur is usually less than a few centimeter at the ground surface, considering the relation between the upper structure and the soils.

We investigated the modulus of subgrade reaction k to be used in practical design of piles from the preceding point of view, on the bases of the following assumption; piles are supported in the lateral direction by the simplest condition, i. e., by independent springs with same constant (the so called Winkler's hypothesis). In this case, the behavior of a pile is expressed by

$$E_pI_p\frac{d^4y}{dx^4}+ky=0 \tag{8}$$

From Eq. (8), the deflection of a free head pile at the ground surface, y_f, is given by

$$y_f=\frac{H}{2E_pI_p\beta^3}(1+\beta h) \tag{9}$$

where

$$\beta=\sqrt[4]{\frac{kB}{4E_pI_p}}$$

The characteristics of the modulus k in Eq. (8) and Eq. (9) could be obtained by applying the relation of each corresponding value of the loading tests.

The piles tested consisted of steel pipe piles, prestressed concrete piles and cast-in-place concrete piles of 30 cm to 150 cm in diameter, and were installed in various test sites in Japan.

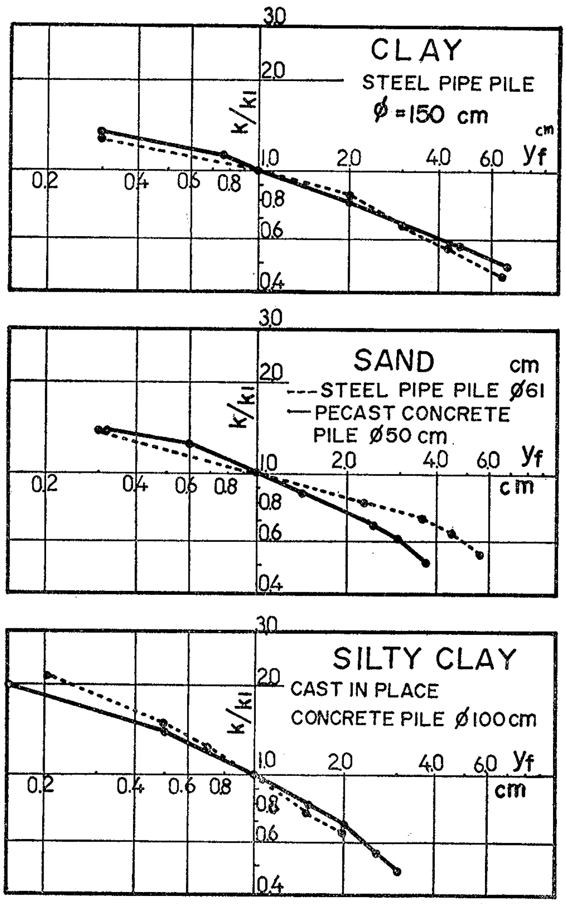

Fig. 8. Relation between (k/k_1) and y_f

Fig. 8 in the logarithmic expression shows the relations between the ratio of k to k_1 (at $y_f=1$ cm) and the deflection y_f of piles at the ground surface.

It is concluded clearly from Fig. 8 that the modulus k has a functional relation with the deflection y_f within its limited range.

Thus, an experimental relation is established as follows:

$$\log\left(\frac{k}{k_1}\right)=-m\log y_f \tag{10}$$

and in a general form,

$$k=\alpha y_f^{-m} \tag{11}$$

where, α and m are constants.

It seems that the constant m represents the intensity of plastic and non-elastic properties of subgrade within the effective depth, and the adhesive force between the pile and the surrounding soil.

The values of m obtained from the lateral loading tests are summarized in Table 1.

It should be noted that the modulus k discussed in the above, strictly speaking, can only be applied for an estimation of the deflection at the pile top.

Table 1. Experimental values of m

m		0~0.3	0.3~0.4	0.4~0.5	0.5~0.7	0.7~1.0
soil conditions	driven pile	(elastic)	diluvial sediments, or clay and clayey soil	sand and sandy soil	—	(plastic)
	cast-in place pile	—	—	generally	in case of $y_f>20$ mm	—

ESTIMATION OF k-VALUE

Derivation of k

The modulus of horizontal subgrade reaction k to be used for the practical design of piles can be obtained from the facts described above.

When the load intensities acting on soil by plates, B_0 and B in diameter (average load on the rigid plates) are both p, and the settlement caused by this load in case of B_0 is y_0, then the settlement of the plate with the diameter of B is, from Eq. (5),

$$y=y_0\left(\frac{B}{B_0}\right)^{-l} \tag{12}$$

If the relation concerning the settlement of loading plates is applied to piles, the following derivation on k may be possible.

The modulus k for B_0 is, from Eq (11),

$$k_0=\alpha_0 y_{f0}^{-m} \tag{13}$$

and Eq. (13) holds for other piles of B in the same supporting condition, where α_0 is a constant and y_{f0} is the lateral deflection of pile of B_0 at the ground surface.

By substituting Eq. (5) in Eq. (13),

$$k=\alpha_0 y_{f0}^{-m}\left(\frac{B}{B_0}\right)^{-l} \tag{14}$$

and similarly from Eq. (12) to Eq. (14), then

$$k=\alpha_0\left(\frac{B}{B_0}\right)^{l(m-1)}\cdot y_f^{-m} \tag{15}$$

Thus, the modulus k can be obtained by Eq. (15), using the deflection properties of a standard pile of B_0 driven into the ground under the same condition.

On the other hand, when we estimate the modulus k by Eq. (15), applying the results of soil investigation, it is necessary to determine the constant α_0 in Eq. (15).

If the modulus k for piles is given in the form of Eq. (16) as for an infinite beam on elastic foundation defined by Vesić (1961) as described later, that is,

$$kB=\alpha' E_s \tag{16}$$

By substituting Eq. (7) in Eq. (16), the following equation is obtained;

$$k=\alpha' B_0^{l-1} B^{-l} E_{so} \tag{17}$$

Thus, it may be said that Eq. (17) is the relation taking into account the properties of soil as compared with k for the elastic foundation by Vesić. However the k value by Eq. (17) corresponds to the strain condition defining E_{so}.

Since Eq. (15) and Eq. (17) become equivalent respectively under the conditions of $B=B_0$ and $y_f=1$, α_0 in Eq. (15) is;

$$\alpha_0=\alpha' B_0^{-1}E_{so} \tag{18}$$

By substituting Eq. (18) in Eq. (15), the general equation for k (in kg per cubic cm) is obtained,

$$k=\alpha' B_0^{-1}E_{so}\left(\frac{B}{B_0}\right)^{l(m-1)}\cdot y_f^{-m} \tag{19}$$

$$=3\alpha' E_b B_0^{-1}\left(\frac{B}{B_0}\right)^{l(m-1)}\cdot y_f^{-m} \tag{19'}$$

where α' is the constant determined from the bending characteristics of piles.

The details about α' will be discussed later. Fig. 9 shows the effects of the deflection y_f and the diameter of piles on the modulus k with parameters l and m.

The value of m obtained from experiments are summarized in Table 1.

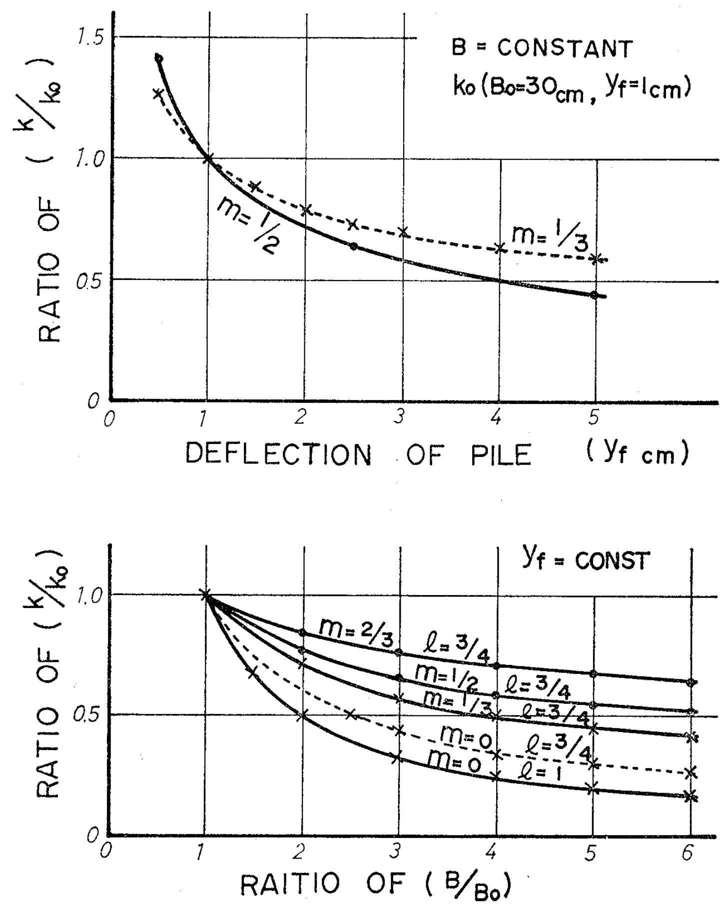

Fig. 9. Effect of y_f and B on k-value

The Constant α' *of Eq.* (19)

1) *On the theoretical values*

The spring constant for a beam with a concentrated load, placed on the infinite-elastic foundation, has been studied by Vesić (1961), (1963).

According to his results, when a beam satisfies the requirement for infinite length, $\lambda L>2.25$, the modulus for the beam becomes constant, and is determined by

$$kB=0.65\sqrt[12]{\frac{E_sB^4}{E_pI_p}}\cdot\frac{E_s}{1-\mu_s^2} \tag{20}$$

Broms (1964), Francis (1964) and Bolows (1968) et al., suggested that the relation expressed by Eq. (20) can be applied to piles, and they have used Eq. (20) for estimating the modulus k for the lateral resistance of piles.

However, Francis considered that the pile condition differs from the beam mentioned above, that is, piles are completely surrounded by foundation materials, and therefore the modulus k for piles should be twice as much as the k value obtained by Eq. (20), i. e.,

$$kB=1.30\sqrt[12]{\frac{E_sB^4}{E_pI_p}}\cdot\frac{E_s}{1-\mu_s^2} \tag{21}$$

For the practical use, Eq. (21) may be simplified since the effect of $\sqrt[12]{E_s}$ is relatively small, then,

$$kB=1.30\sqrt[12]{\frac{B^4}{I_p}}\cdot E_p^{-12}\cdot\frac{E_s}{1-\mu_s^2} \tag{21'}$$

The range of $\sqrt[12]{\frac{B^4}{I_p}}$ is limited due to high order of the function, and Francis (1964) showed that this value was about 1.2—1.3.

It has recently been pointed out, however, that this value appears to range from 1.4 to 1.6 in many cases of concrete and steel pipe piles.

The modulus of elasticity of piles is 2.1×10^6 kg/cm^2 (steel pile) and 2.0—3.5×10^5 kg/cm^2 (concrete pile), so the term of $\sqrt[12]{E_p}$ becomes about 3.4 (the former), and 2.3 (the latter).

Now, assuming Poisson's ratio of soils to be 0.3, then Eq. (21) is further simplified as follows:

$$kB=\alpha' E_s \tag{22}$$

in which

$$\alpha'=1.30\sqrt[12]{\frac{B^4}{I_p}}\cdot\frac{1}{1-\mu_s^2}\quad\begin{cases}:1/1.6 \text{ for steel pipe piles}\\ :1/1.4 \text{ for concrete piles}\end{cases}$$

Hence, the modulus k by Eq. (22) can be applied if the foundation is elastic. Studies such as by Bergfelt (1957) on a beam contacted on the both sides by elastic half spaces has been conducted by Gramholm (1929), Biot (1937) and Jampel (1947) et al.

Biot considered a sinusoidal loading condition for a beam, and assumed that half of the load acted on the beam was supported by each side, giving the following relationship:

$$kB=\frac{8\pi G_s'/1.1(1-\mu_s)}{\ln\frac{E_pI_p}{B^4}+7.85-\ln kB} \tag{23}$$

Applying the general dimension of piles and the following relation held in an elastic body,

$$G_s'=\frac{E_s}{2(1+\mu_s)} \tag{24}$$

then Eq. (23) is simplified as

$$kB=(1/1.6\sim1/2.0)E_s \tag{25}$$

By Jampel (1947), the following equation was obtained:

$$kB=\frac{8\pi E_s/B}{\ln\frac{E_pI_p}{B^4}+7.96-\ln kB} \tag{26}$$

In the same manner, we obtain

$$kB=(1/1.03\sim1/1.88)E_s \tag{27}$$

From the consideration mentioned above, it seems that when a beam is considered as a pile, the spring constant (kB) can approximately be expressed in the form of Eq. (22), and the value of α' is about $1.0\sim1/2.0$ depending upon the assumption for calculations.

2) *On the experimental values*

It is possible to examine the constant α' by applying the experimental deflection curves of lateral loading tests for a pile to Eq. (19).

When Eq. (19) gives the modulus k for piles, the value α' of the reverse calculation mentioned above must have a tendency converging to a special value.

The deformation modulus E_s to be used in Eq. (19) is obtained by averaging the values between the ground surface and the depth near the point of maximum bending moment of piles. This range is considered to be the effective depth on lateral deflection.

As the first step to examine, the constant α' was calculated under the condition in which no influence of deflection y_f on the modulus k was considered and the deflection was assumed approximately elastic being in the range $y_f \leqq 10$ mm.

Strictly, the amount of deflection y_f to be in equivalent strain to each pile due to the difference of diameter of pile is not equal as shown by Eq. (12), but it may be permitted to examine the applicability of Eq. (19), even though by this rough condition.

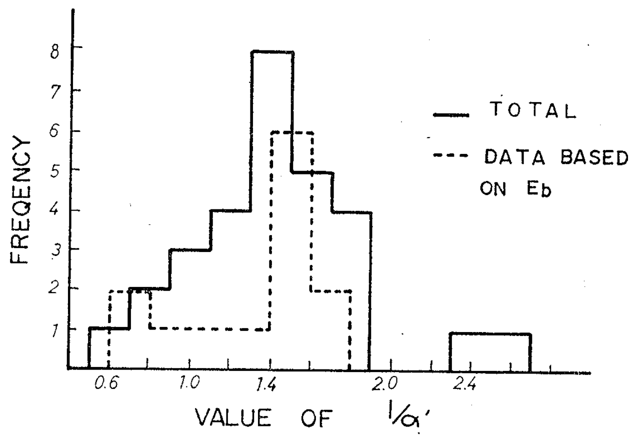

Fig. 10. Distribution of experimental value of α'

The results of calculation (Yoshinaka (1967)) are shown in Fig. 10. In the figure, the solid line shows all the values using E_s from test results by many different methods, and the dotted line shows only the values by the bore hole deformation tests which are considered relatively reliable.

It seems from Fig. 10 that the measured values have a predominant one although scattering in the manner of the normal distribution, and the value α' is about $1/1.3\sim1/1.6$.

Next, we examined the value α' by considering the effect on the modulus k, according to Eq. (15) and Eq. (18), on the experimental data based on the bore-hole deformation modulus E_b and N-value.

In the calculation process, m was determined by using the k-y_f curve of each pile. The results are shown in Table 2 and Fig. 11.

Fig. 11 shows the relationship between the experimental values from Table 2 and the theoretical value by Eq. (22).

Consideration

In the preceding paragraphs the value of α' to be used in Eq. (19) was considered.

From those considerations, it is clear that the value α' can be taken constant judging from the experimental results, and that this value is almost equal to the value calculated by Eq. (22).

That is, α' obtained from thirty five examples converged to the value between $1/1.3 \sim 1/$

Table 2. Summary of the calculated results

PILE	B	E_{SO}	α	α'	m	l	I_p	soil	α'**	test
steel pipe pile	81.28	33	2.14	1/1.54	0.50	3/4	$2\text{-}3\times10^5$	sand	1/1.65	Ministry of Construction
	101.6	33	2.95	1/1.30	0.50	3/4	$4\text{-}6\times10^5$	sand	1/1.53	
	101.6	9	0.48	1/1.85	0.56	3/4	5×10^5	clay	1/1.53	
	152.4	9	0.63	1/1.46	0.56	3/4	1.7×10^6	clay	1/1.74	
	60.96	25	1.36	1/1.86	0.28	3/4	1.1×10^5	fine sand	1/1.54	
precast concrete pile	50.0	25	2.70	1/0.93	0.43	3/4	1.3×10^5	fine sand	1/1.45	

* $E_{SO}=3E_b$ the average over the effective depth

** theoretical value

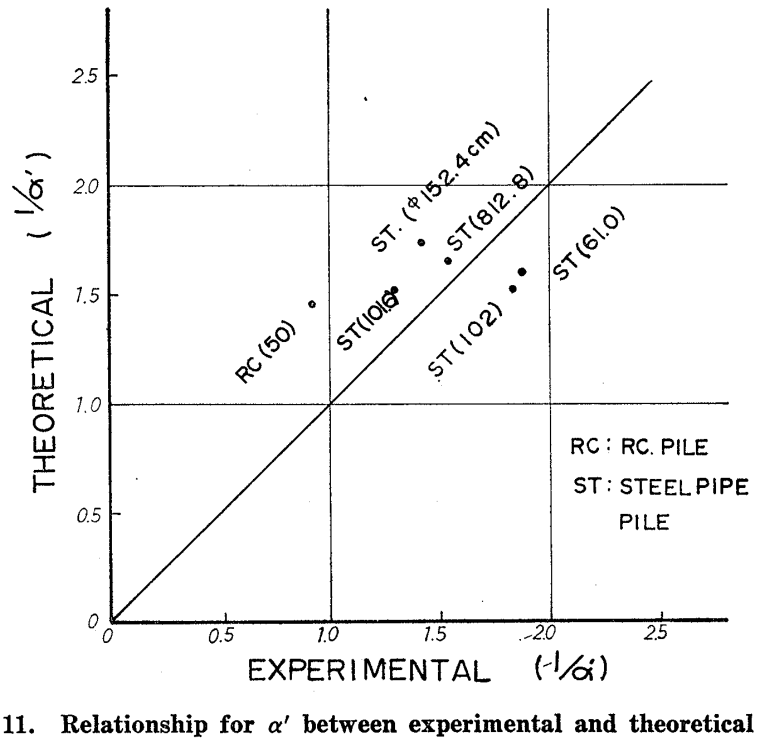

Fig. 11. Relationship for α' between experimental and theoretical value

1.6. But in this case the data were not classified with respect to the materials of piles: steel pipe pile, precast concrete pile, cast-in-place pile, etc., considering the accuracy of the modulus of deformation to be used for analysis. The theoretical values of α' are approximately 1/1.6 in case of steel piles, and 1/1.4 in concrete piles, whicn are generally used. It may be concluded from the above described results that the value α' are;

1/1.6	for steel pipe piles
1/1.4	for precast concrete piles

or the calculated value by

$$\alpha'=1.3\sqrt[12]{\frac{B^4}{I_p}}\cdot E_p^{-12}\cdot(1-\mu_s)^{-1}$$

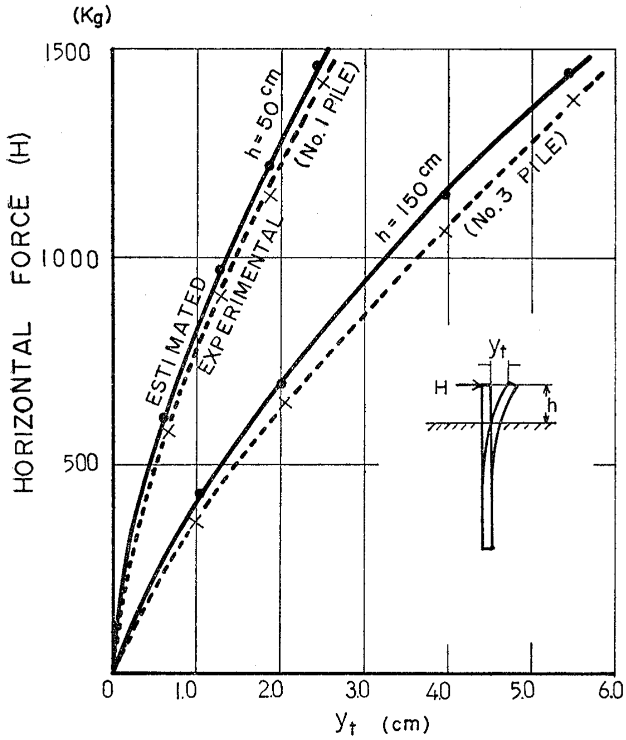

Fig. 12. Comparison of estimated and experimental pile deflection y_f

Finally, we will show an example in which the results obtained in the above were applied to the deflection behavior of a pile tested by R. Miyamoto, M. Sawaguchi (1968) et al., in Fig. 12. The driven piles tested were 126.3 mm in diameter, 15.5 m in length, and the moment of inertia of pile section was 2,910 cm^4.

The foundation soil was soft alluvial clay extending to the depth of 30 m. The effective depth ($0\sim1/3\sim1/4\, l_{m1}$) was about $0\sim1.8$ m, and the modulus of deformation E_s (secant modulus corresponding to half the ultimate compression strength), though gradually increasing with depth, 5.7 kg/cm^2 at the depth of 1.8 m, obtained by unconfined compression tests on undisturbed samples.

CONCLUSIONS

From this experimental study on the effect of testing methods and the loading width to the measured value of the deformation properties, and on the lateral deflection behavior of a pile under static loading, the following results are obtained;

(1) The measured values of the modulus of deformation of soils vary depending upon the methods used. It is found that the following relations hold:

$$E_c \doteqdot E_b$$
$$E_{so} \doteqdot 3E_b$$
$$E_b \doteqdot 7N$$

where E_c is the modulus of deformation measured by uniaxial or triaxial compression tests on undisturbed samples, E_b by the borehole deformation test, E_s by the plate loading test and N by Standard Penetration Test.

(2) The modulus of subgrade reaction k_b in the horizontal direction by plate loading

tests is influenced by the width of loading.

From the results of horizontal loading tests with various diameter, the following equation is obtained

$$k=k_0\left(\frac{B}{B_0}\right)^{-l}$$

where $l \doteqdot 3/4$.

(3) Analyzing many data on lateral loading tests for a pile, the following result on the modulus of subgrade reaction for a pile is obtained. This is valid when the lateral deflection of the pile at the ground surface is relatively small (approximately less than a few centimeter), and it assumes that the lateral behavior of the pile follows the Winkler's hypothesis.

The modulus k by reverse calculation (from Eq. 8) used the stress-strain curves of loading tests at each deflection y_f, can be expressed by the equation on an experimental basis,

$$k=\alpha_0 y_f^{-m} \ (\mathrm{kg/cm^3})$$

where α_0 and m are the constants determined from the properties of soils and the pile.

(4) By using the results presented in (1) to (3) the equation for estimation of the modulus of subgrade reaction k for the practical design can be obtained by

$$k=3\alpha' E_b\left(\frac{1}{B_0}\right)\left(\frac{B}{B_0}\right)^{(3/4)(m-1)} \cdot y_f^{-m} \ (\mathrm{kg/cm^3})$$

where B_0 is the standard width of a pile (30 cm), the constant α' equals about 1/1.3~1/1.6 from experimental results. The value α' almost agrees with the theoretical value given by the following equation proposed by Vesić (1961), Francis (1964), Broms (1964), et al.,

$$\alpha'=1.3\sqrt[12]{\frac{B^4}{E_p I_p}}\cdot\frac{1}{1-\mu_s^2}$$

E_b is the average deformation modulus over the effective depth (approximately, from the ground surface to the depth of the maximum bending moment of piles).

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NOTATION

B=diameter of loading plate or pile
B_0=B of 30 cm in diameter
E_c=modulus of deformation of soil by a compression test on undisturbed sample
E_b=modulus of deformation of soil by a bore-hole deformation test
E_s=modulus of deformation of soil by a plate loading test
E_{s0}=E_s for 30 cm in diameter
E_p=modulus of elasticity of pile
G_s=specific gravity of soil
H=lateral load
I_p=moment of inertia of pile section

k=modulus of horizontal subgrade reaction
L=length of beam
N=number of blows by Standard Penetration Test
w=water content
x=coordinate distance along pile length
y=lateral deflection of pile
$y_f=y$ at ground surface
l, m, n, α, and α'=constants
$\beta=\sqrt[4]{\frac{kB}{4E_pI_p}}$
γ_d=dry density of soil
$\lambda=\beta$ of beam
μ_s=Poisson's ratio of soil
l_{ml}=depth of first zero moment

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