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Effects of Instructional Methods to Teach Quadratic Functions Using Cross-Subject and Ordinary Events

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Abstract

In March 2009, the Course of Study for High Schools was announced. This revision has the goals of increasing the abilities to consider and express various events mathematically, appreciating values of mathematics and actively applying it. Moreover, a survey on the State of Implementation of High School Curriculum conducted in 2005 (National Institute for Educational Policy Research), emphasized the need to actively promote mathematical activities in ordinary and social life in general, so that the students can realize the roles that mathematics play and its utilities, as well as the significance of learning mathematics. Considering these trends, this study was intended to examine effects of instructional methods in which students could realize the significance of learning maximum and minimum values in quadratic functions through mathematical activities related to real life, taking the content about maximum and minimum values in quadratic functions as an example from the first year in high school. The descriptive analysis of pre-and post-tests conducted before and after the lessons that were performed. The results showed that mathematical abilities in examining changes and reactions and predicting unknown conditions were enhanced when the problem of quadratic functions of ordinary and familiar events in other subjects, such as contemporary society (social studies) and physics (science), were used as students apply maximum and minimum values in quadratic functions to cross-subject and ordinary events.

Keywords：Quadratic functions, Mathematical activities, Cross-subject

1. Problem and Purpose

In March 2009, a revision of the Course of Study for High Schools (hereafter New Course of 11Study) was announced. It provides that the purpose of high school math education is "through mathematical activities, to deepen systematic understanding of basic concepts and principles in math, develop the abilities to consider and express various events mathematically, and to cultivate a foundation for creativity, while nurturing attitudes to make judgments based on mathematical bases appreciating values of math and actively applying it." Compared to the current Course of Study, as in those for elementary and junior high schools, it seems to aim to further improve mathematical activities by first mentioning "through mathematical activities."

In addition, in the mathematics section of Survey of the State of Implementation of High School Curricula of 2007 (Ministry of Education, Culture, Sports, Science, and Technology), only about 30% of the respondents endorsed "Classes are taught using operational and experiential activities." Similarly, "Classes are taught with the intention of relating to various events in real life" was endorsed only by approximately 30% of the respondents. These results are not quite consistent with the goals that are addressed in the Course of Study. In high school math education, active use of mathematical activities is needed for students to realize the roles that math plays and its utilities in ordinary events and societal life in general and to recognize the significance of learning math. It is necessary to review objectives of math education and lesson development (Miyama, 2009).

Conventional textbooks do not deal with familiar materials from daily living and the society. In reality, however, functions are an abstract concept to be utilized in considering dynamic subject matters. It works effectively in grasping the relation of two quantities that are found in events not only in the realm of math but also in the real world (Arai, 2006; Iwasaki & Aiki, 2004). Such examples include the trajectory of water of fountains and the balls of home runs. Further, parabolas have characteristics to make parallel rays and radio waves parallel, and satellite dishes and flash lights are examples of their application. Learning quadratic functions is of great significance because parabolas and their characteristics are utilized in ordinary events and items, such as these, among other reasons (Hino, 2008; Nishimura & Nagasaki, 2008). Nevertheless, sections about maximum and minimum values of quadratic functions in the conventional textbooks do not include exercises that use ordinary materials. Understandably, mathematic consideration of matters, abilities to apply math, and usefulness in math do not increase as a result. Thus, this study was intended to examine effects of instructional methods to facilitate to consider maximum and minimum values in various ordinary events by using mathematical activities in teaching maximum and minimum values of quadratic functions, rather than following the instructional methods seen in conventional textbooks.

2. Method

2-1. Study Participants and Lesson Content

A first-year class of 17 students of a private high school in Kanagawa Prefecture participated in the study. Out of those, two students who were absent from the classes were excluded from the analysis. Classes were conducted for seven hours in total on the unit of maximum and minimum values of quadratic functions in the high school first grade Math 1 as presented in TABLE 1. Incorporating contents from other subjects, such as contemporary society (social studies) and physics (science), into the lessons on quadratic functions, the purposes for the students were to: (1) recognize the utility of expressing changing quantities with quadratic functions through understanding the unit of maximum and minimum values of quadratic functions in the first year in high school, and (2) acquire to apply these. In addition, to examine changes in comprehension of concepts related to quadratic functions, pre-and post-tests were conducted. The post-test and explanation were conducted in the seventh class.

TABLE 1 Lesson Plans

1・2h

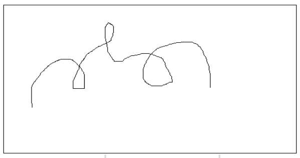
Determining Maximum and Minimum Points Using Graph

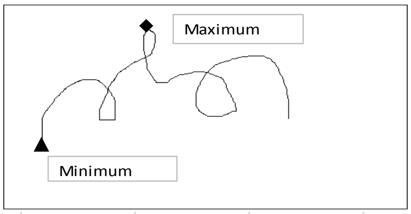
Example: The Highest Temperatures of the Week

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Highest Temperature [°c]	26.5	24.3	24.7	21.5	19.9	22.2	24.0

<Teaching Objective>
Recognize that presenting in a bar graph visually facilitates understanding on which day the day' s highest temperature was the highest and on which day it was the lowest.

Example: Graphs Used in Class



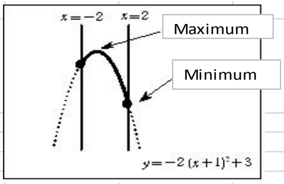


<Teaching Objective>
Assuming a complex curve to be a graph, find its maximum and minimum points.

3・4h

Maximum and Minimum Values in Quadratic Function

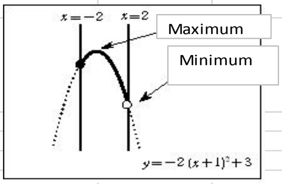
Example: $y=-2(x+1)^2+3 \quad (-2\leq x\leq 2)$



<Teaching Objective>
By identifying where the vertex is located in relation to the domain of definition, find the maximum and minimum points. Understand that in this example, the maximum point is the vertex, and the minimum point is $x=2$.

Maximum and Minimum Values in Quadratic Function Example:

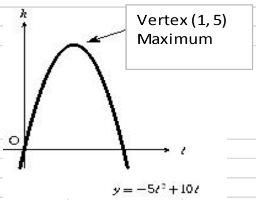
$y=-2(x+1)^2+3 \quad (-2\leq x<2)$ の場合



<Teaching objective>
By identifying where the vertex is located in relation to the domain of definition, find the maximum and minimum points. Understand that in this example, the maximum point is vertex, but the minimum point is nonexistent because $x=2$ is not included in the domain.

5・6h

Application of maximum and minimum values in a quadratic function Example: Trimetric throw of an object (Physics)



$$h=-5t^2+10t$$
$$=-5(t-1)^2+5$$

<Teaching Objective>
Understand that although quadratic functions that are dealt with in math are typically expressed in equations with y and x, maximum and minimum values can be calculated as long as the equation is quadratic even when other letters are used.

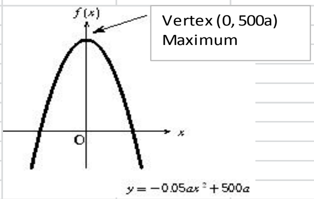
Application of maximum and minimum values in quadratic functions Example: Sales and Profits of Products (Economics)

[Setting]
The number of sales when a product is sold for ¥500 is a ($a>0$). The sales amount when the product is reduced at x% is $f(x)$[¥].

[When the price is reduced at x%]
Sales per products: $(500-5x)$[¥]

Number of sales: $(a+0.01ax)$

$$f(x)=(500-5x)(a+0.01ax)$$
$$=-0.05ax^2+500a$$



<Teaching Objective>
① Express word problems in equations.
② Understand that although quadratic functions that are dealt with in math are typically expressed in equations with y and x, maximum and minimum values can be calculated as long as the equation is quadratic even when other letters are used.

2-2. Pre- and Post-Tests

The pre- and post-tests were conducted with the identical content and format to directly compare the changing conceptual understanding. It required about 30 minutes to complete the test. There were five questions using an identical function equation.

The first question asked to determine the maximum and minimum values of the quadratic function $y = 2x^2 + 4x + 2$, and was a problem to identify maximum and minimum values of a quadratic function without a specific domain of definition (with the domain of all real numbers). It was also a problem to check the understanding of direction of opening, how to identify a vertex (confirmation of completion of a square), and rough shape of the graph. It was to check if the students understood that the vertex is the maximum point when the rough shape of the graph is open downward with no minimum value, and that the vertex is the minimum point when it is open upward with no maximum point based on the understanding of these three features.

The second question asked to determine the maximum and minimum values of the quadratic function $y = -2x^2 + 4x + 2$ in the closed interval $-2 \le x \le 2$, and was a problem to identify maximum and minimum values of a quadratic function with a specific domain of definition. It was also a problem to check the understanding of relative locations of a domain and the vertex (axis) in addition to direction of opening, how to identify a vertex (confirmation of completion of a square), and rough shape of the graph. It was to check if the students understood the point of the greatest y (maximum point) and the point of the smallest y (minimum point) within the given domain based on the understanding of these four features.

The third question asked to determine the maximum and minimum values of the quadratic function $y = -2x^2 + 4x + 2$ in the closed interval $a \le x \le a + 2$ (where $2 < a$), and was a problem to identify maximum and minimum values of a quadratic function with a specific domain that include a letter. It was also a problem to check the understanding of solutions contingent on values of a letter in addition to direction of opening, how to identify a vertex (confirmation of completion of a square), rough shape of the graph, and relative locations of a domain and the vertex (axis). It was to check if the students understood that the point of the greatest y (maximum point) and the point of the smallest y (minimum point) within a given domain can be determined and that even when the domain included a letter, they can be determined by substituting the letter based on the understanding of these five features.

The fourth question is an application of maximum value of a quadratic function concerning contemporary society (social studies). "Suppose you are selling a product whose list price is ¥20,000 (cost price ¥10,000). According to market research, the number of sales of this product will increase by 2% every percent by which the price is reduced. To maximize the profit, for what price do you want to sell this product?" In terms of mathematical understanding, it required the students to comprehend the word problem, generate a function equation on their own, and determine the maximum value of the quadratic function. Thus, it was to check if the students understood to set a letter as needed and to set a domain of definition in addition to direction of opening, how to identify a vertex (confirmation of completion of a square), rough shape of the graph, and relative locations of a domain and the vertex (axis). In terms of economic understanding, it is a problem that highlights "amount of sales ≠ profit." Solving this problem helps to ascertain that the pricing for the maximum sales amount does not necessarily accord with the pricing for the maximum profit. It translates to "annual sales ≠ annual profit" in the context of individual business, which should help students to realize that math is necessary in social realms such as economics and management.

The fifth question is an application of maximum value of a quadratic function concerning physics (science). "Suppose you throw an object upward. The height of the object after t s from the location where it was thrown is given by the formula $y = v_0 t - 4.9t^2[\mathrm{m}]$ where $v_0[\mathrm{m/s}]$ is the acceleration of the object." By using this formula, the problem asked to calculate the height of the highest point that the object achieves and the time lapses until the object strikes the ground for the first time when it is thrown at the initial velocity 19.6 m/s. This physics problem allows to determine the vertex, the height of the highest point, the time it is achieved, and the time it falls to the ground (height 0) for the first time by graphing the function equation, as well as examining the trajectory of the object.

3. Results and Discussion

3-1. Changes in the Level of Understanding of Quadratic Function Concepts

The difference in the levels of understanding in the descriptive analysis of pre-and post-tests were analyzed to examine the change in degrees of understanding of maximum and minimum values in quadratic functions. What level of understanding the student had attained was determined comprehensively based on his or her entire response to Questions 1 to 5, instead of answers to individual questions. TABLE 2 displays the number of students for each understanding level in pre- and post-tests. In addition, TABLE 3 summarizes the assessment criteria for each understanding level and examples of correct and incorrect answers.

TABLE 2　Change in Understanding Level of Concepts

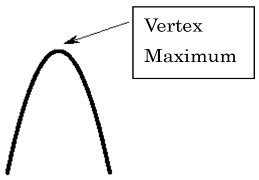
	Proportion of number of students in each category in total number of students (%) in parentheses					
	Level of Understanding of Quadratic Function Concepts					Total
	0	I	II	III	IV	
Pre-test	10(66.7)	5(33.3)	0(0.0)	0(0.0)	0(0.0)	15(100.0)
Post-test	0(0.0)	1(6.7)	5(33.3)	6(40.0)	3(20.0)	15(100.0)

Quantitative analyses based on TABLE 2 showed that the variance in the number of students in each understanding level in pre-test was significant (χ^2 (4)＝228.571, p<.01), many students falling into Level 0 (Level 0: 10 students (67%)). The variance in the number of students in each understanding level in post-test was also found significant (χ^2 (4)＝74.286, p<.01）, many students falling into Level 3 (Level 3: 6 students (40%)).

The variance in number of students in each understanding level between pre- and post-tests was analyzed with the number of students in pre-test as expected frequency. Because there was a cell with 0 for expected frequency (pre-test), descriptive analysis was performed after adding 0.5 to all the cells expediently, and the results showed a variance (χ^2 (2)＝229.361, p<.01). The observed values and the results of residual analyses demonstrated that in the pre-test, there were significantly greater numbers of students in Levels 0 and 1, and in post-test, there were significantly greater numbers of students in Levels 2,3, and 4.

Based on the above results, in the pre-test, the majority of students were in Level 0 in terms of the level of understanding on quadratic functions, whereas in the post-test, the numbers of students in Levels 2,3 and 4 increased, with a number of students achieving Level 3.

TABLE 3 Analysis of Incorrect Answers of Learners

1 . Maximum and Minimum Values in Quadratic Functions (without Defined Domains)	
<u>Problem</u> Determine the maximum and minimum values of $y=-2x^2+4x+2$	
<u>Level 0: Tranforming a quadratic equation by completing the square</u>	
Students at this level cannot complete the square, or determine the direction of opening of the graph and the vertex.	
(Examle of Correct Answer)	(Example of Incorrect Answer of Learners)
To complete the square of the equation $y=-2x^2+4x+2$: $y=-2x^2+4x+2$ $=-2(x^2-2x)+2$ $=-2\{(x-1)^2-1\}+2$ $=-2(x-1)^2+2+2$ $=-2(x-1)^2+4$ By transforming to the form $y=a(x-p)^2+q$, the following can be determined: ・the direction of opening of the graph from the value of a ・vertex $(p,\ q)$ Therefore, for $a=-2$ and $a<0$, the graph is open downward. For $p=1$ and $q=4$, the coordinate of the vertex is $(1, 4)$.	To complete the square of the equation $y=-2x^2+4x+2$: $y=-2x^2+4x+2$ $=2(-x^2+2x+1)$ $=2\{-(x+1)^2\}$ $=-2(x-1)^2$ Therefore, the graph is open downward, and the vertex is $(1, 0)$. **[Analysis of Incorrect Answer]** Because the student identified the common factor 2, it is clear that he/she understood that completion of the square is an application of factorization. But he/she did not understand that the equation must be factorized with the coefficient of x^2 as a common factor to transform it to the form $y=a\left(x^2+\frac{b}{a}x\right)+c$.
<u>Level I: Drawing a Graph Based on a Quadratic Equation</u>	
Students at this level can by competing the square, determine the direction of opening of the graph and the vertex, graph an equation, and find maximum and minimum points.	
(Examle of Correct Answer)	(Example of Incorrect Answer of Learners)
For $y=-2x^2+4x+2=-2(x-1)^2+4$, The graph is a parabola that is open downward with the vertex $(1, 4)$. Therefore, the rough shape of the graph is as follows:  Based on the graph, the maximum point is the vertex, and for the graph continues downward, a minimum point does not exist. Therefore, maximum value: 4 ($x=1$) ; minimum value: nonexistent.	To complete the square of $y=-2x^2+4x+2$: $y=-2(x-1)^2+4$ Therefore, the graph is open upward with the vertex $(-1, 4)$.  Therefore, maximum value: nonexistent; minimum value: 4 ($x=-1$) . **[Analysis of Incorrect Answer]** Because the student completed the square and graphed the equation, it is clear that he/she understood the procedures for solution. But he/she failed to understand that when an equation is obtained after transformation in the form $y=a(x-p)^2+q$, if $a<0$, the graph is open downward.

2. Maximum and Minimum Values in Quadratic Functions (with Domains Defined by Numbers)

<u>Problem</u> Determine the maximum and minimum values of $y = -2x^2 + 4x + 2 \quad (-2 \le x \le 2)$.

<u>Level II: Relative locations of a quadratic graph and a domain defined by numbers</u>

Students at this level can add a domain of definition correctly to the graph that has been drawn at Level I and locate maximum and minimum points.

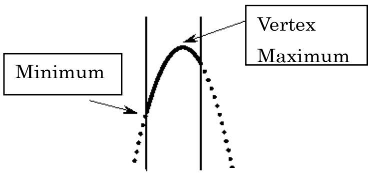
(Examle of Correct Answer)

For $y = -2x^2 + 4x + 2 = -2(x-1)^2 + 4$,

The graph is a parabola that is open downward with the vertex $(1, 4)$.

For $-2 < 1 < 2$, the vertex is located within the domain of definition.

Therefore, the rough shape of the graph is as follows:



Based on the graph, the maximum point is the vertex, and the minimum point is $x = -2$.

When $x = -2$ is substituted in $y = -2(x-1)^2 + 4$,

$y = -2(-2-1)^2 + 4 = -14$

Therefore, maximum: 4 ($x = 1$)

Minimum: -14 ($x = -2$)

(Example of Incorrect Answer of Learners)

For the domain is defined as $-2 \le x \le 2$,

when $-2x^2 + 4x + 2 = f(x)$,

$f(-2) = -2 \times (-2)^2 + 4 \times (-2) + 2 = -14$

$f(2) = -2 \times 2^2 + 4 \times 2 + 2 = 2$

Therefore, maximum: 2 ($x = 2$)

minimum: -14 ($x = -2$)

[Analysis of Incorrect Answer]

In this answer, the student simply substituted $x = -2$ and $x = 2$ in the equation $-2x^2 + 4x + 2$ judging from the definition of domain, instead of utilizing the visual information from the graph. This was presumably based on the experience of solving problems to calculate the codomain (range) of y in "Direct and Inverse Proportion" and "Linear Function" from the first and second grades in junior high school which did not require graphing as they are monotonic increase or decrease.

3. Maximum and Minimum Values in Quadratic Functions (with Domains Defined by Letters)

<u>Problem</u> Determine the maximum and minimum values of $y = -2x^2 + 4x + 2 \quad (a \le x \le a+2)$ when $2 < a$.

<u>Level III: Relative Locations of Quadratic Graphs and Domains Defined by Letters</u>

Students at this level can add a domain defined by a letter correctly to the graph that has been drawn at Level I and locate maximum and minimum points.

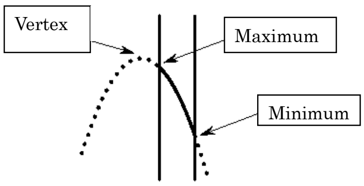
(Examle of Correct Answer)

For $y = -2x^2 + 4x + 2 = -2(x-1)^2 + 4$, the graph is a parabola that is open downward with the vertex $(1, 4)$.

For $2 < a$, $4 < a + 2$.

Therefore, for $1 < a < a + 2$, the vertex is located to the left, outside of the domain.

Therefore, the rough shape of the graph is as follows:



Based on this graph, the maximum point is when $x = a$, and the minimum point is when $x = a + 2$.

When $x = a$ is substituted in $y = -2x^2 + 4x + 2$,

$y = -2a^2 + 4a + 2$

When $x = a + 2$ is substituted in $y = -2x^2 + 4x + 2$,

$y = -2(a+2)^2 + 4(a+2) + 2 = -2a^2 - 4a + 2$

Therefore, maximum value: $-2a^2 + 4a + 2$ ($x = a$)

minimum value: $-2a^2 - 4a + 2$ ($x = a + 2$)

(Example of Incorrect Answer of Learners)

Because the domain is defined as $a \le x \le a + 2$, when

$-2x^2 + 4x + 2 = f(x)$,

$f(a) = -2a^2 + 4a + 2$

$f(a+2) = -2(a+2)^2 + 4(a+2) + 2 = -2a^2 - 4a + 2$

$f(a+2) - f(a) = (-2a^2 - 4a + 2) - (-2a^2 + 4a + 2)$

$= -8a < 0 \quad (\because 2 < a)$

$f(a+2) - f(a) < 0$ より $f(a+2) < f(a)$

$y = -2(a+2)^2 + 4(a+2) + 2$ $y = -2a^2 - 4a + 2$

Therefore, maximum value: $-2a^2 + 4a + 2$ ($x = a$)

minimum value: $-2a^2 - 4a + 2$ ($x = a + 2$)

[Analysis of Incorrect Answer]

Because the domain was defined by a letter instead of numbers, the student was not able to examine relative locations of the vertex and the domain, and because the vertex was included within the domain in the previous problem, he/she assumed that the vertex should be located outside of the domain in this problem. He/she substituted $x = a$ and $x = a + 2$ in $-2x^2 + 4x + 2$, and based on the relation between these values, determined that the maximum value was when $x = a$, and the minimum value was when $x = a + 2$. The results appear to be correct, but because his/her process does not always lead to correct answers, it was decided that he/she did not achieve Level III.

4. Maximum and Minimum Values in Quadratic Functions (Application to Other Subjects: Economics)

Problem Suppose you are selling a product whose list price is ¥20,000 (cost price ¥10,000). According to market research, the number of sales of this product will increase by 2% every percent by which the price is reduced. To maximize the profit, for what price do you want to sell this product?

Level IV: Applicaion of Knowlegde of Quadratic Function

Students at this level can apply the knowledge of maximum and minimum values in quadratic functions to problems that include materials from other subjects.

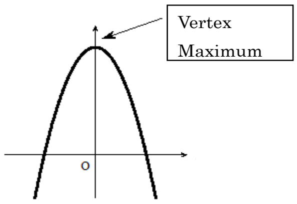
(Examle of Correct Answer)

If the number of sales is a ($a>0$) when the price is ¥20,000 and the profit is $f(x)$ [¥] when the price is reduced by x%,

$$f(x)=(10000-200x)(a+0.02ax)$$
$$=-4ax^2+10000a$$

For a price reduction at x% decreases the profit per product by $200x$ yielding the profit per product of $(10000-200x)$ [¥], while it increases the number of sales by $0.02ax$, yielding the number of sales of $(a+0.02ax)$.

The following is the graph of this equation:



Based on this graph, $f(x)$ is maximum when $x=0$.

Therefore, the profit will be maximum when the price is reduced by 0% in other words, when the products are sold for ¥20,000.

(Example of Incorrect Answer of Learners)

If the number of sales is a ($a>0$) when the price is ¥20,000 and the profit is $f(x)$ [¥] when the price is reduced by x%,

$$f(x)=(20000-200x)(a+0.02ax)$$
$$=-4ax^2+200ax+20000a$$
$$=-4a(x-25)^2+22500a$$

for a price reduction at x% decreases the sales per product by $200x$ yielding the sales per product of $(20000-200x)$ [¥], while it increases the number of sales by $0.02ax$, yielding the number of sales of $(a+0.02ax)$.

Therefore, $f(x)$ is maximum when $x=25$.

[Analysis of Incorrect Answer]

It is clear that the student understood that the reduction at x% yields the sales reduction of $200x$ [¥] per product. If the problem asked about amount of sales, rather than profit, the answer of this student was correct, and since he/she refers to "sales per product" in the response, it is likely that he/she misinterpreted the problem.

5. Maximum and Minimum Values in Quadratic Functions (Application to Other Subjects: Physics)

Problem Suppose you throw an object upward. The height of the object after t s based on the location at which it was thrown is given by the following formula where v_0 [m/s] is the acceleration of the object:

$$y=v_0t-4.9t^2$$

By using this formula, determine the height of the highest point that the object achieves and the time until the object strikes the ground for the first time when it is thrown at the initial velocity 19.6 m/s.

Level IV: Applicaion of Knowlegde of Quadratic Function

Students at this level can apply the knowledge of maximum and minimum values in quadratic functions to problems that include materials from other subjects.

(Examle of Correct Answer)

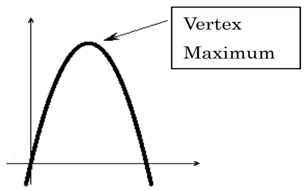
For $v_0=19.6$,

$$y=-4.9t^2+19.6t$$

To complete the square of this equation:

$$y=-4.9t^2+19.6t$$
$$=-4.9(t-2)^2+19.6$$

The following is the graph of this equation;



Based on the graph, y is maximum when $t=2$.

Therefore, the height of the highest point is 19.6 m.

For the height of the ground is 0 m, substitute $y=0$ in $y=-4.9t^2+19.6t$, and an equation $0=-4.9t^2+19.6t$ is obtained.

The solutions of this quadratic equation are $t=0, 4$.

For $t=0$ represents the moment when the object is thrown, it strikes the ground for the first time when $t=4$. In other words, it takes 4 s until the object first strikes the ground.

(Example of Incorrect Answer of Learners)

For $v_0=19.6$,

$$y=-4.9t^2+19.6t$$

$$y=-4.9t^2+19.6t$$
$$=-4.9t(t-4)$$

The following is the graph of this equation.

[Analysis of Incorrect Answer]

The strategy of substituting $v_0=19.6$ from the problem is correct. However, there is no constant term in the equation $y=-4.9t^2+19.6t$, and the student was drawn to the common factor $-4.9t$, factorizing the equation with the common factor, instead of completing the square. If he/she realizes that the highest point that the object achieves can be conceptualized as the maximum value of y, he/she will recognize the need to complete the square.

3-2. Analysis of Factors Causing Difference in Understanding Level

The following is the results of a qualitative analysis based on responses (see TABLE 3).

Students at Levels 0 and 1 are able to determine the direction of opening of a graph, transform the given equation by completing the square, and graph the equation. Common to the responses to the problems of the learners at these levels was failure in the process of completing the square. Specifically, while completion of the square involved first factoring out the coefficient of x^2 of the members x^2 and x to transform the given function $ax^2 + bx + c$ to the form $a\left(x^2 + \frac{b}{a}x\right) + c$, the learners had factored out the given function $-2x^2 + 4x + 2$ with the common factor -2, obtaining $-x^2 + 2x + 1$, and then transformed it to $-(x + 1)^2$. This was presumably because they had had more experience in factorizing with a common factor than in completion of the square, and they reached incorrect answers as a result of using formulas such as $x^2 + 2ax + a^2 = (x + a)^2$.

Students at Levels 1 and 2 are able to determine maximum and minimum values in quadratic functions when domains are defined by specific numbers. Common to the incorrect answers to the problem at these levels was attempting to calculate the minimum and maximum values of the given function by substituting the values from the given domains directly into the function equation. This was presumably based on the experience of solving problems to calculate codomain (range) of y in direct and inverse proportion and linear function in junior high school math which did not require graphing but substituting values from the domain of *x*. It is conceivable that inaccurate knowledge about functions, such as this, that had originated in junior high school math was the factor (Marubayashi & Kawakami, 2003). However, for the correct answers, the learners could identify the problem that the rough shapes of quadratic functions (open downward or upward) are different from those of linear functions (upward or downward slope), or monotonic increase or decrease, and the relative location of a vertex within the domain.

Students at Levels 2 and 3 are able to determine maximum and minimum values in quadratic functions when domains are defined not by specific numbers but by letters representing variables. In this problem, for the incorrect answers, the relative locations of the vertex and the domain is of importance, and the fact that the vertex (1, 4) is located to the left of the domain $a \leq x \leq a + 2$, based on the condition $2 < a$, must be understood. However, based on the domain $a \leq x \leq a + 2$ and the fact that the graph is open downward, learners commonly assumed that the maximum point was the vertex and the minimum point was either when $x = a$ or $x = a + 2$. This was presumably because they failed to identify the location of the vertex relative to the domain based on the condition $2 < a$, and instead, assumed that the vertex was located within the domain, leading to a judgment that vertex should mark the maximum point based on the rough shape of the graph (Inenaga, 2007). However, for the correct answers, the learners could identify the problem that the point of the greatest y (maximum point) and the point of the smallest y (minimum point) within a given domain can be determined and that even when the domain included a letter, they can be determined by substituting the letter. The results showed that it was effective to teach the solutions contingent on values of a letter in addition to direction of opening, how to identify a vertex (confirmation of completion of a square), rough shape of the graph, and relative locations of a domain and the vertex (axis).

Students at Levels 3 and 4 are able to apply maximum and minimum values in quadratic functions in daily situations. In the problem about profit (civics), learners were required to generate an equation to represent the word problem, defining the profit when the price was reduced by x % as y[¥] and the number of sales before reduction as a . Common to the incorrect answers, the learners at these levels was misinterpreting $(20000 - 200x)$[¥] as the profit per product even when they understood the concept (price per product) x (number of product) = (amount of sales) and that the profit decreases by $200x$[¥] when the price was reduced by x%. The amount $(20000 - 200x)$[¥] is the amount of sales per product, instead of the profit per product. As Nakamura (2007) stated, this was likely due to the lack of abilities in comprehension and application in this type of problems, such as setting letters on their own, because classes tend to be given in the lecture format, in which the teacher explains example problems from the textbook and solve many exercise problems. The needs for teaching methods that cut across subjects, such as math and social studies, are valuable although the students do have experience in elementary school social studies in learning terms such as cost price, cost, and profit. However, for the correct answers, the learners could identify $f(x)$ is maximum when. $x = 0$ Thus, the profit will be maximum when the price is reduced by 0%, in other words, when the products are sold for ¥20,000. The results showed that it was effective to teach the word problem, generate a function equation on their own, and determine the maximum value of the quadratic function. Thus, the students could understand to set a letter as needed and to set a domain of definition in addition to direction of opening, how to identify a vertex (confirmation of completion of a square), rough shape of the graph, and relative locations of a domain and the vertex (axis). Therefore, students at Levels 3 are able to apply maximum and minimum values in quadratic functions to cross-subject and ordinary events.

More students answered correctly the problem about throwing (science) than the problem about benefit partly because a quadratic equation was given in the problem. It appeared that some learners were confused because many of them had an understanding that quadratic functions were relational expression between y and x, whereas in this problem, the equation included t instead of x as a variable. In this problem, the typical ways to determine the maximum value is to judge from the vertex in the graph, and to calculate the time lapses until the object hits the ground is to solve the quadratic equation $0 = 19.6t - 4.9t^2$ for the height of the ground can be expressed as $y = 0$. Common to the incorrect answers to the problem of the learners at these levels was "For the highest point is $t = 2$ and it should take 2 s as well to fall from the highest point to the ground, a total 4 s is the answer," and "The solutions to the equation $0 = 19.6t - 4.9t^2$ are $t = 0, 4$, and therefore, their middle point $t = 2$ should signify the highest point, achieving the height $y = 19.6 \times 2 - 4.9 \times 2^2 = 19.6$." However, for the correct answers, the students realize that the highest point that the object achieves can be conceptualized as the maximum value of , they will recognize the need to complete the square. Therefore, the height of the highest point is 19.6m. For the height of the ground is 0m, substitute $y = 0$ in $y = -4.9t^2 + 19.6t$, and an equation $0 = -4.9t^2 + 19.6t$ is obtained. The solutions of this quadratic equation are $t = 0, 4$.

For $t = 0$ represents the moment when the object is thrown, it strikes the ground for the first time when $t = 4$. Thus, the learners at these levels were able to determine the vertex, the height of the highest point, the time it is achieved, and the time it falls to the ground (height 0) for the first time by graphing the function equation, as well as predicting the unknown trajectory of the object. The results showed that it was effective to teach the application of the rough shape of quadratic graphs, symmetric with respect to a vertical line through the vertex, and likely due to the factor that the problem could be approached with the basic knowledge about quadratic functions. Therefore, students at Levels 4 are able to apply maximum and minimum values in quadratic functions to cross-subject and ordinary events, moreover predict unknown conditions.

4. Conclusion and Future Issues

As stated in the section of Problem and Purpose, a revision of the Course of Study for Schools was announced, and the Survey of the State of Implementation of High School Curricula of 2005 illustrated the problems in math education in Japan. Considering these problems and issues into account, classes based on this project assisted students in examining the changing values in functions through understanding maximum and minimum values in quadratic functions in the first grade in high school, using materials from other subjects, such as contemporary society (social studies) and physics (science), in the lessons on quadratic functions. In these classes, the objectives were for the students to, in addition to learning how to determine maximum and minimum values in functions, recognize the utility of expressing changing quantities using quadratic functions, and acquire abilities to apply them.

Traditionally, there has not been much research conducted using daily situations and materials from other subjects in high school math education. More specifically, how the learners develop concepts after such classes has not been studied fully (Shimizu, 2006, 2007). In this respect, this study conducted an exploratory analysis from the aspect of conceptual understanding of quadratic functions in high school math. It was demonstrated that as a result of the implemented lessons in accordance with this project, many students advanced to higher conceptual levels, such as Levels 2,3 and 4, and especially many fell into Level 3.

Future topics include the improvement of instructional methods, so that most students may attain at Level 4. It is so difficult to attain Level 4 (i.e., Learners apply maximum and minimum values to quadratic functions in daily situations, and predict unknown conditions.), that examination is needed in the future on measures to counteract this issue. Furthermore, there is a possibility of bias of using materials in math from other subjects, such as civics (contemporary society) and science (physics), for learners who think they are weak in these subjects, so the instructional methods of quadratic functions using cross-subject and ordinary events need to be explored to keep close collaboration between other subjects.

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