

The Study of Open-ended Approach in Mathematics Teaching Using Jigsaw Method:

A Case Study of the Water Beaker Problem

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Abstract

The aim of study was to (1) create a lesson plan of an open-ended problem using jigsaw method (2) experiment the lesson plan in junior high school in Japan and (3) compare between expected and empirical result in class experiment. This study was conducted by reviewing the related literatures, creating the lesson plan and other instruments, experiment the lesson plan in junior high school in Japan and summarizing the result of teaching.

The process of Mathematics teaching using the open-ended problem is the method that allows students to encounter Mathematics problems having the openness in processes, answers and developing of problem. The students will find the solution by using Mathematics knowledge based on their prior knowledge and their abilities. The jigsaw activity provides students learn to collaborate with others and improves their communication skills in order to explain the findings to others. The lesson plan about the water-beaker problem tilted with one fixed edge, was created along with jigsaw method which are as follows:(1) Expert activity was divided into 3 steps: 1.1) Brainstorming: Students are divided into groups to brainstorm what they want to examine the beaker from only one view (Front/Top/Side-view), 1.2) Sharing: Students regroup to share viewpoints of brainstorming group in own view, 1.3) Experiment: Students regroup to examine viewpoints by experiment in own view, (2) Jigsaw activity: Students regroup to group of all views (Front, Top, Side-view) to explain all findings. The lesson plan was tried out with students in 2nd year junior high school class of Saitama prefecture junior high school for 2 periods by Japanese teacher and the researcher.

The results of experiment showed that students perceived the volume of water is constant and discovered a variation of water height which is the height of water at the tilted side increases but the opposite side decreases. Those are important foundation conditions to generate other amounts. Most students collected data in experiment by measuring so the teachers should encourage students draw a model of tilted beaker to help seeing the perspective in order to compute other amounts. It should be noted that the students were 2nd year junior high student and still have not study about irrational number such as square root of 2, Pythagorean theory. Therefore they could not find some exactly detail. On the other hand, they could find many changes about ratio, directly and inverse proportional which they have already studied. It means they could integrate their mathematical knowledge. Furthermore, most of them could not explain ideas clearly and accurate-

ly so the teacher should emphasize students to add more detail of change clearly.

From this study, it shows that students' thinking ability is satisfactory in flexibility but unsatisfactory in fluency and originality. They should be offered the improvement because these skills are important and beneficial effect on their lives. Teacher should pay attention to make familiar relationship with the students at the beginning and must be able to communicate with students naturally and explain simply in order that the learning process could be done smoothly and successfully. Communication skill is a very indispensable and important skill that teacher have to improve. About learning activities, the more openness of question teacher asked may be the greater of the challenge and interesting of question. Moreover, if teacher provided students make a condition of question by themselves, students will be engaged to the problem gradually and can understand the problem more deeply.

Keywords: Open-ended problem, Open-ended approach, Jigsaw method

1. Introduction

1.1 Background of the study

One of the teaching approaches which improve students' thinking skills is the open-ended approach. The students in Mathematics classroom adapting open-ended approach learn Mathematics more meaningfully by themselves based on sharing mathematical ideas in order to create the shared meaning and leading to shared goal.(Thinwiangthong T., Inprasitha M., Loipha S., 2012) Moreover, jigsaw method, a teaching technique focusing on the interaction among students, is one of powerful strategy in helping students learn to understand subjects from different perspectives and discussing with others is an aid to critical thinking and helps practice problem solving. For this reason, the researcher is interested to study the Mathematics teaching process that applies the open-ended approach and jigsaw method in order to create an example lesson plan.

1.2 Purpose of the study

- 1) To create a lesson plan of an open-ended problem using jigsaw method
- 2) To experiment the lesson plan in junior high school in Japan
- 3) To compare between expected and empirical result in class experiment

1.3 Study questions

- 1) What are the characteristics of open-ended approach and jigsaw method?
- 2) How to create lesson plan of open-ended problem using jigsaw method is?

1.4 Definition of terms

- 1) The open-ended problem means a water-beaker problem in case it is tilted with one fixed edge.
- 2) The water-beaker means a transparent rectangular prism container with 12 cm long, 5 cm wide, and 12 cm high and marked on the sides with lines to indicate the height of water.

1.5 Expected outcome

- 1) The researcher understands the open-ended approach and the teaching by using jigsaw method.
- 2) The researcher can create the lesson plan in the way of the open-ended approach.
- 3) The researcher experiences and learns in class experiment.

2. Review of Related Literatures

2.1 Open-ended problem

Open-ended problems or incomplete problems are the problems formulated to have multiple correct answers and asked to focus on different methods, ways, or approaches to getting an answer. (Isoda, 2007) The open-ended problems were classified into three types: (1) Finding relations problems: students are asked to find some mathematical rules or relations, (2) Classifying problems: students are asked to classify according to characteristics, and (3) Measuring problems: students are asked to measure a certain phenomenon. (Sawada, 1997)

2.2 Open-ended approach

Open-ended approach is an approach of learning which an open-ended problem is presented first and then many correct answers of it will provide experience in finding something new in the process. This can be done through combining students' own knowledge, skills, or ways of thinking that have been learned. (Shimada, S., & Becker, J.P., 1997)

2.3 Jigsaw method

Jigsaw method is one of technique in cooperative learning. The learning process was started by placing the students in groups of 5-6 students which are called "Jigsaw groups". The material is separated into 5-6 parts for each member. Students studying the same part will join in groups which are called "Expert groups" and then start helping each other to understand the material and decide how to teach the material to the jigsaw groups. In this way, each student becomes an expert in different content. After finished working in expert group, the students reassemble in their jigsaw groups and teach them each other. The students have to depend on one another to learn all their material. This process made it imperative that the students treat each other as resources. (Aronson and Patnoe, 1997)

2.4 Creative thinking

Three criteria are regarded as the basis of creative thinking as follows: (1) Fluency: the ability to generate quantities of ideas, (2) Flexibility: the ability to create different categories of ideas, and (3) Originality: the ability to generate new, different, and unique ideas. (Karakelle, 2009)

3. Methodology

3.1 Instruments Creation

The Lesson plan of a water-beaker problem is about solving the water-beaker problem,

“When the beaker whose dimension is 5 cm by 12 cm by 12 cm with water 4 cm high was tilted with one fixed edge, what would happen with water?” for 1 period (50 minutes) and use the jigsaw method to carry on the class activity. The teaching process consists of 4 steps: 1) Introduction and problem presentation (10 minutes): Students predict to problem by individual, 2) Expert activity (20 minutes) separated to 2.1) Brainstorming (5 minutes): Students are divided into groups of 4 or 5 students to brainstorm what they want to examine for change or consistency of water from only one view (Front/Top/Side-view) depended on color of worksheet. 2.2) Sharing (5 minutes): Students regroup to groups of 6-8 students to share viewpoints of their expert group. 2.3) Experiment (10 minutes): Students regroup to groups of 3 students to examine by experiment, 3) Jigsaw activity (10 minutes): Students regroup to group of 3 students of 3 views (Front, Top, Side-view) to explain their findings and 4) Summary (10 minutes)

Worksheets are 2 types of worksheets: 1) Individual worksheets for 3 kinds of expert group: F-worksheet for expert in front view, T-worksheet for expert in top view and S-worksheet for expert in side view. Students have to write: Predictions, Viewpoints and data, Conclusions of 1 view. 2) Group worksheet for jigsaw group. Students have to write: Conclusions of 3 views, Conclusions of problem.

Microsoft Powerpoint file is a tool to facilitate teaching process and show seat diagrams of expert group which consist of 3 sub-activities: Brainstorming, Sharing and Experiment, and seat diagram of jigsaw group.

3.2 Lesson plan experiment

The researcher tried out the lesson plan of water-beaker problem with 2nd year junior high students of Saitama prefecture junior high school for 2 periods: Japanese teacher and the researcher.

3.3 Result summarization

Result was considered from 2 evidences: worksheet and video tape in 2 issues: (1) Comparison of expected result and empirical result (2) Comparison of difference of 2 classes

4. Result

4.1 Comparison of expected result and empirical result

About prediction, students must have the capacity to fully understand the problem. They may create a drawing of the problem, creating a visual representation of the problem and predict what the answer to the problem will be. In class, students perceived the volume of water is constant and discovered a variation of water height which is the height of water at the tilted side increases but the opposite side decreases. They are important foundation conditions to generate others. But few students could not predict. It may be caused by they didn't understand the situation of problem and need more time to think.

About observation, students are divided to observe the beaker from only one view: front or top or side view depending on the color of their worksheet but most students often observed the

beaker in front view. Thus the teachers should emphasize students to realize their observed direction and should make the colored scale at the beaker along with the colored worksheet.

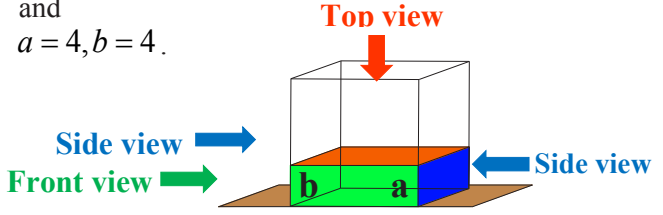
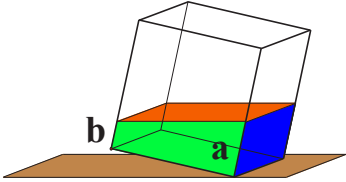
About viewpoint, students have to brainstorm to find viewpoints for experiment from their predictions, write them in the first column of the table and then share and write down the different viewpoints. But students confused about where to write the viewpoints so the teacher should adjust the worksheet to specify places for answer clearly.

About experiment, students are required to do the experiment and collect data which may be obtained by measuring or computing. In class, most students collected data by measuring so that teachers should encourage students draw a model of tilted beaker to help seeing the perspective in order to compute others. Furthermore, it should be noted that the students were 2nd year junior high student and still haven't studied about irrational number such as square root of 2, Pythagorean theory. Therefore they could not find some exactly detail. If they have known about them, they can find something more. On the other hand, they could find many changes about ratio, directly and inverse proportional which they have already studied. It means they could integrate their mathematical knowledge.

About conclusion, students are expected to summarize the results about what changed or unchanged and how it changed in thoroughly sentence or mathematical expression but in class, they could not explain ideas clearly and finely so the teacher should emphasize students to add the detail of change clearly and should extend this lesson into 2 periods in order to there are enough time to explain all findings thoroughly. The expected result and students' conclusion of problem were shown in Table 1 and 2.

About creative thinking skills, the result shows that students' thinking ability is satisfactory in flexibility but unsatisfactory in fluency and originality. They should be offered the improvement because these skills are important and beneficial effect on their lives.

Table 1. Expected Result

Experiment	Expected results
<p>1. The beaker with 12 cm long, 5 cm wide, and 12 cm high filled with water 4 cm high is placed on the table and $a = 4, b = 4$.</p> 	<p>1. Water gradually changes from a rectangular prism to a trapezoidal prism to a right triangular prism. 2. Water volume is 240 cm^3. 3. Observe the front: 1) Shape changes from a rectangle to a trapezoid to a right triangle to an isosceles triangle and a right triangle. 2) $a + b$ is 8 when $4 < a < 8, 0 < b < 4$. 3) Area always is 48 cm^2. 4) a and c is inversely proportional when $a > 8$. 5) Shape is an isosceles triangle when $a = c = \sqrt{96} = 4\sqrt{6}$.</p>
<p>2. The beaker with water is tilted with fixed right edge, angle of tilt is very small and $4 < a < 8, 0 < b < 4$.</p> 	<p>4. Observe the top: 1) Shape always is a rectangle.</p>

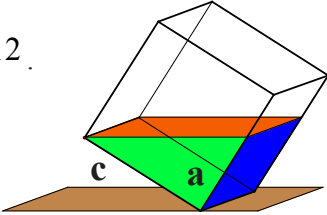
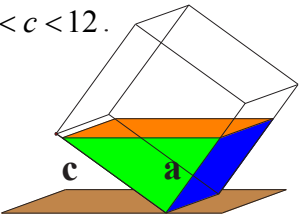
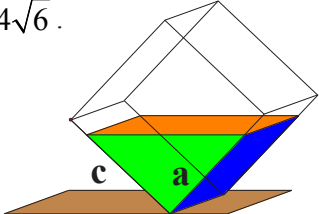
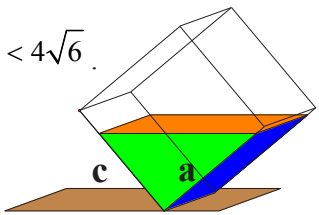
Experiment	Expected results
<p>3. The beaker with water is tilted with fixed right edge until $a = 8, b = 0, c = 12$.</p> 	<p>2) Area increases when $4 < a < 8$. 3) Area is a maximum when $a = 8$. 4) Area decreases when $8 < a < 4\sqrt{6}$. 5) Area is a minimum when $a = 4\sqrt{6}$. 6) Area increases if $a > 4\sqrt{6}$</p>
<p>4. The beaker with water is tilted with fixed right edge until $8 < a < 4\sqrt{6}, 4\sqrt{6} < c < 12$.</p> 	<p>5. Observe the side: 1) Shape in right changes from a rectangle to a square to a rectangle. 2) Shape in left changes from a rectangle to a line and then it disappears. 3) Summation of area of both side planes is a constant when $4 < a < 8, 0 < b < 4$.</p>
<p>5. The beaker with water is tilted with fixed right edge until $a = 4\sqrt{6}, c = 4\sqrt{6}$.</p> 	<p>4) When $a > 8, c = \frac{96}{a}$ and $d = 12 - \frac{96}{a}$ so $b = \frac{a \cdot d}{c} = \frac{a^2}{8} - a$.</p>
<p>6. The beaker with water is tilted with fixed right edge until $4\sqrt{6} < a < 12, 0 < c < 4\sqrt{6}$.</p> 	

Table 2. Conclusion of Problem

Group	2 nd Year Class 1: Japanese teacher	2 nd Year 8 Class 4: Researcher
1	<p>Unchangeable thing</p> <ul style="list-style-type: none"> - The volume of water - The area of water (The area of front plane) <p>Changeable thing</p> <ul style="list-style-type: none"> - The area (The area of surface) - The ratio of the height of both sides - The height of both sides 	<p>Area of front plane changes because it increases by 6cm. Area of top plane does not change. Area of two side planes changes because the area of side-plane (a) increases 5 cm. and side-plane (b) decreases.</p>
2	<p>The area changes in various viewpoints. The shapes of water in top view and side view do not change but it becomes the triangle in front view. The ratio of area in side view and the ratio of depth of water are same.</p>	<p>From the front view, the area does not change. If the water tilts by 1 cm, the angle of tilt increases by 10 degrees. In the middle of it, it increases by 5 degrees. From the top view, the area changes from 9 cm.</p>

Group	2 nd Year Class 1: Japanese teacher	2 nd Year 8 Class 4: Researcher
2		<p>From the side view, the right-scale (a) increases when the beaker is tilted. The area of right-side-plane increases too.</p> <p>From the side view, the left-scale (b) decreases when the beaker is tilted. The area of left side-plane decreases too.</p> <p>From the side view, the summation of right-side area and left-side area is same.</p>
3	<p>The ratio of area of both sides and the ratio of height of both sides are same.</p> <p>The volume of water is always constant.</p>	
4	<p>Changeable things : The shape of water, a+b, The angle of tilt</p> <p>Unchangeable things : The area of surface, The volume of water</p>	<p>The area of water in top view</p> <p>The total of (a) scale and (b) scale when the beaker is tilted with the right edge</p> <p>From the side view, if the height changes and then the area will change.</p>
5	<p>The summation of right and left scale is 8 cm.</p> <p>From top view, the shape of surface does not change but from other sides, the shape of water planes change.</p> <p>From side view, the area of water planes change.</p> <p>The volume of water does not change.</p>	
6	<p>From top view, the area of surface does not change. From front view, the shape of front plane changes to a triangle at 8 cm. From side view, the shape of side planes becomes a rectangular shape at 4cm.-7cm. From top view, the shape of water does not change even if the beaker is tilted.</p>	<p>The area in front view does not change.</p> <p>The area in top view is constant until 35 degrees but it becomes small at 40 degrees.</p>
7	<p>Unchangeable thing</p> <p>From top view, all about water do not change. From front view, a+b and the area of water plane From side view, the shape and the area of water planes</p>	<p>The area in front view does not change.</p> <p>The volume in front view does not change.</p> <p>The shape of water in front view changes from a rectangle to a trapezoid and a right triangle.</p> <p>The surface area in top view does not change.</p> <p>The shape of water in top view does not change.</p> <p>The tilted side</p>
8	<p>Unchangeable thing</p> <ul style="list-style-type: none"> - The summation of right scale and left scale when looking from front and top view. - The area of surface does not change. <p>Changeable thing</p> <ul style="list-style-type: none"> - From side view, left scale and the summation of right and left scale - The right scale increases by 1cm. when the beaker is tilted. - The left scale decreases in order of 0.5cm. → 1.5cm. → 0.5cm. → 1.5cm. 	<p>From the top view, the area changes.</p> <p>Seeing from the right, the area increases by 5 cm²</p> <p>Seeing from the left, the area decreases by 5 cm²</p> <p>It becomes 8cm by all means.</p>

Group	2 nd Year Class 1: Japanese teacher	2 nd Year 8 Class 4: Researcher
8	- The summation of right and left scale increases 0.5 cm. and decreases 0.5 cm. and increases 0.5 cm. and decreases 0.5 cm.	
9	Unchangeable thing is the volume of water.	
10	Changeable thing - The shape of water in front view and side view Unchangeable thing - The area of surface - The shape of water in top view	Although the beaker is tiled, the area in front view does not change. It is 48 cm ³ . The shape of surface does not change. It is a rectangle.
11	The area and a+b are not change. The shape in front view and side view changes. The shape in top view does not change. B decreases by 1cm. when A increases 1cm. in a single direction.	From front view, top view or side view The length of right scale and left scale does not change. - The angle of tilt increases by 10 degrees. - The volume of water does not change.
12	Changeable thing Top: The area of surface changes. Front: The summation of a and b changes from 9 cm. Side: The length of a+b changes in the range 8cm.-9cm. Unchangeable thing	From the front view, a+b = 8. From the front view, The angle of tilt increases by 10 degrees. From the front view, the distance between the beaker and the desk increases by 2 cm. From the front view, the area of beaker without water is constant.
12	Top: The shape does not change. It remains be the rectangle. Front: a+b does not change within 8cm. Side: The shape of except 6cm. does not change.	From the side view, a+b = 8. From the side view, the volume of water in one side increases but the other one decreases. (The area)

Table 3. The Comparison of Expected and Empirical Result

Consideration	Expected result	Empirical result	Cause	Improvement
1. Prediction	- Understand problem - Create drawing of problem - Predict answers	- Wrote 1-2 predictions about volume and variation of water height - Some students couldn't predict anything.	- Don't understand problem - Time is too short.	- Explain problem several time and use guiding questions - Extend to 2 periods
2. Observation	- Observe from own direction.	- Observed from front view mostly	- The front plane is observed most easily.	- Emphasize student to explore in own direction - Scale should be same color with worksheet.
3. Viewpoint	- Identify many viewpoints completely	- Wrote 1-4 viewpoints about shape, area, volume and angle of incline mostly.	- Those are observed most easily.	- Emphasize student to use previous mathematical knowledge

Consideration	Expected result	Empirical result	Cause	Improvement
4. Experiment	Be able to do experiment by measuring and mathematical computing	- Mostly explored by measuring	- Always use tools such as protractor and ruler	- Encourage them to draw model of tilted beaker to help seeing the perspective in order to compute others
5. Conclusion	Be able to conclude results of experiments by using sentence or mathematical expression	- Founded out what's changed and what has not changed but could not explain how to change or do not change.	- Have not enough fundamental knowledge - Time is too short.	- Emphasize student to explain the change or consistency clearly - Extend to 2 periods

4.3 Comparison of differences between two classes

About relationship and communication, the obliging relationship between teacher and students will make the class atmosphere be relaxed and students could show their opinions freely and enjoy in class. Teacher should pay attention to make familiar relationship with the students at the beginning and must be able to communicate with students naturally and explain simply so the learning process could be done smoothly. Communication skill is a very indispensable and important skill that teacher have to improve.

About learning activities, teacher showed a beaker with water and asked what you want to do with this beaker. The question is more opened question. The more openness of question teacher asked may be the greater of the challenge and interesting of question. Moreover, if teacher provided students make a condition of question by themselves, students were engaged to the problem gradually and could understand the problem more deeply. In addition, teacher also should repeat about prediction that don't write anything is the thing you should not do because writing the predictions provides students an opportunity to examine the problem thoroughly. Japanese teacher always repeated the question several times and wrote it on the blackboard in order to keep students' mind to concentrate on the problem. Furthermore, he always repeated student's answers aloud in order to all students could get completely all viewpoints. In each regrouping, students had specified to spend only 10 seconds to regroup so they didn't waste more time for it. Eventually, Japanese teacher chose some groups as representative to announce the answers and repeated all answers in order to let students to understand them thoroughly.

Table 4. The Comparison of Differences between Two Classes

Aspect	Consideration	Researcher	Japanese teacher	Necessity
Teacher and student	1. Relationship	- Feel unfamiliar and be nervous in class	- Feel familiar and be relax in class	focus on making familiarity
	2. Communication	- Explanations were not fine and thorough.	- Communicate naturally and explain simply	focus on improving communication skill

Aspect	Consideration	Researcher	Japanese teacher	Necessity
Learning activity	1. Introduction and prediction	<ul style="list-style-type: none"> - Problem was not created from idea of student. - Tilted beaker was shown. 	<ul style="list-style-type: none"> - Problem was created from idea of student. - Tilted beaker was not shown. - What you should do 	Clearly explanation
Learning activity		<ul style="list-style-type: none"> - What happens if beaker is tilted like that? - Did not suggest student to think about mathematical concepts. - Did not reiterate student to write prediction 	<ul style="list-style-type: none"> with the beaker? - Suggested student to think about mathematical concepts. - Reiterated student to write prediction 	
Learning activity	2. Jigsaw activity	<ul style="list-style-type: none"> - Did not reiterated about what students have to do - Do not specify clearly time for each activity. - Did not repeated student's answers out loud 	<ul style="list-style-type: none"> - Reiterated about what students have to do and wrote it on the blackboard - Specify clearly time for each activity. - Repeated answers out loud 	Repeat the goal of lesson and give feedback immediately.
	3. Summarization	<ul style="list-style-type: none"> - Did not summarize the answers 	<ul style="list-style-type: none"> - Some groups as representative to announce the answer - repeated all answer 	Repeat the conclusion

5. Conclusion

One of the researcher's findings in the study on the open-ended approach was that open-ended Mathematics problems provide students tasks which are enjoyable and challenging which in turn gives them desire to solve the problems because they have an opportunity to find their own way of thinking. Thus this approach motivates students' learning as well. Furthermore, the open-ended problem mostly relates with real life which can encourage students to live curiously in everything. They not only learn in class but also learn in real life. On the contrary, there are some obstacles in this approach as whether some students cannot think any idea or ideas do not correspond with expectations, the teacher should prepare various expected answers agree with students' ability and suggest them consider prior mathematical knowledge. Students can learn up to any level depend on their ability. When they have studied more foundation knowledge, they can solve the problem thoroughly. After students present their answers, teacher should repeat all answers in order to confirm the understanding of all students.

Regarding the jigsaw activity, the researcher found that students were happy and enjoyable in learning by themselves because they learn by doing instead of listening to lectures. Students have learnt to collaborate with others and improve their communication skills: speaking and listening.

In addition they have to consolidate their understanding and thought in order to explain the findings to others. On the other hand, there are some obstacles in the jigsaw activity. For example, students spend more time to move to their group so the teacher should make an agreement about limited time for grouping, students not only talk and play a lot in group but also don't attend to activity. The teacher should create an interesting problem in order to motivate them. Moreover some students don't like to speak in front of the class. Teacher should provide them an explained time for a short. If they are accustomed to speak in class, they can speak finely.

Mathematics teaching in Thailand should be adjusted to utilize the open-ended approach in order to provide students generate method to find answers by themselves instead of describing the method by teacher which students cannot improve their thinking skill. Moreover teacher must practice to create the open-ended problem by changing the question word to "if....then what happens can be?"

References

- Aronson, E., & Patnoe, S. (1997). *The jigsaw classroom building cooperation in the classroom*. (2nd ed.). New York: Addison Wesley Longman.
- Ikeda, T. (2010). Roots of the open-ended approach. *Journal of Japan society of mathematical education*, 92, 88-89.
- Inprasitha, M. (2006). Open-ended approach and teacher education. *Tsukuba Journal of Educational Study in Mathematics*, 25, 169-177. Retrieved October 8, 2014, from <http://www.human.tsukuba.ac.jp/~mathedu/2514.pdf>
- IPST. (2008). *The basic education core Curriculum (Mathematics)*. Retrieved October 5, 2014, from http://www.ipst.ac.th/web/images/stories/files/Curriculum/MathCur_2008_EngVersion.pdf
- IPST. (2012). *PISA 2012 Assessment in mathematics reading and science : an executive summary*. Retrieved October 5, 2014, from <http://pisathailand.ipst.ac.th/report/pisa-2012/pisa2012ExecutiveSummary.pdf>
- Isoda, M. (2007). Section 2: a brief history of Mathematics lesson study in Japan. In M. Isoda, M. Stephens, Y. Ohara, & T. Miyakawa (Eds.), *Japanese lesson study in Mathematics Its impact, diversity and potential for educational improvement* (pp.8-15). Singapore: World Scientific Publishing.
- Isoda, M. (2012). Introductory chapter: problem solving approach to develop mathematical thinking. In K. Stacey, D. Tall, M. Isoda, & M. Inprasitha (Series Ed.), *Monographs on lesson study for teaching mathematics and sciences: Vol.1. Mathematical thinking how to develop it in the classroom* (pp.1-28). Singapore: World Scientific Publishing.
- Karakelle, S. (2009). Enhancing fluent and flexible thinking through the creative drama process. *Thinking Skills and Creativity*, 2009, 124-129. Retrieved December 25, 2014, from <http://www.tlsfrankfurt.com/tls/wp-content/uploads/2011/08/Enhance-Flexible-Thinking-Adults.pdf>
- Mengduo, Q., & Xiaoling, J. (2010). Jigsaw strategy as a cooperative learning technique: Focusing on the language learners. *Chinese Journal of Applied Linguistics (Bimonthly)*, 33, 113-125. Retrieved December 25, 2014, from <http://www.celea.org.cn/teic/92/10120608.pdf>
- Murni. (2013). Open ended approach in learning to improve students thinking skill in Banda Aceh. *International Journal of Independent Research and Studies*, 2, 95-101. Retrieved October 5, 2014, from

- <http://pakacademicsearch.com/pdf-files/art/68/95-101%20Vol.%202,%20No.2%20> (April,%202013).pdf
- Rand, A. (2002). *Education*. Retrieved December 25, 2014, from <http://aynrandlexicon.com/lexicon/education.html>
- Sarnat, M. (2011). *The powerful fours of creative thinking*. Retrieved January 19,2015, from <http://www.jrimagination.com/blog/2011/11/11/the-powerful-fours-of-creative-thinking.html>
- Sawada, T. (1997).Developing Lesson Plans.In S. Shimada, & J.P. Becker. *The open-ended approach: A new proposal for teaching mathematics* (pp.23-35). Virginia: National Council of Teachers of Mathematics.
- Secretariat of Education Council. (2007). *A report of synthesis in situation and factor affecting educational quality of thailand*. Bangkok: Office of Education.
- Shimada, S., & Becker, J.P. (1997). *The open-ended approach: A new proposal for teaching mathematics*. Virginia: National Council of Teachers of Mathematics.
- Thinwiangthong, S., Inprasitha, M., &Loipha, S. (2012). Adaptation of lesson study and open approach for sustainable development of students' mathematical learning process. *Scientific Research*, 3, 906-911. Retrieved October 8, 2014, from <http://dx.doi.org/10.4236/psych.2012.310136>

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