

# A Velocity Measurement Method for Acceleration Control

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**Abstract**— This paper proposes a novel method of velocity measurement for motor drives with optical encoders. Acceleration control is indispensable for advanced motion control while it requires accurate velocity information. Although many methods on velocity measurement have been proposed, accurate measurement was not achieved in a high-speed range. We therefore propose “synchronous-measurement method (S method)” that measures the velocity synchronized with alteration of pulse numbers. Accurate velocity measurement is achieved in all speed ranges with this method. Furthermore, other estimation methods are applicable in addition since the method does not require any model of a control object. Simulation and experimental results verify the validity of the proposed method.

## I. INTRODUCTION

Motion control is a fundamental subject for robots, vehicles and so on. There are a lot of strong demands on the control system with rapid response and high robustness. Acceleration control is indispensable for constructing this kind of system. Hence, many studies on motion control have focused on acceleration control. Disturbance observer[1] is useful to develop an acceleration control system with simple parameter modification.

An accurate and rapid velocity measurement is vital for high-performance acceleration control. The authors have proposed a multi-rate sampling method with a shorter output sampling period to acquire velocity information rapidly[2]. The velocity is often derived from position obtained by an optical encoder. There are many results on velocity measurement or estimation by an optical encoder. Among them, there are two commonly used methods: M method and T method. M method, also called fixed-time method, counts the number of pulses from the optical encoder during a fixed interval of time and calculate velocity by finite-difference derivative. On the other hand, T method, or fixed-position method, calculates velocity as the interpulse angle divided by the time between sequential pulses. Accuracy deteriorates in a low-speed range with M method, while T method achieves high accuracy. T method, however, is applicable only to the low-speed range. Ohmae, Matsuda, Kamiyama and Tachikawa [3] proposed M/T method which works in all speed ranges and has a high accuracy in the low-speed range. The method has been applied in many studies since it is effective for practical use. This method is extended to a system termed constant sample-time digital tachometer (CSDT) [4]. It is

more easily incorporated into a controller operating with a constant sample time.

Velocity estimation method with Kalman filter improves the velocity standard deviations [5]. Instantaneous speed observer [6], a discrete-time observer to grasp the velocity between the encoder pulses, is an effective tool for accurate velocity estimation. These methods, however, require plant models.

The velocity measurement method in this study should satisfy the following terms.

- high accuracy
- wide speed range
- rapid response
- measurement without any models

Above all, this study lays weight on “high accuracy” because acceleration control badly requires accurate velocity measurement. Although M/T method almost satisfies the terms, it only has high accuracy in a low speed range. It was believed for long time that no one estimator algorithm is best for a system with a large dynamic range of speeds, large transients, and an imperfect encoder[7]. This paper, however, proposes a velocity measurement method with high accuracy in all speed ranges.

Contents of this paper are as follows: Section II is a description of the experimental setup in this study. In Section III, the mechanism of acceleration control is described to show why acceleration control badly requires accurate velocity measurement. Section IV shows conventional methods of velocity measurement. Their resolution and measurement time are also introduced. The proposed method is described in Section V. The validity of the proposed method is verified from simulation and experimental results in Section VI and Section VII. Finally, this paper is concluded in Section VIII.

## II. EXPERIMENTAL SETUP

Fig. 1 shows an overview of the experimental system in this study. It is a 1 DOF flywheel driven by a DC motor. An optical encoder generates pulses in proportion to arm displacement. A counter board counts pulses from the optical encoder and PC reads a pulse number from the board. Here, pulse number denotes the number of pulse signals on every sampling period  $T_s$ .  $T_s$  is controlled to be constant with the real-time architecture of RT-Linux. We compared

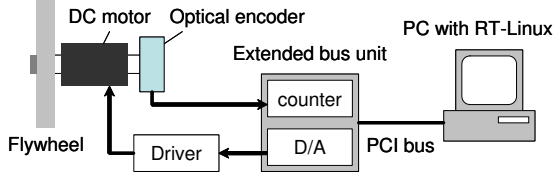


Fig. 1. Overview of experimental setup

TABLE I  
EXPERIMENTAL PARAMETERS

Sampling period	[ms]	1.0
Flywheel MOI	[Kg m <sup>2</sup> ]	0.003
Type of motor		Maxon RE40
Stall torque	[mNm]	2500
Torque constant	[mNm/A]	60.3
Type of optical encoder		Maxon HEDS 5540
Pulse per revolution	[pulse]	500

performance of the proposed method with other methods on this system.

The parameters on the experimental setup are shown in Table I.

### III. ACCELERATION CONTROL

#### A. Acceleration Control System with Disturbance Observer

This subsection describes a mechanism of acceleration control. This study applies disturbance observer as basic technique for acceleration control. Fig. 2 is the block diagram of disturbance observer. Here,  $\tau_l$  is a mechanical load,  $\hat{\tau}_{dis}$  is estimated disturbance torque,  $G_{dis}$  is a cut-off frequency of disturbance observer,  $G_v$  is a cut-off frequency of the low-pass filter (LPF) for measured velocity,  $I_a$  is input current,  $K_t$  is a torque constant,  $\theta$  is position response of the controlled object,  $\omega$  is velocity response,  $J$  is inertia,  $s$  denotes a Laplace operator, a bar over a variable denotes a calculated value, a subscript  $n$  denotes a nominal value, a superscript  $ref$  denotes a reference value, and a superscript  $cmp$  denotes a compensation value.

The total disturbance torque  $\tau_{dis}$  contains a mechanical load  $\tau_l$ , varied self-inertia torque  $\Delta J\ddot{\theta}$ , and torque ripple from motor  $\Delta K_t I_a^{ref}$ . The disturbance torque  $\tau_{dis}$  is represented as follows:

$$\tau_{dis} = \tau_l + \Delta J\ddot{\theta} - \Delta K_t I_a^{ref}. \quad (1)$$

This disturbance torque is figured out from input and output values as shown in (2).

$$\tau_{dis} = K_{tn} I_a^{ref} - J_n \omega s \quad (2)$$

here, the Laplace operator  $s$  denotes a derivative calculation. The first term  $K_{tn} I_a^{ref}$  in (2) is based on input information, and the second term  $J_n \omega s$  is based on output information. Disturbance torque is estimated through the LPF as shown in (3) in order to reduce noise.

$$\hat{\tau}_{dis} = \frac{G_{dis}}{s + G_{dis}} \left( K_{tn} I_a^{ref} - \frac{G_v}{s + G_v} J_n \omega s \right) \quad (3)$$

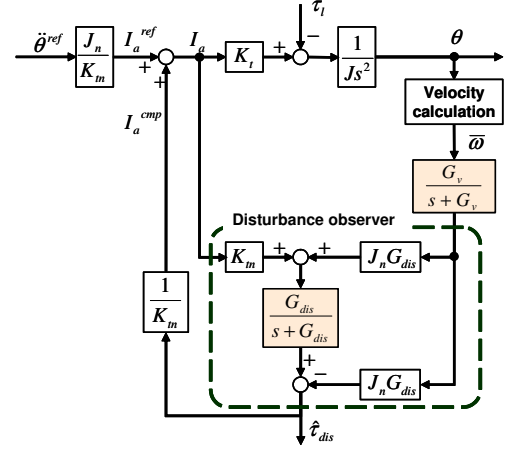


Fig. 2. Disturbance Observer

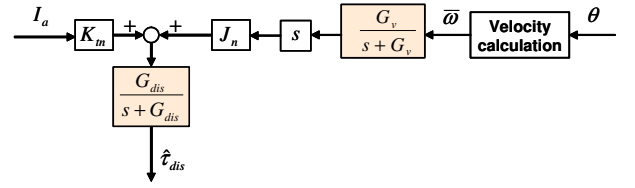


Fig. 3. Equivalent transformation form of disturbance observer

Disturbance observer estimates disturbance on the control system and compensates it. This disturbance estimation is based on the acceleration  $\omega s$  derived from the output of the optical encoder. In other words, the control system has acceleration feedback in essence. Consequently the control system with disturbance observer is an acceleration control system in essence.

#### B. Quantization error in acceleration control

This subsection describes the influence of a quantization error on optical encoders in order to show the importance of velocity measurement accuracy in acceleration control. Fig. 3 shows the equivalent transformation form of disturbance observer. Equation (4) shows  $\hat{\tau}_{dis}$  estimated in practice.

$$\hat{\tau}_{dis} = \frac{G_{dis}}{s + G_{dis}} \left( K_t I_a^{ref} - \frac{G_v}{s + G_v} J_n \bar{\omega} s \right) \quad (4)$$

$\bar{\omega}$  includes a certain amount of noise due to the quantization error on the optical encoder. Its accuracy and delay depend on the calculation method. Finite-difference derivative amplifies the noise. Fig. 4 shows velocity and acceleration values in simulation. Velocity values are derived by finite-difference derivative of position values. At the same time, acceleration values are derived by finite-difference derivative of the velocity values. Parameters in this simulation such as encoder resolution and sampling period are equal to experimental parameters. The result shows how large the noise on measured acceleration is, compared to the true value. Two LPFs are introduced to reduce this noise while they cause a delay on disturbance estimation. The cutoff

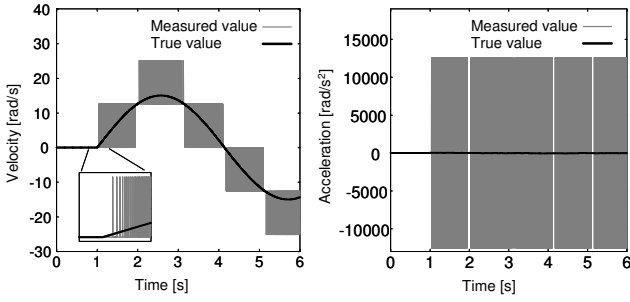


Fig. 4. Velocity values and acceleration values derived by finite-difference derivative

frequency of the LPFs should be high since this delay may deteriorate the performance of the control system. Accurate velocity measurement is indispensable to heighten the cutoff frequency.

#### IV. RELATED RESEARCH

##### A. M method

M method is the most widely used method to measure velocity from encoder pulses. The principle is shown in Fig. 5.  $m_e$  is the number of pulses during a fixed sampling period  $T_s$ .  $m_e$  is utilized for velocity calculation by finite-difference derivative.

Measured velocity  $\bar{\omega}$  is figured out by (5). Following equations show velocity resolution and measurement time.

$$\bar{\omega} = \frac{2\pi m_e}{PT_s} \quad (5)$$

$$Q_V = \frac{2\pi}{PT_s} \quad (6)$$

$$\frac{Q_V}{V} = \frac{2\pi}{PT_s V} \quad (7)$$

$$T_m = T_s \quad (8)$$

where  $P$  denotes the encoder pulse number per rotation.  $Q_V$  stands for absolute velocity resolution and  $\frac{Q_V}{V}$  stands for relative velocity resolution.  $V$  denotes actual angular velocity. Measurement time  $T_m$  is equal to the sampling period  $T_s$ . Here, a sampling period stands for interval time to count a pulse number periodically. On the other hand, measurement time stands for interval time to calculate velocity based on the pulse number counted in single or multiple sampling periods.

As shown in (6) and (7), velocity resolution becomes larger as the sampling period becomes shorter. However,

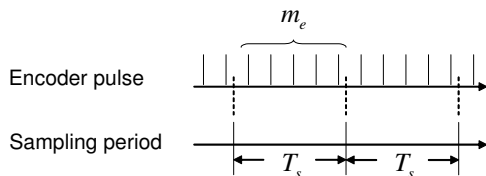


Fig. 5. Principle of M method

acceleration control requires both a short sampling period and accurate velocity measurement.

The easiest way to improve accuracy is to average the velocity values. The average of  $n$  sampling periods is acquired by (9). The absolute and relative resolutions are shown in Eqs. (10) and (11). Measurement time is prolonged as shown in (12). These equations show that averaging improves the accuracy to the  $n$ th part of the resolution while measurement time becomes  $n$  times longer.

$$\bar{\omega}(i) = \frac{2\pi \sum_{j=0}^n m_e(i-j)}{nPT_s} \quad (9)$$

$$Q_V = \frac{2\pi}{nPT_s} \quad (10)$$

$$\frac{Q_V}{V} = \frac{2\pi}{nPT_s V} \quad (11)$$

$$T_m = nT_s \quad (12)$$

##### B. T method

T method measures velocity by dividing the interpulse angle by the pulse interval time as shown in Fig. 6. In the figure, encoder pulses come successively with  $T_e$  intervals. Assuming that interpulse angle of the optical encoder is completely accurate, accuracy of this method only depends on the measurement of interval time  $T_e$ .  $T_e$  is substituted by  $T_m = m_s T_s$ . Here,  $m_s$  is the number of sampling periods during pulse interval. Although  $T_m$  is an approximate value of  $T_e$ , it contains an error less than  $T_s$  since  $m_s$  is an integer.

Velocity is calculated by (13). Equations (14), (15) and (16) show the performance.

$$\bar{\omega} = \frac{2\pi}{m_s PT_s} \quad (13)$$

$$Q_V = \frac{2\pi}{m_s(m_s-1)PT_s} \quad (14)$$

$$\frac{Q_V}{V} = \frac{2\pi}{m_s(m_s-1)PT_s V} \quad (15)$$

$$T_m = m_s T_s \quad (16)$$

It is shown from the equations that T method reduces the maximum error inversely proportional to  $m_s(m_s-1)$  while measurement time becomes  $m_s$  times longer. It is obvious that accuracy improves much more than averaging. However, there are several problems on this method. The first problem is that the pulse interval is not measurable if it is shorter than the sampling period  $T_s$ . In other words, this method is only applicable to a low-speed range. The other problem on this method is that the measurement time

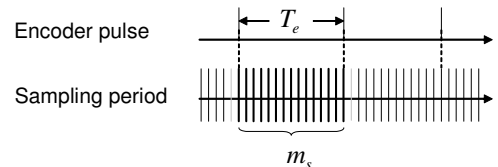


Fig. 6. Principle of T method

$T_m$  is fluctuant and it depends on the velocity. Therefore the measurement delay may become large.

### C. M/T method

M/T method is an effective measurement method that combines M method and T method. It improves the measurement accuracy in a low-speed range and furthermore, it works in all speed ranges. Its performance is in proportion to T method in the low-speed range and M method with averaging in the high-speed range.

## V. PROPOSED METHOD

### A. Principle of proposed method

Principle and procedure of the proposed method are shown in this subsection. The object of the proposed method is to acquire high accuracy in all speed ranges. In order to acquire high accuracy, this subsection discusses why T method achieves high accuracy. Then the measurement method that achieves the accuracy of T method in all speed ranges is proposed.

Equation (13) shows that T method calculates velocity by dividing interpulse angle  $\frac{2\pi}{P}$  by interval time  $m_s T_s$ . In other point of view, it averages the velocity synchronized with pulses.

Velocity measurement is more accurate in T method since velocity calculation is synchronized with the timing of encoder pulses. Fig. 7(a) shows an example when averaging calculation is not synchronized. Average velocity is irregular since sum of pulse numbers in  $n$  samples fluctuates depending on the calculation timing. Fig. 7(b) shows the other example when averaging calculation is synchronized with pulses. Average velocity is smooth in this case since sum of pulse numbers does not fluctuate. Consequently, the synchronization in T method reduces the noise in calculated velocity. T method is, however, limited in a low-speed range because it is impossible to measure the interval time if pulses occur in every sampling period.

Fig. 8 shows pulse numbers on respective sampling periods when velocity varies from low speed to high speed. In fact, it is impossible to measure the interval time between sequential pulses. However, the patterns of the waveform

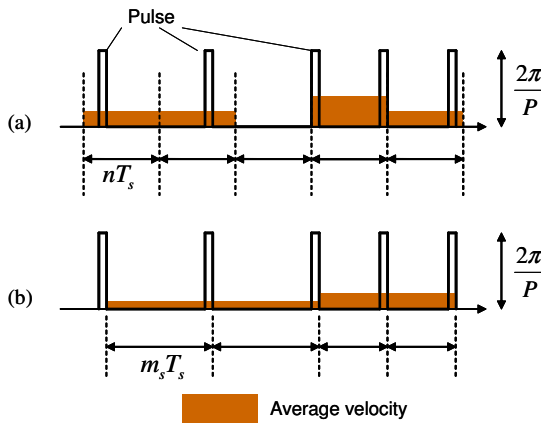


Fig. 7. Pulse pattern in low-speed range

in a high-speed range are quite similar to that in a low-speed range. We can presume the patterns as occasional pulses with a certain amount of offset. Hence the alteration of pulse numbers, as shown in Fig. 9, should be detected so as to synchronize the velocity calculation with it. We call the alteration “pulse alteration”.

The procedures of velocity measurement are shown as follows:

1. Count the pulse number  $m_e(i)$  in each sampling period
2. Do not update the velocity value while the pulse numbers are constant
3. Calculate and update the velocity value if the pulse numbers alter (i.e. if pulse alteration occurs)

The velocity value is derived from (17).

$$\bar{\omega}(i) = \frac{2\pi \sum_{j=0}^{m_s} m_e(i-j)}{m_s P T_s} \quad (17)$$

Here,  $m_s$  is the sample number during interval of pulse alteration. The equation is quite similar to M method with averaging. The difference is that averaging calculation of the proposed method is synchronized with pulse alteration. Equations (18), (19) and (20) show that the performance is equivalent to T method. Furthermore, this method works in all speed ranges.

$$Q_V = \frac{2\pi}{m_s(m_s-1)P T_s} \quad (18)$$

$$\frac{Q_V}{V} = \frac{2\pi}{m_s(m_s-1)P T_s V} \quad (19)$$

$$T_m = m_s T_s \quad (20)$$

The proposed method is named “synchronous-measurement method (S method)” since its calculation is

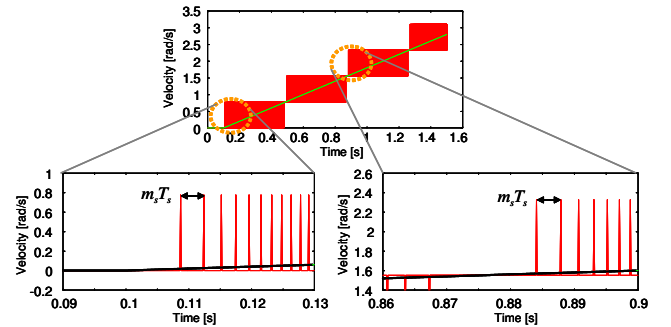


Fig. 8. Pulse pattern in low and high-speed range

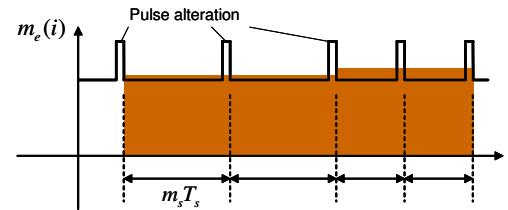


Fig. 9. Pulse pattern in high-speed range

synchronous with pulse alteration. High accuracy of T method, which is synchronous with pulses but not with pulse alteration, is expanded to a high-speed range with this method.

The procedures of S method is carried out on a PC with RT-Linux in this study. One of the advantages of this method is that it is applicable to a system with a relatively long sampling period since it works in all speed ranges. Note that T method often requires a short sampling period for extension of its measurable speed range. Furthermore, this method is also applicable to a system with an auxiliary processor to acquire a shorter sampling period. Accuracy of the velocity measurement improves with the shorter sampling period.

### B. Performance of velocity measurement methods

This subsection compares the theoretical performance of each method.

Fig. 10 shows the measurement time on each method. M method acquires the velocity information in every sampling period. Averaging makes the measurement time longer while it improves the measurement accuracy. Measurement time depends on pulse interval in M/T method. The measurement time is long in the low-speed range while the measurement time is almost constant in the high-speed range. Measurement time is fluctuant in the proposed method. Measurement time becomes long when pulse alteration does not often occur. In fact, the delay becomes large in particular velocity. This deteriorates control performance when adverse effect of delay is larger than that of the quantization error. Therefore a method to modify the measurement time is applied [8]. In this method, velocity is calculated compulsory if measurement time exceeds a threshold.

Fig. 11 compares the measurement resolution on each method. M/T method is more accurate than T method in a velocity range from 3.2 rad/s to 12.5 rad/s since its measurement time is modified to be longer than  $T_c = 0.004$  while the measurement time of T method is shorter in the range. Meanwhile, M/T method has large  $Q_v$ , absolute velocity resolution, in a high-speed range. Accuracy of velocity measurement methods is often compared on  $Q_v/V$ , relative velocity resolution. Although M/T method has large  $Q_v$  in a high-speed range,  $Q_v/V$  is kept small in all

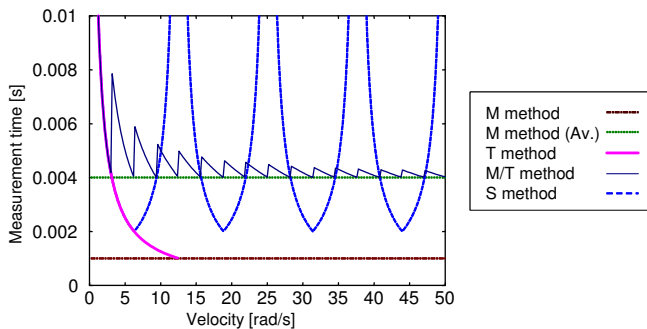


Fig. 10. Measurement time on each method

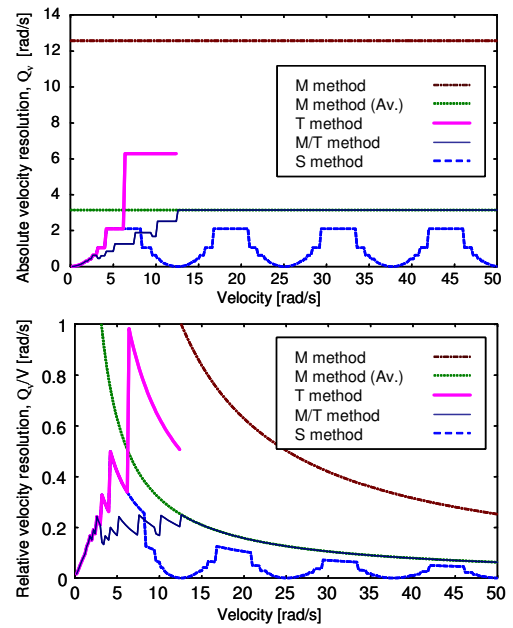


Fig. 11. Measurement resolution on each method

speed ranges. In this point of view, M/T method is effective for many motion control systems. However, acceleration control regularly requires accuracy in all speed ranges. The accuracy in the acceleration dimension is in proportion to  $Q_v$ , absolute velocity resolution. Hence small  $Q_v$  is indispensable for acceleration control. The result show that S method achieves small  $Q_v$  in all speed ranges.

## VI. SIMULATION RESULT

Simulations were executed to compare the accuracy of velocity measurement methods in the completely same conditions. Parameters were equal to those of the experiment. In each simulation, the same sequence of external force input was applied without any control input in order to compare just the accuracy of measurement.

Fig. 12 shows the velocity measurement results of M method, M/T method and S method respectively. M method was averaged and the sample number for averaging was 4. Velocity resolution is constant and large in M method. On the other hand, M/T method achieves high resolution in the low-speed range while the resolution is still low in the high-speed range. The result shows that high resolution in all speed ranges was achieved by S method.

## VII. EXPERIMENT

This section shows experimental results to verify the validity of S method. The main focus is how robustness of a motion control system improves with the method.

We applied PD control with disturbance observer to the experimental system. Command velocity was kept constant after it was raised from 0 rad/s to 40 rad/s in 5 seconds. Virtual disturbance of 0.2 Nm was given for 0.5 second when 7 seconds passed from the beginning of the experiment. The disturbance was given by adding a cur-

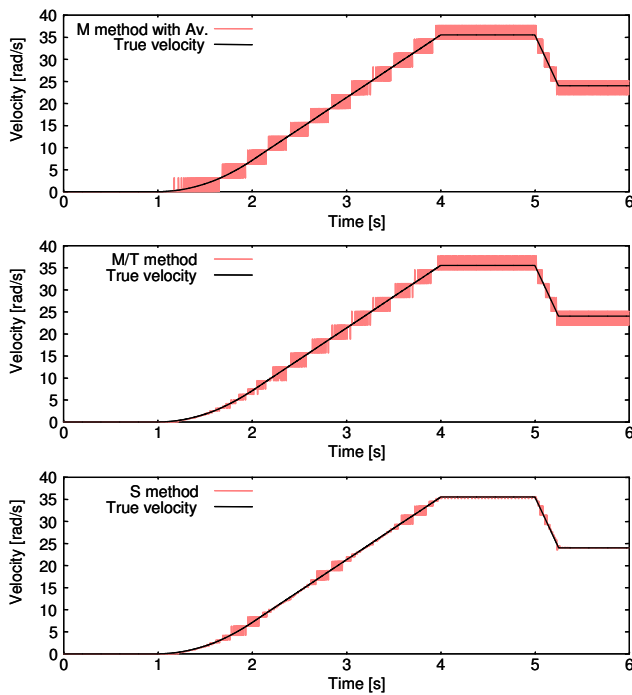


Fig. 12. Velocity measurement on each method

rent input to the control system. Fig. 13 shows the results. Here,  $k_v$  denotes a derivative gain of the PD controller. In order to show a fair comparison, all of the response values in the figure are derived by M method with a LPF. In other words, the values in the figure are just to show and they are not used for control.  $G_v, G_{dis}$  and  $k_v$  were changed in each experiment while  $k_p$ , a proportional gain, was 20.0 all the time.

The motor made a sound noise when  $G_v$  and  $G_{dis}$  were larger than 20.0  $rad/s$  in M method. A large oscillation is confirmed in the result when  $G_v = G_{dis} = 50.0$  in M method. Acceleration control with  $G_v = G_{dis} = 50.0$  was achieved without a sound noise in S method. The result shows that the performance of acceleration control improved with S method. Furthermore,  $k_v$  could be heighten more without any destabilization because of the high resolution of measured velocity. It is confirmed from the result that an error on velocity control reduced with higher  $k_v$ .

We did not multiply the pulse signal in this experiment since the performance improvement due to S method became smaller. This result indicates that irregularity of interpulse angle adversely affects the accuracy of S method. Our future work is to cope with this problem.

### VIII. CONCLUSION

This paper proposed S method, a novel method of velocity measurement for motor drives with optical encoders. In S method, the velocity measurement is synchronized with alteration of pulse numbers. High accuracy of T method is acquired in all speed ranges due to this synchronization. Furthermore, other estimation methods are applicable in addition since the method does not require any model of

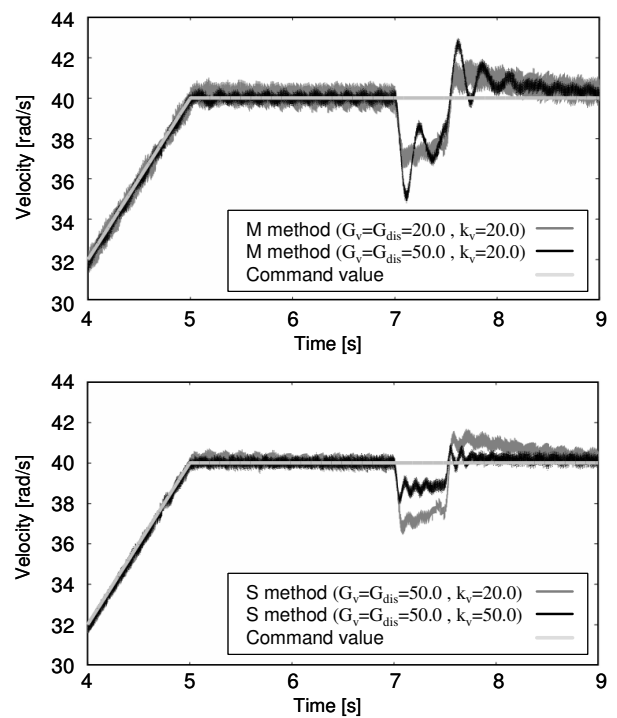


Fig. 13. Experimental result of PD control with disturbance observer

a control object. Simulation and experimental results verified the validity of the method. Although the method is a fundamental technique for all motion control systems with optical encoder, it is particularly effective for acceleration control systems.

### REFERENCES

- [1] K. Ohnishi, M. Shibata, T. Murakami: "Motion Control for Advanced Mechatronics," IEEE/ASME Trans. on Mechatronics, Vol. 42, No. 2, pp. 123–130 (1995)
- [2] M. Mizuochi, T. Tsuji, K. Ohnishi: "Multirate Sampling Method for Acceleration Control System," IEEE Intr. Symp. on Industrial Electronics, (2005)
- [3] T. Ohmae, T. Matsuda, K. Kamiyama and M. Tachikawa: "A Microprocessor-Controlled High-Accuracy Wide-Range Speed Regulator for Motor Drives," IEEE Trans. on Industrial Electronics, Vol. 29, No. 3, pp. 207–211, (1982)
- [4] Richard C. Kavanagh: "Improved Digital Tachometer with Reduced Sensitivity to Sensor Nonideality," IEEE Trans. on Industrial Electronics, Vol. 47, No. 4, pp. 890–897, (2000)
- [5] P. R. Belanger: "Estimation of Angular Velocity and Acceleration from Shaft Encoder Measurements," Proc. of the 1992 IEEE Int. Conf. on Robotics and Automation, pp. 585–592 (1992)
- [6] Y. Okamura, Y. Chun, Y. Hori: "Robust and adaptive control of servomotor with low resolution shaft encoder by average speed type instantaneous speed observer," Proc. of IPEC-Yokohama'95, pp. 705–711 (1995)
- [7] R. H. Brown, S. C. Schneider, M. G. Mulligan: "Analysis of Algorithms for Velocity Estimation from Discrete Position Versus Time Data," IEEE Trans. on Industrial Electronics, Vol. 39, No. 1, pp. 11–19, (1992)
- [8] T. Tsuji, M. Mizuochi, K. Ohnishi: "Modification of Measurement Time for Digital Tachometers," Japan Industry Applications Society Conference, (2005)
- [9] K. Fujita, K. Sado: "Instantaneous Speed Detection with Parameter Identification for AC Servo Systems," IEEE Trans. on Industry Applications, Vol. 28, No. 4, pp. 864–872, (1992)
- [10] S. H. Lee, J. B. Song: "Acceleration Estimator for Low-Velocity and Low-Acceleration Regions Based on Encoder Position Data," IEEE/ASME Trans. on Mechatronics, Vol. 6, No. 1, pp. 58–64, (2001)