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The Development of Educational Methods Using Manipulative Activities to Promote the Understanding of Positive and Negative Integers

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Abstract

In March 2008 a revision of the Course of Study for Lower Secondary Schools was announced, calling for more mathematical activities to be added to the traditional curriculum. These were designed to promote the fun and pleasure of mathematics though connecting mathematical activities to actual life. Going beyond traditional educational methods, this study discusses materials for teaching positive and negative integers to seventh grade students. It focuses on card games, a mathematical activity connected to actual life, that promote understanding of the principles underlying the manipulation of the signs of positive and negative in arithmetic. The card games are used in a series of practical lessons focusing on mathematical activities.

Results of pre- and post-tests suggest that the games aid in discovering mathematical principles and promote fun and pleasure in learning, as well as meaningful understanding through formal comprehension.

Key Words: Positive integers, Mathematical activities, Conceptual understanding, Card games, Seventh grade

1. Introduction

In March 2008 a revision of the Course of Study for Lower Secondary Schools was announced, calling for "the use of mathematical activities to deepen the understanding of the basic concepts, principles, and rules of figures and quantities, to promote the learning of mathematical processing and expression, and to increase the abilities for mathematical representation of phenomena through realizing the fun of mathematical activities and the pleasure of mathematics, and thereby cultivate a demeanor that can apply these through thinking and assessment." To the prior formulation of the Course of Study for Lower Secondary Schools (December 2008), this revision added “using mathematical activities” and “abilities for mathematical representation” and changed “knowing the pleasure of mathematical viewpoints and thinking” to “realizing…the pleasure of mathematics,” and “cultivating a demeanor that can apply these” to “apply these through thinking and assessment.”

These revised points, aimed to engage motivated learning, call for the development of the ability for clear and concise expression, efficient processing, and reflective consciousness of learning processes. These revisions aim to go beyond class-work that simply communicates mathematics, to promote observation of and rule application to other phenomena, as well as encourage mathematical induction through concrete manipulative activities. They anticipate that the experience of learning mathematics through “activities promoting development of figures and numbers” will expand lessons based on the fun of experiencing the creativity, wonder, and excitement of learning mathematics (Ohta, 2009; Yamasaki, 2009; Kunimoto & Yamamoto, 2004).

The 2009 National Survey on the Situation of Academic Ability and Learning (carried out in April 2008), given to sixth graders at Japanese public and private elementary and special education schools and ninth graders at junior high and special education schools, revealed that the average score for students on “knowledge” was 62.7%, but that only a very low 30.6% of students answered that “the things learned in mathematics class will be helpful in the future when I go out into society.” These results show that awareness of the utility of mathematics for tasks in various everyday life situations is insufficient, and suggest that the problem is the lack of teaching methods for the enriching this line of study. That is, the problem concerns not the amount of mathematical knowledge, but mathematical activities connected to everyday life.

This study uses “positive and negative integers,” part of the first-year program of the Course of Study for Lower Secondary Schools, to examine teaching methods designed to enrich mathematical activities connected to actual life that will promote the understanding of arithmetic rules and principles of positive and negative integers. The “positive and negative integers” examined here are learned in mathematics classes at the beginning of junior high school. Many teaching units include algebraic calculations, which form the bridge between elementary and junior high school mathematics education. Elementary school mathematics establishes and utilizes an understanding of the number zero, positive integers, and positive fractions, and addresses the meaning and significance of doing arithmetic with integers. Junior high school mathematics builds on what was learned at elementary school, expanding the range of numbers to include negative fractions. After reviewing integers as a mathematical concept, their arithmetic possibilities are introduced.

However, when the numbers students know are expanded as they move from elementary to junior high school, several problems emerge in the practical education of positive integers. The degree of simplicity in explaining the meaning of and introducing positive integers to students are of central importance in the teaching unit. Previous textbooks have used a number line to introduce positive integers as a vector, with the plus side (+), or the temporal future, moving towards the right from a cardinal point, and the minus side (-), or the temporal past, moving towards the left. Explanation of the computational principles of integers is based on vector computational principles

(Todani, 2007). Expressed concretely, the instruction of methods of adding and subtracting positive integers uses the directionality and size of vectors, and multiplying and dividing uses velocity. As it is, students have difficulty grasping the conceptual meaning of positive and negative through this method, and calculation principles are difficult to learn (Yanagimoto, 1990). Pointing to the positive integers in the calculation exercises of the teaching units, Kuroda (2004) argues that that even if the students understand how to "answer" the calculations expanding on class-work, they do not grasp the "formal" meaning or "things expressing that relationship."

The present study attempts to rectify the above problem in applied education by taking into account Kuroda's (2004) point that "formulas are the language of mathematics, and learning begins when there is an appropriate context for expressing that language," and suggesting card games that express the importance of formulas to promote the understanding of positive integers. Working with the "positive and negative integers" section of first year of the New Course of Study for Lower Secondary Schools, the goal of the study is to design mathematical activities linked to everyday life through devising teaching methods using card games to promote the understanding of the principles and rules of signs used in arithmetic with positive and negative integers.

2. Method

2.1 Participants

The participants consisted of a class (20 people) of seventh grade junior high school students at a private school in Kanagawa Prefecture. The study was carried out over the five hours of the instructional unit on "positive and negative integers" taught from the middle to the end of July.

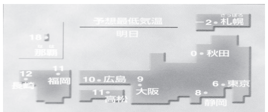
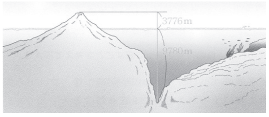
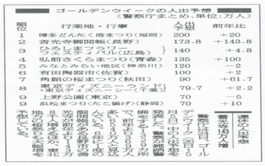
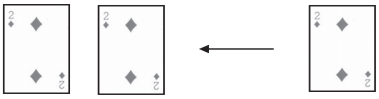
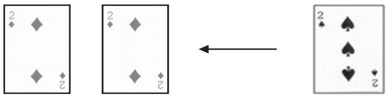
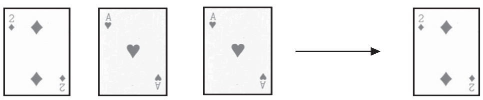
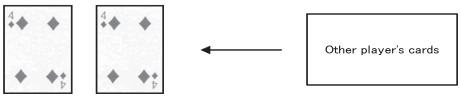
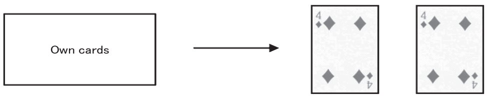
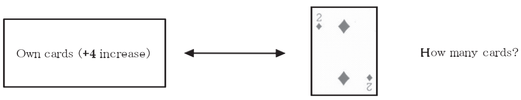
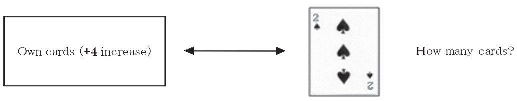
2.2 Teaching method

The instructional method engaged the attribution of meaning to and the handling of the numerical formulas in the arithmetic manipulation of positive and negative integers through card games. An example of the teaching method is displayed on Table 1. First, participants were divided into groups of four and decks of twenty cards using the ace through five of every suit were prepared. Five cards were given to each participant. The red suits (hearts and diamonds) were defined as plus and the black suits (clubs and spades) were defined as minus, and the participants were to take one card each, following along the lines of the card game "old maid." After five rounds were finished, the students were told to stop, and the participant with the largest number of points on their cards was declared the winner. Modifying the game to include mathematical expressions, drawing a card from another person was expressed as "plus" (+) and having a card taken by another person was expressed as "minus" (-). Next, the cards were used for mathematical computations applying the significance of mathematical expressions.

(1) For addition, when the total number of points held was +3, the player took four red cards from the neighboring player, yielding +3 (+4). After taking the cards, the player revealed them, showing +7.

(2) For subtraction, when the total number of points held was +3, four black cards were taken by the

Table 1. An Outline of a Educational Method for Promoting the Understanding of Positive

1h	Creating the connection between positive and negative integers Educational Goal: Thinking about how positive and negative integers are used in everyday life. Example: thermometer markers, depth tables, yearly comparisons of expectations of crowding during holidays    ゴールデンウイークの人出予想 （警察庁まとめ、単位：万人） 順位 | 行楽地・行事 | 人出予想 | 前年比 1 | 博多どんたく港まつり（福岡） | 200 | +20 2 | 善光寺御開帳（長野） | 173.8 | +143.8 3 | ひろしまフラワーフェスティバル（広島） | 140 | +4.8 4 | 弘前さくらまつり（青森） | 135 | +100 5 | みなとみらい地区（神奈川） | 120 | −2 6 | 有田陶器市（佐賀） | 100 | +2 7 | 角館の桜まつり（秋田） | 90 | +61.7 8 | 東京ディズニーランド・東京ディズニーシー（千葉） | 79.7 | +2.2 9 | 上野公園（東京） | 70 | −5 9 | 浜松まつり〈たこ揚げ〉（静岡） | 70 | +10 連休の人出予想 善光寺143万人増 警察庁は24日、ゴールデンウイーク（26日～5月5日）中の人出予想を発表した。警察が雑踏警備をする1329カ所で、前年より369万人多い6540万人と見込んでいる。6年に1度の御開帳がある長野市の善光寺が前年より143万8千人増えるほか、東北地方の桜祭りも大幅増を予想している。 A standard is decided for quantities that can move in opposite directions, and one side can be expressed using positive numbers and the other can be expressed using negative numbers.	
2h	Addition total (same sign) of positive and negative integers Example: $(+4) + (+2)$  Educational goal: Using a game to learn that the value of the sum of two numbers with the same sign through the commonality of the symbols, is the sum of the absolute value of the two. Also learned through the game is that negation occurs in the case of differing signs for the same numbers.	Addition total (different signs) of positive and negative integers Example: $(+4) + (-2)$  Educational goal: Using a game to learn that the value of the sum of two numbers with different signs takes the sign of the number with the largest absolute value and that the result is derived by subtracting the absolute value of the smaller number from the absolute value of the larger number.
3h	Subtraction total (same sign) of positive and negative integers Example: $(+4) - (+2)$  Educational goal: Using a game to learn that it is possible to calculate the subtraction of two numbers with the same sign by changing the sign of the subtracted number and adding them. Concerectely, the case above shows that the formula $(+4) - (+2)$ can be calculated by changing it to $(+4) + (-2)$.	Subtraction total (different signs) of positive and negative integers Example: $(+4) - (-2)$  Educational goal: Using a game to learn that it is possible to calculate the subtraction of two numbers with different signs by changing the sign of the subtracted number and additing them. Concretely, the case above shows that the formula $(+4) - (-2)$ can be calculated by changing it to $(+4) + (+2)$
4h	Multiplication total (same sign) of positive and negative integers Example: $(+4) \times (+2)$  Educational goal: Using a game to learn that the value of multiplying two numbers with the same sign is always positive and is equal to multiplying the absolute value of the two numbers. In the above case, two +4 cards are taken and the total of the cards held is + (plus).	Multiplication total (different signs) of positive and negative integers Example: $(+4) \times (-2)$  Educational goal: Using a card game to learn that the value of multiplying two numbers with different signs is always negative and the value is equal to multiplying the absolute value of the two numbers. In the above case, two +4 cars are taken away and the total of the cards held is − (minus).
5h	Division total (same sign) of positive and negative integers Example: $(+4) \div (+2)$  Educational goal: Using a game to learn that the value of dividing two numbers with the same sign is always positive and is equal to dividing the absolute value of the two numbers. Concretely, if $(+4) \div (+2)$ is understood multiplicatively, it can be changed to $(+4) = (+2) \times [\ ]$ of [ ] to	Division total (different signs) of positive and negative integers Example: $(+4) \div (-2)$  Educational goal: Using a game to learn that the value of dividing two numbers with different signs is always negative and the value is equal to diving the absolute value of the two numbers. Concretely, if $(+4) \div (-2)$ is understood multiplicatively, it can be changed to $(+4) = (-2) \times [\ ]$

neighboring player, yielding +3 – (-4). After the cards were taken, the player revealed the hand, showing +7. Through this card game, the students not only cultivated the sense that when minus cards are taken away, the result is a plus score for the player, but also the sense of the equivalence of +3 – (-4) is the same as +3 + (+4). Concretely, they learn that "taking four black (minus) cards" and "having four red (plus) cards taken" are equivalent, and that "having four black (minus) cards taken" and "taking four red (plus) cards" are equivalent. Expressed mathematically, this is – (-4) =

+ (+4), + (-4) = - (+4). This method both attributes an easy-to-understand significance to and promotes the learning of the principle rules of arithmetic with positive and negative integers. Furthermore, abbreviating the plus signs in the total amount is connected to algebraic summation, and because each card is taken as one item and because items of the same kind are interchangeable, the exercise is connected to a mathematical way of thinking about the interchangeability of items.

(3) The game rules were changeb for multiplication, and the cards were taken twice to convey the mathematical meaning of multiplication. For example, having -4 cards taken twice by a neighboring player, mathematically expressed, is -4 × (-2), and having the minus cards taken two times lends to the main player the experience that the total becomes a plus. Thus, they learn that same signs become a plus in multiplication (in this case, the negative integer × the positive integer).

(4) For division, to achieve the value of (+4) ÷ (+1), the equivalence of (+4) = (+1) × [ ] of [ ] was used. As in the case of significance attribution in multiplication, the meaning of the mathematical process is based on understanding that an increase of the score by 4 is equivalent to receiving four +1 cards.

In this way, a mathematical activity using cards, based on thinking about the movement of things and experimenting with them to see how they work, is connected to increased intellectual capacity. In this real-life situation, manipulating physical objects, thinking about the activity and explaining it are connected, and reciprocal activation becomes possible.

2.3 Pre- and post-tests

Pre- and post-tests were used to directly compare understanding before and after the session. The tests used the same contents and form, and required about ten minutes to answer. The content of the test aimed to examine the transferal of the subject of the lesson to the students. Furthermore, the pre- and post-test questions were designed to assess conceptual understanding by creating a venue using subtraction and multiplication with a combination of plus and minus signs. The first question was "explain the mathematical expression 2 - (-1) using familiar things," and the second question was "explain the mathematical expression 2 × (-1) using familiar things."

3. Results and Analysis

3.1 Quantitative analysis

A pre- and post-test assessing the difference in level of conceptual understanding (hereafter, "understanding level") of positive and negative integers was used to examine the degree of change in the understanding of positive and negative integers (subtraction, multiplication). The understanding level of each student was assessed using the answers on the pre- and post-test. What level of understanding the student had attained was determined synthetically based on their answers to questions 1 and 2. Table 2 displays the assessment criteria for each understanding level and examples of correct and incorrect answers, as well as the pre- and post-test understanding levels for each student.

Table 2-1. Change in Understanding Level

Pre-test

	Understanding level of positive and negative integer concepts						total	
	0		I		II			
Question 1 (subtraction)	19	(95.0)	1	(5.0)	0	(0.0)	20	(100.0)
Question 2 (multiplication)	20	(100.0)	0	(0.0)	0	(0.0)	20	(100.0)

Note: the percentage of students in each group is denoted within the parentheses

Post-test

	Understanding level of positive and negative integer concepts						total	
	0		I		II			
Question 1 (subtraction)	4	(20.0)	5	(25.0)	11	(55.0)	20	(100.0)
Question 2 (multiplication)	1	(5.0)	2	(10.0)	17	(85.0)	20	(100.0)

Note: the percentage of students in each group is denoted within the parentheses

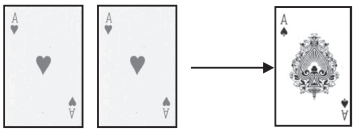
Table 2-2. Change in Understanding Level

Student descriptive examples

<subtraction example> form explanation of $(+2)-(-1)$

level 0 : before understanding

descriptive example



The total of cards held is +2, consisting of two +1 cards. However, when three cards are distributed to one person, three cards have to be lost. Looking only at the formula from this, the background (rule) is not understood.

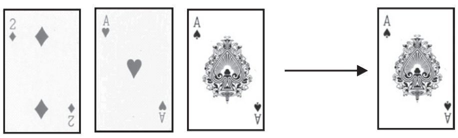
level I: formal understanding

(descriptive example)

A -1 card is taken from another player and the effect on the total of one's own cards is an increase of $+1$.
Introducing relational forms, the same results can be expressed by the formula $(+2)-(-1)=(+2)+(+1)$.

level II: meaningful understanding

(descriptive example)

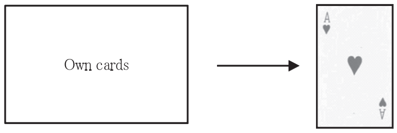


A −1 card is taken by another player, leaving a +2 card and a +1 card for a total of +3 in one's own hand.

multiplication example form explanation of $(+2)\times(-1)$

level 0 : before understanding

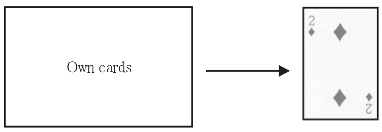
(descriptive example)



Understanding that the total in the previous move is achieved by a +2 addition/subtraction method, it is understood that the other player takes +2 cards from the total in one's own hand as an overall numerical formula. But the prior move and the following move are not understood as a numerical formula.

level I: formal understanding

(descriptive example)

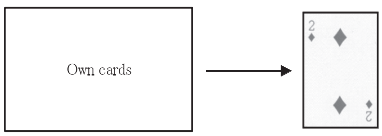


One card

One +2 card is taken by another player and the effect on one's own cards is a decrase by −2.

level II: meaningful understanding

(descriptive example)



One card

One +2 card is taken by another player, effecting the total of cards in one's own hand by a decrease of −2.

Examination of bias in the headcount for the understanding levels in the pre-test yielded significant results (question 1: $\chi^2 = 34.4$, $p < 0.01$, question 2 $\chi^2 = 40.0$, $p < 0.01$), and for each question, level 0 had the most respondents (question 1: 19 people (95%), question 2: 20 people (100%)). Examination of bias in the headcount for understanding levels in the post-test yielded significant results for question 2 (multiplication) (question 2: $\chi^2 = 24.10$, $p < 0.01$), and level II had the most respondents (question 2: 17 people (85%)). Furthermore, in order to investigate bias in the headcount for conceptual levels in each of the questions in the pre- and post-tests, a χ^2 test was carried out on the expected frequency of people in the pre-test and the observed frequency in the post-test. The results revealed a bias in the headcount (question 1: $\chi^2 = 264.21$; $p < 0.01$, question 2: $\chi^2 = 603.61$, $p < 0.01$; because there was a cell with 0 people in the expected frequency (pre-test), 0.5 was added to all cells for convenience before carrying out the test)). For the level of conceptual understanding of positive integers, the results revealed that level 0 had the greatest number of people for both question 1 (subtraction) and question 2 (multiplication), that there was no bias in the headcount for the conceptual level on question 1 (subtraction) in the post-test, and that level II had the greatest number of people for question 2 (multiplication). Furthermore, the results of examining bias in the headcount of the understanding level between the pre- and post-tests revealed that a great number of students had reached level II on question 1 (subtraction) and question 2 (multiplication).

3.2 Qualitative analysis

Moving from level 0 to level 1 captures the shift in students' understanding of the application of mathematical expressions to phenomena in everyday life and the ability to grasp the mathematical principles from everyday phenomena. The results of the students' answers revealed some inconsistency in applying mathematical expressions to phenomena. Take, for example, a descriptive example of subtraction: "In applying the mathematical expression (+2) – (-1) to everyday phenomena, taking the example of cards, the idea of the mathematical expression can be seen as another player taking a -1 card from a person with a total of +2 cards. However, if one looks at the students' descriptive answers, they do not write that the total of the cards held is +2, but they write that two of the +1 cards are chosen, and do not include that the -1 cards the other player is supposed to take." See also a descriptive example of multiplication: "In applying the mathematical expression (-2) × (-1) to everyday phenomena, taking the example of cards, the mathematical meaning is that another person takes one -2 card. However, looking at the descriptive answers, from the -2 total of the cards held, +1 goes to the other player." These descriptive examples show that in mathematical activities using cards, the meaning of the mathematical expression in antecedent mathematical expressions is determined based on mathematical operations, and that the level of recognition of mathematical operations by students is low.

In the shift from level I to level II, principles are adduced via everyday phenomena, and the correct interpretation of mathematical expressions can be shown. In looking at the students' answers, taking + (plus) and – (minus) cards from another player in a game, and having those cards being taking by another player, the mental situation in calculating the total amount of the cards employs pictures and words in establishing a descriptive answer. See the following descriptive example of subtraction: "Expressing the mathematical formula (+2) – (-1) with playing cards means that another player takes a -2 card. Another player holding a -2 card then influences the total of the cards held by oneself, which increases by +2, and thus the physical circumstances are one of a plus." From this descriptive example, students discover mathematical principles through a mathematical activity of a card game, which can be seen an important learning activity.

This study introduced a card game to promote understanding of arithmetic principles, and suggests that this method promotes a shift from formal understanding to meaningful understanding. In this process, students learn both the mathematical formulation of phenomena in everyday life and society, and come to understand the results of processing through mathematical methods while experiencing the fun and pleasure of mathematics. However, the study did not capture how students transfer to everyday activities the knowledge and concepts gained after a class using a card game as a mathematical activity. This problem needs to be the subject of further research.

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