A Control of Biped Robot which Applies Inverted Pendulum Mode with Virtual Supporting Point

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Abstract: We applied inverted pendulum mode to the control of biped robot in order to plan a trajectory on real time. Though inverted pendulum mode is useful to plan a trajectory, the stability margin of biped robot is very narrow because of its structural limit. Virtual Supporting Point (VSP), a new indicator of biped robot control, is suggested with a view to broaden the stability margin of biped robot. Walking parameters could be adjusted by setting VSP without changing other parameters. The mobility of biped robot will be improved compare to usual method if proper VSP is set. The stable range of VSP is figured out by revealing the relationship between ZMP and VSP. Additionary, biped locomotion could be designed simply and its stability could be determined clearly beforehand by applying VSP. Results of the simulation and the experiment shows the validity of the suggested method.

1 Introduction

It is desirable that biped robot works in the human surroundings in the future. Biped robot has a structural ability to reply to complex environments compare to other mobile robots. Additionally, its physical structure is similar to human. To improve the mobility of biped robot in human surroundings is the main purpose in this paper.

There are many researches about the control of biped robot taking environment into account. For example, there are methods which deals with the force interaction[1], which sets the command trajectory of ZMP[10], and which stabilizes the locomotion by trunk motion[2].

In human surroundings, large amount of disturbances like collision, slipping and walking on unleveled ground will occur. The trajectory should be planned passively to these kinds of disturbances so that the robot can keep its stability. Inverted pendulum mode is applied [3, 4, 5, 6] to calculate the trajectory which assures the stability of biped locomotion on real time.

Virtual Supporting Point (VSP), a new indicator of inverted pendulum mode locomotion, is suggested to improve the mobility of biped robot. It is shown how VSP could be applicated. The stability margin of biped locomotion using VSP is also shown.

The results of simulation and experiment show the validity of suggested method.

2 Modeling

The model of robot used in this paper is shown in Fig.1. x-coordinate denotes lateral direction, y-coordinate denotes traveling direction and z-coordinate denotes vertical direction. The measured parameters of the robot are shown in Table 1.

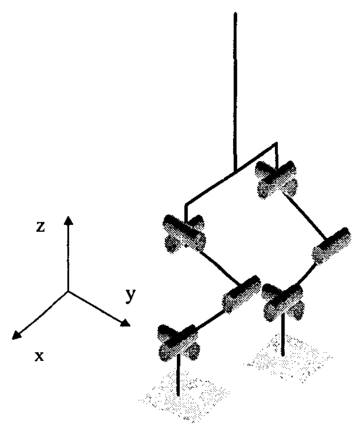

Fig.1: Model of biped robot

Table 1: Values of robot parameters

	Mass (kg)	Length (m)	CoG from upper joint (m)
Trunk	15.0	0.40	0.200
Waist joint link	2.6	0.00	-0.020
Upper link of leg	3.5	0.30	0.126
Lower link of leg	3.4	0.30	0.095
Ankle joint link	2.6	0.00	-0.020
Foot link	2.0	0.12	0.023

We used the 3D biped robot which has 6 DoF in sagittal plane and 4 DoF in frontal plane. The degree of freedom adds up to be 10 DoF. The robot is assumed as a mass system which has its mass on the CoG of each link. In the usual method, the foot link of the support

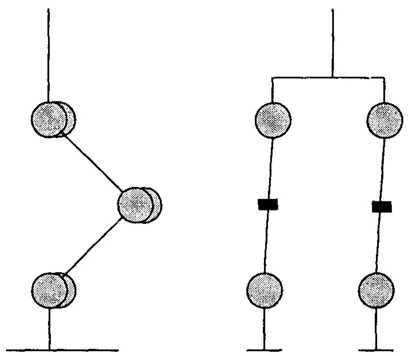

Fig.2: Sagittal plane and frontal plane

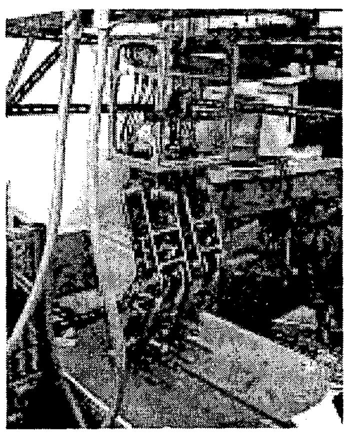

Fig.3: Photo of biped robot

leg is assumed as a base link fixed to the ground. In this paper, the trunk link is set to be a base link and floor is regarded as a part of environment in order to realize a falling in simulation. Its dynamics is calculated on Eq.(1)[8].

$$\begin{bmatrix} \boldsymbol{H}_b & \boldsymbol{H}_{bm} \\ \boldsymbol{H}_{bm}^T & \boldsymbol{H}_m \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{x}}_b \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} \boldsymbol{c}_b \\ \boldsymbol{c}_m \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_b \\ \tau \end{bmatrix} + \begin{bmatrix} \boldsymbol{J}_b^T \\ \boldsymbol{J}_m^T \end{bmatrix} \boldsymbol{F}_h \tag{1}$$

where $\boldsymbol{H}_b$, $\boldsymbol{H}_{bm}$, $\boldsymbol{H}_m$ are inertia matrix, $\boldsymbol{x}_b$ is the absolute coordinate of base link, $\boldsymbol{c}_b$, $\boldsymbol{c}_m$ are non-linear terms, τ is torque on joints, ϕ is angle of joints, $\boldsymbol{J}_b$, $\boldsymbol{J}_m$ are the Jacobean matrix of base link and joints respectively, $\boldsymbol{F}_b$, $\boldsymbol{F}_h$ are forces occur to the base point and end effector.

3 DoF of swing leg tip position, 2 DoF of swing leg tip posture, 3 DoF of CoG position, 2 DoF of trunk posture are controlled respectively. The postures of base link and swing leg tip have only 2 DoF despite the posture in 3D space should have 3 DoF. The robot in this paper doesn't have the degree of freedom on the turning direction. Therefore we control the 2 DoF except the turning direction.

Many biped robot controls which apply inverted pendulum mode let the ankle joint of support leg free. On the contrary, we control every joint including ankle joint of support foot so that no torque should occur on the supporting point of the pendulum.

3 Linear Inverted Pendulum Mode

This paper aims to improve the mobility of biped robot locomotion in human surroundings. In human surroundings, large disturbances exist. To keep the posture stable against disturbances, its trajectory should be planned on real time. Linear inverted pendulum mode(LIPM)[3] is applied in this method so as to plan a trajectory on real time.

The robot is modeled as an inverted pendulum. The pendulum has its mass on the CoG. In the usual method, the supporting point of the pendulum is set on the ankle joint of support leg. Though our main insistence is to set the support point to somewhere else virtually, we explain the usual method of LIPM first. The trajectory is controlled so that the height of the pendulum becomes constant. At the same time, the trajectory is controlled so that no torque occurs on the supporting point. This kind of a trajectory could be figured out uniquely from the dynamics of inverted pendulum.

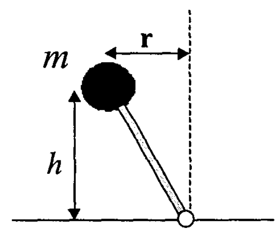

Fig.4: Inverted pendulum

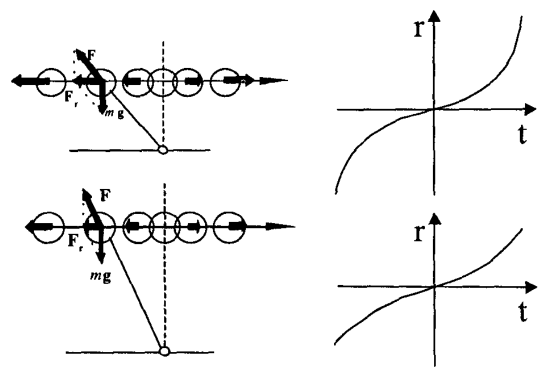

Fig.5: Length and acceleration

$$r = \frac{r_0 - \sqrt{\frac{h}{g}}\dot{r}_0}{2} e^{-\sqrt{\frac{g}{h}}t} + \frac{r_0 + \sqrt{\frac{h}{g}}\dot{r}_0}{2} e^{\sqrt{\frac{g}{h}}t} \tag{2}$$

$$\dot{r} = \frac{\dot{r}_0 - \sqrt{\frac{g}{h}}r_0}{2} e^{-\sqrt{\frac{g}{h}}t} + \frac{\dot{r}_0 + \sqrt{\frac{g}{h}}r_0}{2} e^{\sqrt{\frac{g}{h}}t} \tag{3}$$

where r is the 2 dimensional position vector of CoG on level plane based on supporting point, r_0 is the initial value of r, g is the gravity, h is the height of pendulum, t is time.

If the robot follows this trajectory, the support foot surface can keep the contact with ground, because no torque occurs on the supporting point.

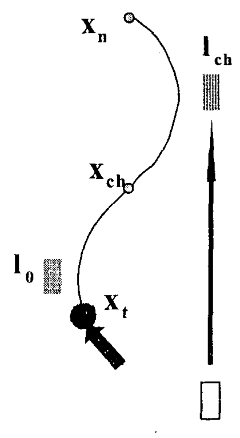

Fig.6: Trajectory of CoG

Essentially, inverted pendulum is a system which is statically unstable. Biped locomotion could be sustained dynamically stable by repeating steps. The trajectory switches after the step because the supporting point changes. The trajectory after the step could be figured out extending Eq.(2) and Eq.(3).

$$\begin{aligned} x_n = {} & \frac{x_{ch} - l_{ch} - \sqrt{\frac{h}{g}}\dot{x}_{ch}}{2} e^{-\sqrt{\frac{g}{h}}(t-t_0)} \\ & + \frac{x_{ch} - l_{ch} + \sqrt{\frac{h}{g}}\dot{x}_{ch}}{2} e^{\sqrt{\frac{g}{h}}(t-t_0)} \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{x}_n = {} & \frac{\dot{x}_{ch} - \sqrt{\frac{g}{h}}(x_{ch} - l_{ch})}{2} e^{-\sqrt{\frac{g}{h}}(t-t_0)} \\ & + \frac{\dot{x}_{ch} + \sqrt{\frac{g}{h}}(x_{ch} - l_{ch})}{2} e^{\sqrt{\frac{g}{h}}(t-t_0)} \end{aligned} \tag{5}$$

where x_n is the absolute position of the CoG after the robot steps, x_{ch} is the absolute position of the CoG when the robot steps, l_0 is current support foot position, l_{ch} is the position that the current swing leg steps on, t_0 is the time of the stepping motion. All the vectors in these equations are 2 dimensional vector on level plane.

Eq.(4) and Eq.(5) show the future trajectory and it depends on l_{ch} which could be controlled from the swing leg trajectory. The proper l_{ch} which satisfies the desired x_n or $\dot{x}_n$ is determined uniquely from Eq.(6) and Eq.(7).

$$l_{ch} = \frac{-\sqrt{\frac{h}{g}}\left(e^{-\sqrt{\frac{g}{h}}(t-t_0)} - e^{\sqrt{\frac{g}{h}}(t-t_0)}\right)\dot{x}_{ch} - 2x_n}{e^{-\sqrt{\frac{g}{h}}(t-t_0)} + e^{\sqrt{\frac{g}{h}}(t-t_0)}} + x_{ch} \tag{6}$$

$$l_{ch} = \frac{-\sqrt{\frac{h}{g}}\left(e^{-\sqrt{\frac{g}{h}}(t-t_0)} + e^{\sqrt{\frac{g}{h}}(t-t_0)}\right)\dot{x}_{ch} - 2\sqrt{\frac{h}{g}}\dot{x}_n}{e^{-\sqrt{\frac{g}{h}}(t-t_0)} - e^{\sqrt{\frac{g}{h}}(t-t_0)}} + x_{ch} \tag{7}$$

Controlling l_{ch}, the robot can achieve the command position or the command velocity discretely. If it is presumed that x-coordinate of x_{ch} is constant in steady locomotion, the walking term T will be figured out as follows.

$$T = -2\sqrt{\frac{h}{g}} ln \left(\frac{r_{0x} - \sqrt{\frac{h}{g}}\dot{r}_{0x}}{r_{0x} + \sqrt{\frac{h}{g}}\dot{r}_{0x}} \right) \tag{8}$$

where r_{0x} denotes the x-coordinate value of r when the robot steps.

4 Virtual Supporting Point

In this section, we suggest the idea of virtual supporting point(VSP), a new indicator of inverted pendulum mode locomotion, and explain the application of VSP.

Differentiating Eq.(3), the acceleration to the CoG inversely relates to the height of CoG. In the inverted pendulum mode locomotion, it is desirable for the pendulum to get higher because it is better that the velocity is constant. However, the height of CoG is limited by the structure.

Therefore we suggest a method which sets the supporting point virtually. The left figure of Fig.7 denote the usual method. As shown in the figure, the supporting point is set on the ankle joint of support leg which becomes free joint. The suggested method is that the supporting point is set somewhere else virtually instead of letting the ankle joint free. For example, the virtual supporting point could be set on the supporting surface or even under the ground, as shown in the center and right figure of Fig.7.

This inverted pendulum model is applied for trajectory planning and it doesn't interfere in the controller. The trajectory is planned so that no torque occurs on the supporting point.

Shifting the position of VSP, the height of the pendulum could be adjusted without the robot changing the posture. This advantage could be applied to speed adjustive locomotion and footstep locating. Furthermore, parameters of inverted pendulum could be adjusted beyond the structural limit. For example, the height of the pendulum could be set higher than the structural limit as shown in Fig.8.

Compare on the same average speed, the acceleration on CoG reduces and the walking term gets longer by using the higher inverted pendulum model. Therefore lower torque are needed and it will be able to walk faster. This shows that the mobility of biped locomotion improves by applying VSP.

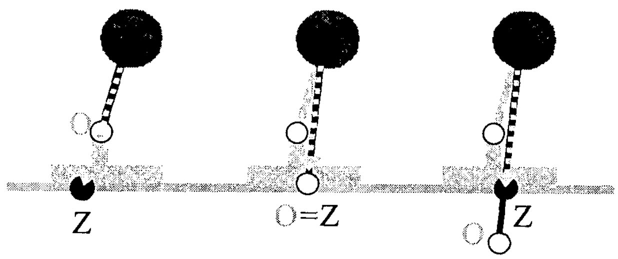

Fig.7: VSP and ZMP

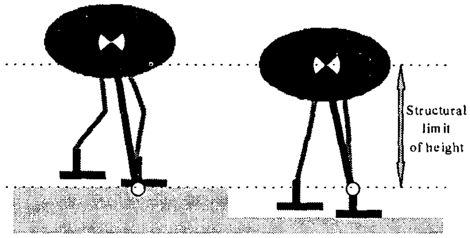

Fig.8: Structural limit

5 VSP and ZMP

The relationship between VSP and ZMP is shown in this section so that the stability of biped locomotion applying VSP could be figured out.

In the usual inverted pendulum mode, ZMP will be the point which extension of pendulum and foot surface cross. Similarly, ZMP will be the point which extension of pendulum and foot surface cross in this method because no torque should occurs on the extension. The theoretical value of $\boldsymbol{Z}$, which is the plane vector of ZMP position based on VSP, will be figured out as follows.

$$\boldsymbol{Z} = \frac{d}{h_g + d}\boldsymbol{r} \tag{9}$$

where h_g is height of CoG and d is depth of VSP.

The biped locomotion gets most stable when VSP is set on the support surface because ZMP and VSP in the model will match. This shows that the stability will be improved compare to the usual method if VSP is set in the proper position.

Fig.9 is the example of foot surface watching from above. ZMP will move as the CoG moves.

ZMP is an indicator to approve the grounding of foot surface. So if ZMP is in the supporting surface, walking stability is assured at the moment. To walk in safety, $\boldsymbol{r}$ shouldn't go further than the line which ZMP comes to the limit as shown in Fig.10. The stable area of CoG is figured out as follows.

$$if\ \ d > 0$$

$$\tfrac{h_g+d}{d} Z_{xmin} \le r_x \le \tfrac{h_g+d}{d} Z_{xmax} \tag{10}$$

$$\tfrac{h_g+d}{d} Z_{ymin} \le r_y \le \tfrac{h_g+d}{d} Z_{ymax} \tag{11}$$

$$if\ \ d < 0$$

$$\tfrac{h_g}{d} Z_{xmax} \le r_x \le \tfrac{h_g}{d} Z_{xmin} \tag{12}$$

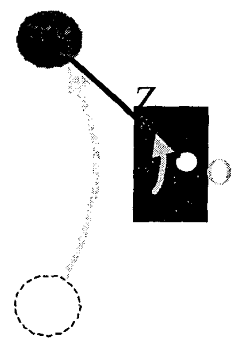

Fig.9: Movement of ZMP

$$\tfrac{h_g}{d} Z_{ymax} \le r_y \le \tfrac{h_g}{d} Z_{ymin} \tag{13}$$

where r_x is x-axis of $\boldsymbol{r}$, r_y is y-axis of $\boldsymbol{r}$, Z_{xmin} is minimum limit of $\boldsymbol{Z}$ x-axis, Z_{xmax} is maximum limit of $\boldsymbol{Z}$ x-axis, Z_{ymin} is minimum limit of $\boldsymbol{Z}$ y-axis, Z_{ymax} is maximum limit of $\boldsymbol{Z}$ y-axis.

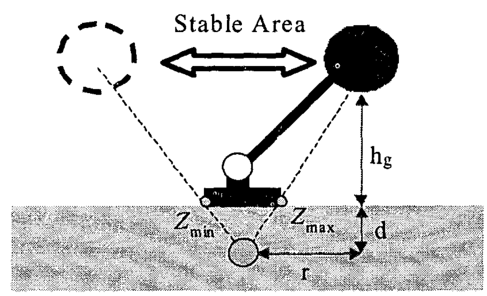

Fig.10: Stable area of CoG

On the contrary, if the step width of locomotion is decided, the area that VSP can be set for a stable locomotion could be figured out. The stable area of VSP which considers the motion in x-coordinate can be figured out as follows.

$$d \ge \frac{h_g Z_{xmax}}{r_{xmin} - Z_{xmax}},\ \ if\ r_{xmin} < Z_{xmax} \tag{14}$$

$$d \le \frac{h_g Z_{xmax}}{r_{xmin} - Z_{xmax}},\ \ if\ r_{xmin} > Z_{xmax} \tag{15}$$

$$d \ge \frac{h_g Z_{xmin}}{r_{xmin} - Z_{xmin}},\ \ if\ r_{xmin} > Z_{xmin} \tag{16}$$

$$d \le \frac{h_g Z_{xmin}}{r_{xmin} - Z_{xmin}},\ \ if\ r_{xmin} < Z_{xmin} \tag{17}$$

$$d \ge \frac{h_g Z_{xmin}}{r_{xmax} - Z_{xmin}},\ \ if\ r_{xmax} > Z_{xmin} \tag{18}$$

$$d \le \frac{h_g Z_{xmin}}{r_{xmax} - Z_{xmin}},\ \ if\ r_{xmax} < Z_{xmin} \tag{19}$$

$$d \ge \frac{h_g Z_{xmax}}{r_{xmax} - Z_{xmax}},\ \ if\ r_{xmax} < Z_{xmax} \tag{20}$$

$$d \le \frac{h_g Z_{xmax}}{r_{xmax} - Z_{xmax}}, \; if \; r_{xmax} > Z_{xmax} \tag{21}$$

The stable area which considers the motion in y-coordinate can be figured out similarly.

As shown in Fig.9, ZMP moves forward in each steps. Essentially, it is similar to the method which sets the command trajectory of ZMP. But there is an advantage that the future trajectory can be figured out and the future stability can be determined easily. Therefore VSP is very useful to plan a trajectory beforehand.

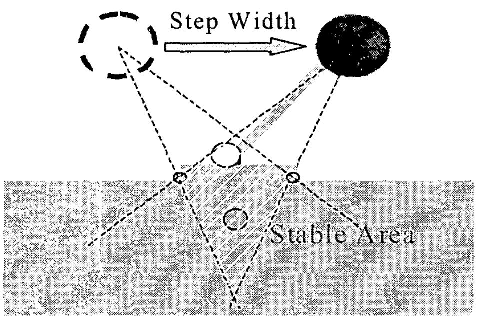

Fig.11: Stable area of VSP

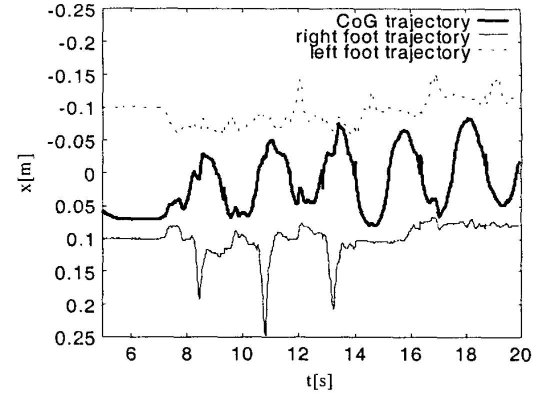

Fig.12: Biped locomotion(d=0.0)

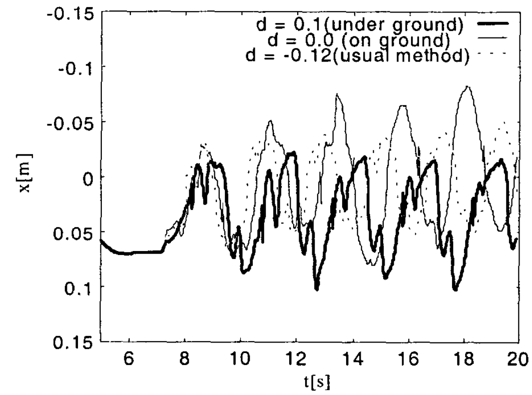

Fig.13: Adjusting term

6 Simulation and Experimental Result

Fig.12 shows the experimental result setting VSP on the support surface. The average speed was controlled to be 0.03[m/s]. Biped locomotion sustained when LIP-M with VSP was applied.

We compare the experimental result that the height of VSP was shifted in Fig.13 . d, the depth of VSP, was set -0.12, 0.0 and 0.1 respectively. By shifting VSP, we adjusted the walking term and other parameters like average speed, structural height of CoG, and so on were set to the same value. d=-0.12 denotes the result of the usual method since the height of ankle joint is 0.12[m]. The height of the inverted pendulum was 0.49[m] then. d=0.0 denotes the result when VSP was set on the ground and the height of the inverted pendulum was 0.61[m]. d=0.1 denotes the result when VSP was set under the ground and the height of the inverted pendulum was 0.71[m], which is beyond the structural limit. The biped locomotion could sustain when VSP was shifted in its safety boundary and walking term could be adjusted.

In order to compare the stability and mobility of the biped locomotion applying VSP, we show the simulation result of high speed walking in Fig.14.

The average speed was set to be 0.10[m/s] on each simulations. We used the same parameters of VSP with the experiment except the height of CoG. d was set to be -0.12, 0.0, 0.1 and the height of inverted pendulum was 0.346, 0.466, 0.566 respectively. The locomotion sustained when VSP was set on the ground and under the ground. On the contrary, the robot fell when it was controlled in the usual method because of the large acceleration which occurs on CoG. This result shows that biped locomotion could get stable when the model of inverted pendulum is adjusted applying VSP without changing its structural posture. Additionally, the inverted pendulum model could be adjusted beyond the structural limit.

The stability of biped locomotion improved when VSP is set to the proper position even if the model of inverted pendulum was same as shown in Fig.15. The robot fell in the simulation of usual method shown in Fig.14. We set the VSP on the ground and shifted CoG lower by changing the posture. As a result, the height of inverted pendulum became 0.346[m], which is same as the fallen simulation. The robot could walk in this simulation as shown in Fig.16. This result shows that walking gets much stable than the usual method setting VSP in the center of stability boundary of VSP.

7 Conclusion

We suggested the idea of virtual supporting point, which is a new indicator of inverted pendulum mode locomotion.

Walking parameters like stepping term could be adjusted without changing any other parameters by setting up VSP. Additionally, these parameters could be set beyond the structural limit of the robot. The stabil-

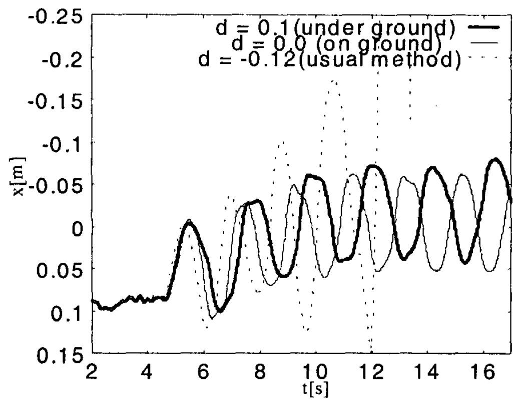

Fig.14: High speed walking simulation

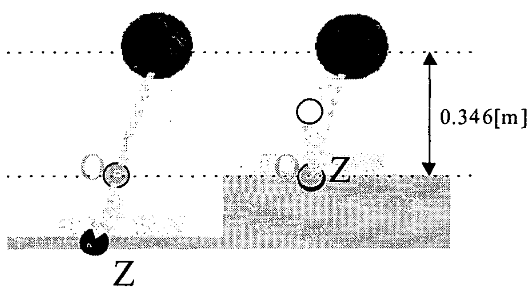

Fig.15: Same pendulum model

ity and mobility improved adjusting the inverted pendulum model.

We evaluated the stability of biped locomotion by formulating the relationship between VSP and ZMP. The biped locomotion got more stable than the usual method when VSP is set in the center of stability boundary of VSP.

VSP is useful to plan the biped locomotion ahead of time because future trajectory and future stability can be figured out easily.

We checked the validity of the suggested method by simulations and experiments.

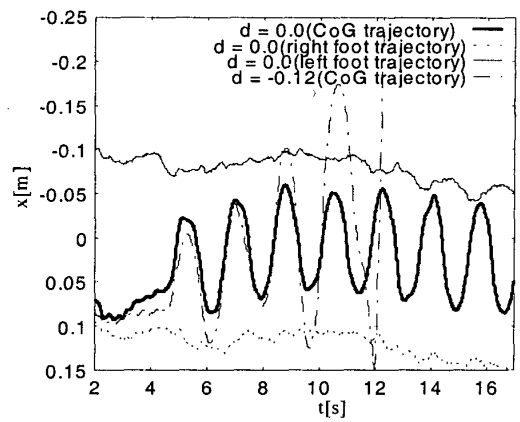

Fig.16: Same pendulum model

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