Force Sensing and Force Control Using Multirate Sampling Method

Mariko Mizuochi, Toshiaki Tsuji and Kouhei Ohnishi

Keio University
Department of System Design Engineering
3-14-1 Hiyoshi, Kohoku-ku, Yokohama, 223-8522, JAPAN
mariko@sum.sd.keio.ac.jp, tsuji@sum.sd.keio.ac.jp, ohnishi@sd.keio.ac.jp

Abstract— **This paper focuses on a bandwidth and a delay of force sensing. High bandwidth force sensing is important for motion control systems to cope with unknown environments. Reaction torque observer is utilized instead of force sensors to detect external force in this paper. The bandwidth of force sensing is limited by the sampling period and data noise. The delay in force information may deteriorate the performance of force control since force information is fed back. According to these considerations, this paper proposes application of a multirate sampling method to the force control system. Reaction torque observer is modified for the multirate system. The bandwidth of force sensing can be improved and the delay in force information can be decreased with the proposed method. The validity of the proposal is verified by stability analysis, simulation and experimental results.**

I. Introduction

According to resent dramatic development in robotics, robots have been utilized in a variety of fields. As robots are required to perform various tasks, contact motion is indispensable. In industrial application, motions such as picking up and holding objects and pressing objects with desired force are important tasks of robots. Force control is also important in application of robots to human society. If only position control is implemented to a robot, the robot may break an object on its way to accomplish the position command. Hence, it may easily harm the human and environments. Force control is a key technique for cooperative tasks due to safety reason. Requirements for force control are therefore increasing. Considering strong demands for applying a robot in open environment, it is important to attain stable contact with unknown environments. Wide bandwidth force sensing is required to cope with any kind of environments [1]. Although force sensors are usually implemented to detect external force, problems such as adverse influence of a narrow bandwidth have been reported [2]. As an alternative way to determine external force or torque imposed on a motor, reaction torque observer has been proposed [3]. This paper focuses on force sensing in a wide bandwidth to achieve stable force control.

In digital control, a high sampling frequency enables acquisition of better performance. The sampling periods have limitations, however, relating to hardware performances. In order to acquire better control performance despite such hardware limitations, the methods for setting sampling periods individually have been proposed. These methods are called multirate sampling control [4]. Many studies have been performed on the system in which output information cannot be acquired fast enough [5], [6]. The computer hard disk drives or systems utilizing visual camera are the examples. Multirate control was applied to force control in [7] to change sampling periods between calculations considering calculation amount. The authors proposed a multirate sampling method for realization of acceleration control [8]. The performance was greatly improved by acquiring output information in a sampling period shorter than that for renewal of actuation input. This paper applies the method to force sensing and control with expansion of observers. This paper aims at the realization of stable contact motion by broadening the bandwidth of force sensing. Simulation on force sensing was performed to verify the improvement in its bandwidth and accuracy. Improvement of stability and performance of force control were confirmed by stability analysis and experiments.

II. Force Control

Block diagram of a force control system is shown in Fig. 1. Here, τ^{cmd} denotes torque command, $\hat{\tau}^{reac}$ denotes sensed external torque, τ^{ext} denotes external torque, J denotes inertia, θ^{res} denotes position response and K_f denotes force gain. The aim of force control is to enable a robot to impose desired force on the object. Thus, the following two requirements are pointed out.

- Attain stable contact with any environments
- Impose exact desired force in rapid response

The first requirement is prerequisite for the second since force cannot be imposed constantly when contact is unstable.

As an alternative method of force sensors to determine external force or torque imposed on a motor, reaction torque observer shown in Fig. 2 has been proposed [3]. Mechanical load τ_l can be expressed as follows:

$$\tau_l = \tau^{ext} + \tau_{int} + \tau_g + F + D(\dot{\theta}) \tag{1}$$

where, τ^{ext} denotes external torque, τ_{int} denotes interference torque, τ_g denotes gravity torque, F denotes coulomb

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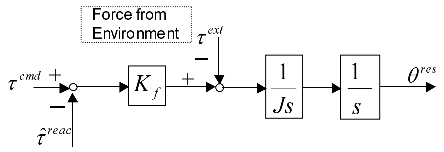


Fig. 1. Force Control System

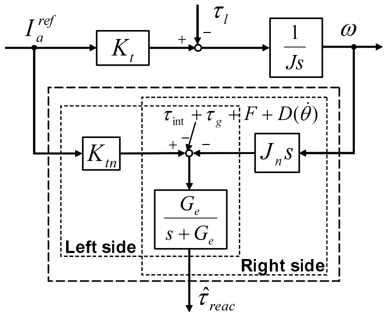


Fig. 2. Reaction Torque Observer

friction and $D(\dot{\theta})$ denotes viscous friction. External torque is calculated as follows:

$$\hat{\tau}^{reac} = \frac{G_e}{s+G_e}(K_{tn}I_a^{ref} - J_n s\omega - (\tau_{int}+\tau_g+F+D(\dot{\theta}))) \quad (2)$$

where, G_e denotes a cut-off frequency of a low pass filter (LPF), K_t denotes a motor coefficient, I_a^{ref} denotes current reference, ω denotes angular velocity and a subscript n denotes nominal value. It is necessary to use real values K_t and J in calculation to acquire the real value of external force. It is also indispensable to detect amounts of τ_{int}, τ_g, F and $D(\dot{\theta})$. The first term of (2) corresponds to the left side of Fig. 2, while the second corresponds to the right. In other words, the first is based on input information, and the second is based on output information. The bandwidth of force sensing depends on the cut-off frequency G_e. It is therefore important to heighten G_e in order to cope with unknown environments. G_e is limited, however, due to the sampling period and the noise of the data. Most of the noise is caused by the quantization error of the encoder and conducting derivative calculation twice in the right side of the observer. Therefore, the limit of G_e depends on the right side rather than on the left. In other words, the output sampling period and noise decide the limit of G_e, since the right side is derived from output information. Considering that external force is estimated based on the information in an acceleration dimension, the multirate sampling method proposed for an acceleration system [8] can be effectively applied to force sensing.

III. Multirate Sampling Method

In multirate sampling methods, sampling periods for input T_u, output T_y and controller T_r are set independently. These methods are useful to acquire better performance under the limitations of sampling periods. Here, output and input periods are defined as follows:

- output sampling period: sampling period for acquisition of sensor information; and
- input sampling period: sampling period for renewal of a current input reference.

T_u is often limited by the processing time of D/A converters or the frequency of PWM. T_y depends on the processing time of counter boards or the processing rate of sensors. Calculation time of a computer limits T_r. The limitations on the input sampling period are often more strict than those on the output in motor control. A multirate sampling method shown in Fig. 3 has then been proposed for an acceleration control system [8]. T_u and T_r are selected as follows:

$$T_u = nT_y, \quad T_r = T_y. \quad (3)$$

Angular or position information of a motor is acquired several times in one input sampling period. Control calculation is performed in every output sampling period.

The continuous-time plant represented by the following equations is then considered.

$$\dot{\boldsymbol{x}}(t) = \mathbf{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t), \quad y(t) = \boldsymbol{c}\boldsymbol{x}(t) \quad (4)$$

Considering that actuation input is renewed only when $t = iT_u$ is satisfied in the method, state-space equations of the multirate system is obtained as follows:

$$\begin{aligned} \boldsymbol{x}[i,k+1] &= \mathbf{A}_m\boldsymbol{x}[i,k] + \boldsymbol{b}_m u[i,0] \quad : k \neq n-1 \quad (5) \\ \boldsymbol{x}[i+1,0] &= \mathbf{A}'_m\boldsymbol{x}[i,n-1] + \boldsymbol{b}'_m u[i,0] : k = n-1 \quad (6) \\ \boldsymbol{y}[i,k] &= \boldsymbol{c}_m\boldsymbol{x}[i,k] \quad (7) \end{aligned}$$

where,

$$\boldsymbol{x}[i,k] = \boldsymbol{x}((i+\frac{k}{n})T_u) = \boldsymbol{x}(iT_u + kT_y) \ (k=0,\cdots,n-1)$$

$$\mathbf{A}_m = e^{\mathbf{A}T_y}, \ \boldsymbol{b}_m = \int_0^{T_y} e^{\mathbf{A}\tau}d\tau\boldsymbol{b}, \quad \boldsymbol{c}_m = \boldsymbol{c}.$$

Expected advantages of applying the multirate sampling method are as follows:

- enhance the cut-off frequency of the reaction torque observer; and
- decrease a delay in force information.

As mentioned in the previous section, the limit on G_e depends on the right side of the observer. Therefore, G_e can be increased as a sampling period of the right, which is an output sampling period, becomes shorter. Another advantage is reduction of the delay in force information. When

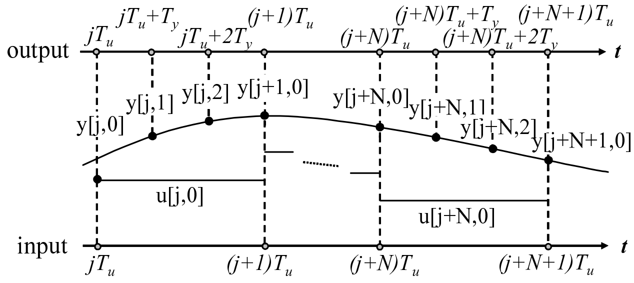


Fig. 3. Multirate Sampling

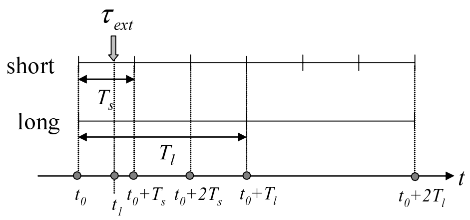


Fig. 4. Recognition External Force

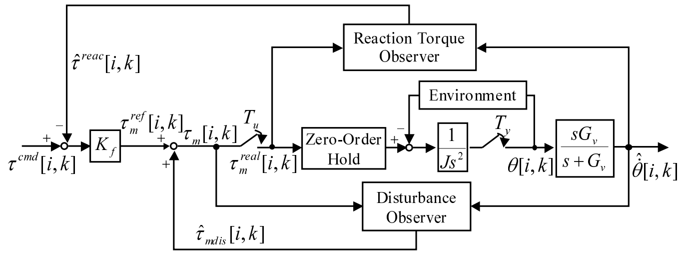


Fig. 5. Multirate Force Control System

only the acquisition of force information is considered, only accuracy of the data is important and the delay is not a large problem. In case of force control, however, the delay may deteriorate the performance since sensed force information is fed back. The delay in the force information can be decreased with the multirate sampling method for two main reasons below.

- Decrease of the delay in a calculation process by heightening G_e
- Early recognition of reaction force

The data acquired through LPF has a sort of delay and magnitude of the delay depends on the cut-off frequency of the LPF. The derivative calculation to acquire the velocity from an angle is often performed with a pseudo-derivative method with LPF as shown in the equation below to reduce the influence of noise.

$$\hat{\dot{\theta}} = \frac{sG_v}{s + G_v}\theta \tag{8}$$

Thus, it is possible to suppress the delay by setting the cut-off frequencies G_v and G_e high. The image of the second reason is shown in Fig. 4. The upper part shows a short output sampling period realized with the multirate control. The lower part shows a long period in single-rate control. Here, it is assumed that the external force is added on $t = t_1$, which satisfies $t_0 \leq t_1 \leq t_0 + T_s$. Influence of external force τ^{ext} appears at $t_0 + T_s$ when an output sampling period is short, while at $t_0 + T_l$ when the period is long. It is clear that external force is recognized earlier with the short output sampling period.

A force control system using the multirate sampling method is shown in Fig. 5. Disturbance observer and reaction torque observer in the multirate system are explained in the following sections.

B.1 Disturbance Observer in Multirate System

This section briefly explains the disturbance observer for the multirate system proposed in [8]. Disturbance observer

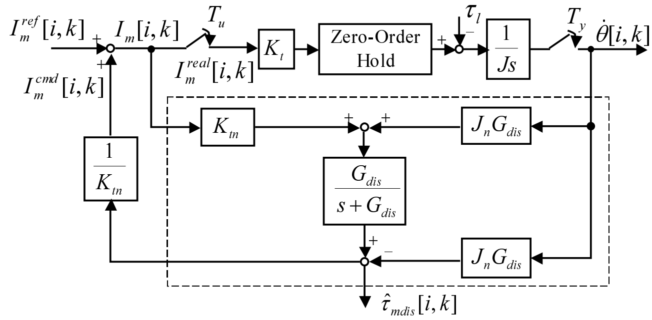


Fig. 6. Disturbance Observer for Multirate System

has been proposed to estimate and compensate disturbance torque and make the system an acceleration control system [9]. The total disturbance torque estimated in disturbance observer contains a mechanical load, varied self-inertia torque, and torque ripple from motor. There is another factor that influences the system as disturbance in the multirate system. It is the deviation between two values of input, a desired input value $I_m[i,k]$ and a real input reference value $I_m^{real}[i,k]$. The former is calculated at an output sampling rate and the latter is a real input reference value to the robot, which is renewed at an input sampling rate. The deviation exists since the input sampling period is longer than the output. So as to compensate the influence of deviation, not $I_m^{real}[i,k]$ but $I_m[i,k]$ is utilized in Fig. 6. The disturbance torque is calculated by the following equation.

$$\hat{\tau}^{mdis}[i,k] = \frac{G_{dis}}{s + G_{dis}}(K_{tn}I_m[i,k] - J_n s\dot{\theta}[i,k]) \tag{9}$$

B.2 Reaction Torque Observer in Multirate System

Reaction torque observer explained in the previous section is expanded to the multirate system. Although reaction torque observer and disturbance observer have almost the same mechanism, their roles differ largely. Since disturbance observer is to estimate and compensate the entire disturbance on the system, $I_m[i,k]$ is utilized. On the other hand, reaction torque observer is expected to estimate only a real external force. It is therefore required to use a real current input to a motor to estimate real external force. Assuming that current feedback loop is fast enough, $I_m^{real}[i,k]$ is considered to be equal to real input current. In order to improve the bandwidth of force sensing, reaction torque observer is expanded, as shown in Fig. 7, by using the multirate sampling method. Reaction torque is estimated through LPF as shown in the equation below.

$$\begin{aligned}\hat{\tau}^{reac}[i,k] = \frac{G_e}{s + G_e}\ (\ &K_{tn}I_m^{real}[i,k] - Js\dot{\theta}[i,k] \\ &- (\tau_{int} + \tau_g + F + D(\dot{\theta}))) \end{aligned} \tag{10}$$

Note that the real input reference value $I_a^{real}[i,k]$ is utilized instead of $I_a[i,k]$.

IV. Accuracy of force sensing

This section verifies how accuracy of force sensing changes with the multirate sampling method. Application

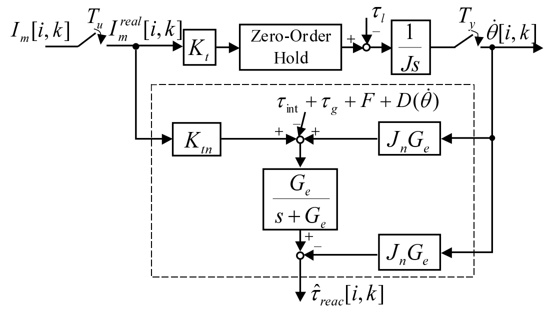


Fig. 7. Reaction Torque Observer for Multirate System

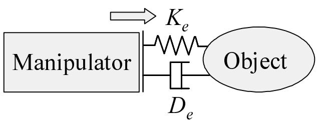


Fig. 8. Model of Environment

of the multirate sampling method shortens both the sampling period for acquisition of output information and that for observer. Force control was performed with $T_u = T_y =$ 0.1 msec in any case and several reaction torque observers were implemented to estimate the force and to compare the accuracy. Simulations were performed based on parameters of the experimental equipment and a quantization model of the encoder was introduced. The environment was modeled as a spring and damper model as shown in Fig. 8. Fig. 9 shows the result with changing the cut-off frequency G_e. τ_{env} is not a value estimated with observer but the real value given in the simulation. A certain value of external force was detected when $t = 1.0$ sec though the manipulator did not touch the environment. It is caused by the fact that the force value calculated with the observer contains inertia torque. The result showed that the estimated value became closer to the real value by heightening G_e. The value oscillated and diverged, however, when G_e was further heightened. The ability of force sensing therefore improves if it is possible to make the maximum value of G_e higher.

Influences of a sampling period on the maximum value are then verified. Fig. 10 compares the results of long and short sampling periods: $T_y = 0.8$ msec and $T_y = 0.2$ msec. In case of a long sampling period, the oscillation was confirmed with G_e higher than 1500. On the other hand, the cut-off frequency G_e could be set much higher without oscillation when a sampling period was short. It showed the advantage of the multirate sampling method with its shorter sampling period of the observer.

There is another advantage: early recognition of external force. Fig. 11 compares the results of various sampling periods to verify the effect of early recognition on force sensing. The cut-off frequency was set to be the same in all cases. The force estimated as negative was also due to inertia torque. Rise time became earlier and convergence to the real value became faster by shortening the output sampling period. The result therefore indicated that the shorter an output sampling period became, the earlier estimation became.

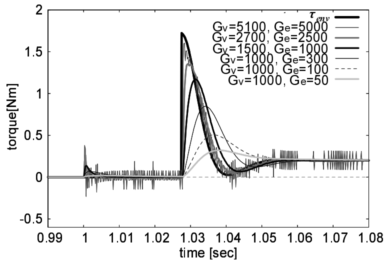


Fig. 9. Change in Accuracy of Force Sensing with G_e

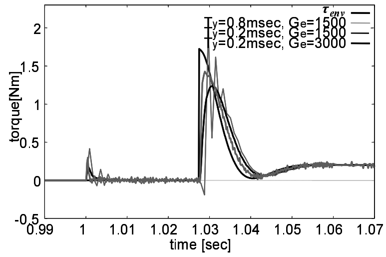


Fig. 10. Change in Limitation on G_e with Sampling Periods

The advantages of applying the multirate sampling method confirmed in the simulation are summarized below. The maximum value of the cut-off frequency became higher with a shortened sampling period of observer. Recognition and estimation became early with a shortened sampling period for acquiring output information. As a result, delay in force information was reduced and accuracy improved.

V. Stability analysis

The effects of application of the multirate sampling method were verified in terms of stability. The main focus in the analysis is not disturbance observer but reaction torque observer and the whole force control system.

A dynamic equation of a 1-DOF manipulator in discrete-time is shown in the following equation.

$$\begin{bmatrix} \theta[i+1] \\ \dot{\theta}[i+1] \\ \tau_{dis}[i+1] \end{bmatrix} = \begin{bmatrix} 1 & T & -\frac{T^2}{2J} \\ 0 & 1 & -\frac{T}{J} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta[i] \\ \dot{\theta}[i] \\ \tau_{dis}[i] \end{bmatrix} + \begin{bmatrix} \frac{T^2}{2J} \\ \frac{T}{J} \\ 0 \end{bmatrix} \tau_m[i] \quad (11)$$

where, τ_m denotes input torque and τ_{dis} denotes disturbance torque, which is assumed to be constant between sampling periods. The environment was modeled as a spring and damper model and assumed to be connected directly to the manipulator as shown in Fig. 8. Stability

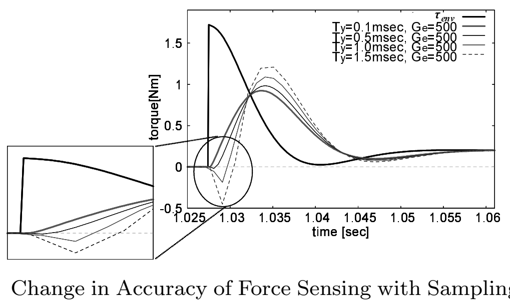


Fig. 11. Change in Accuracy of Force Sensing with Sampling Periods

improvement with the multirate sampling method and disturbance observer shown in Fig. 6 is described in [8]. This paper therefore discusses stability of the system without disturbance observer in order to exclude the effect of disturbance observer and to focus on reaction torque observer and the whole force control system. Control input in the analysis is then represented as the equation below.

$$\tau_m[i] = K_f(\tau_{cmd}[i] - \hat{\tau}^{reac}[i]) \tag{12}$$

Analysis on the multirate system requires rewriting of the system for a longer sampling period. This paper utilized the method proposed in [10].

Nyquist diagram in a continuous domain was attained as in Figs. 12 and 13. The three patterns of sampling periods were selected for the analysis.

- Single-rate (long) : $T_u = 0.3$ msec, $T_y = 0.3$ msec
- Single-rate (short) : $T_u = 0.15$ msec, $T_y = 0.15$ msec
- Multirate : $T_u = 0.3$ msec, $T_y = 0.15$ msec

Following two types of environments were assumed.

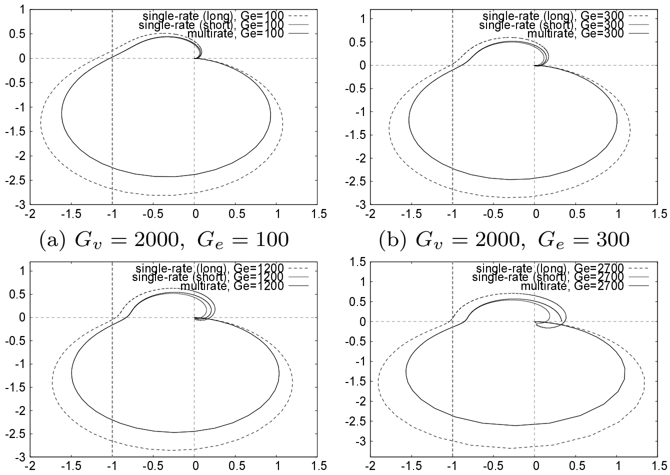
- Soft (Fig.12):$K_e = 1000$Nm/rad, $D_e = 10.0$Nm·s/rad
- Hard (Fig.13):$K_e = 3000$Nm/rad, $D_e = 10.0$Nm·s/rad

Fig. 12 shows that the point crossing real axis moved to right and then moved back to left when G_e was changed from low to high. The results clearly showed that the stability was improved by heightening G_e since accurate force sensing was achieved. Stability was adversely affected when G_e was further heightened because of the influence of discrete time. Comparison between multirate control and single-rate control with a long sampling period showed that the stability was always much better for multirate control than single-rate control even though the input sampling periods were the same. When $G_e = 100$, both single-rate and multirate controls were unstable. Not single-rate but multirate control became stable with $G_e = 300$. $G_e = 1500$ stabilized both single-rate and multirate. Single-rate control became unstable with $G_e = 2700$. Single-rate control was easily destabilized by heightening G_e while multirate control kept stable. Remarkable is that the results of multirate control were almost the same as those of single-rate with a short sampling period though the input sampling period was longer in multirate control. When the environment was harder, both single-rate and multirate controls were unstable with $G_e = 300$. It means that a higher G_e was required for stabilization. As a result, it was impossible to stabilize the system in single-rate control with a long sampling period while it is possible in multirate control.

The results of stability analysis are summarized as follows:

- multirate control improved stability;
- multirate control was stable with higher G_e; and
- hard environment required higher G_e.

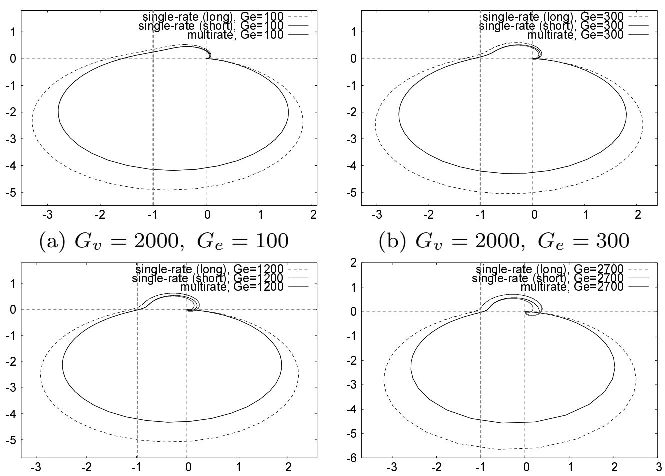
As a result, multirate control enables adaptation to harder environments with higher G_e and improved stability. The most remarkable was that multirate control was almost the same as single-rate control with a short sampling period in



(a) $G_v = 2000,\ G_e = 100$ (b) $G_v = 2000,\ G_e = 300$

(c) $G_v = 2000,\ G_e = 1200$ (d) $G_v = 2800,\ G_e = 2700$

Fig. 12. Nyquist Diagram of Soft Environment



(a) $G_v = 2000,\ G_e = 100$ (b) $G_v = 2000,\ G_e = 300$

(c) $G_v = 2000,\ G_e = 1200$ (d) $G_v = 2800,\ G_e = 2700$

Fig. 13. Nyquist Diagram of Hard Environment

terms of stability even with a longer input sampling period. These results clearly show and strongly support the advantages of applying the multirate sampling method to force control.

VI. Experiment

Experimental equipment is a single-link manipulator composed of a motor, an encoder and a light arm. Gravity term was negligible since the rotational plane of the manipulator was horizontal. Friction term was also negligible since the direct drive manipulator was used. The number of pulses of the encoder is 81,000 pulses/rev and it is multiplied by four in a counter board to improve resolution.

The following four types of experiments were performed.

- Single-rate control: Fig. 14
- Multirate control: Fig. 14
- Multirate control with longer sampling periods: Fig. 15
- Multirate control with higher cut-off frequencies:Fig.16

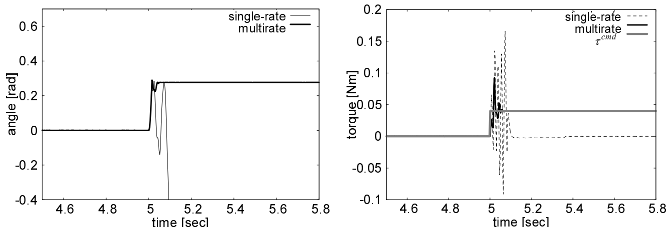
TABLE I
SAMPLING PERIODS AND CUT-OFF FREQUENCIES (EXPERIMENT)

	T_u[ms]	T_y[ms]	G_v	G_{dis}	G_e
Case 4(single-rate)	3.0	3.0	200	150	150
Case 5(multirate)	3.0	1.0	200	150	150
Case 6(long)	4.0	2.0	200	150	150
Case 7(high)	3.0	1.0	300	250	250

Sampling periods and cut-off frequencies of each experiment are presented in TABLE I and a force gain K_f was set to $K_f = 2.0$. The manipulator was controlled to make contact with an aluminum block with a force command given as a step input. In single-rate control, the manipulator repeated contact and noncontact motions several times and finally the response diverged at about $t = 5.1$ sec. Fig. 14 shows that stable contact was attained in the multirate control. In Fig. 15, although hunting was confirmed, stable contact was attained finally even with longer sampling periods. The result strongly supports the validity of the proposed method. Noteworthy is that the difference between Case 4 and Case 5 was only the output sampling period, and cut-off frequencies were not changed. Stable contact motion seemed to be attributable to the following two factors; early recognition of reaction torque and reduced noise. The first factor is explained in III-B. Noise and the bandwidth of force sensing have a trade-off relationship. Both the narrow bandwidth and the noise of force information might affect the performance in single-rate control. The noise might be reduced and its influence might thus be also reduced to attain stable contact in multirate control. Noise reduction in multirate control was confirmed in Fig. 16. The cut-off frequency could be set high in multirate control, while it became unstable and could not be moved in single-rate control. The result shows that the bandwidth of force control was widen by shortening the output sampling period. Accordingly, the multirate sampling method enabled the manipulator to cope with various environments.

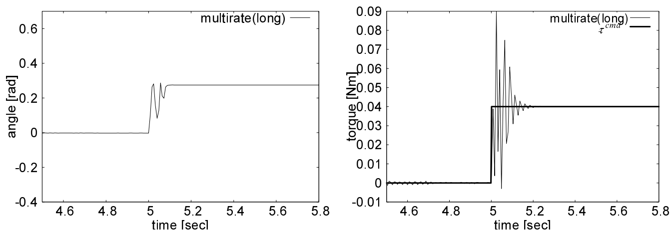
VII. CONCLUSION

This paper proposed a force sensing and force control method using the multirate sampling method. The output sampling period is important in detection of external force. From this point of view, the multirate sampling method was applied to force control. Reaction torque observer was modified for the multirate system. Simulation results showed the improvement in accuracy of force sensing in terms of the bandwidth and delay. Noteworthy improvements were confirmed both in stability analysis and experiments. Stability became almost the same as in single-rate control with a short sampling period, even though an input sampling period was long in multirate control. In experiments, stable contact was achieved even with the same control parameters. With the improved bandwidth, ability of adaptation to unknown environment might improve. This method can be utilized together with conventional force control methods such as force control with velocity loop or impedance control.



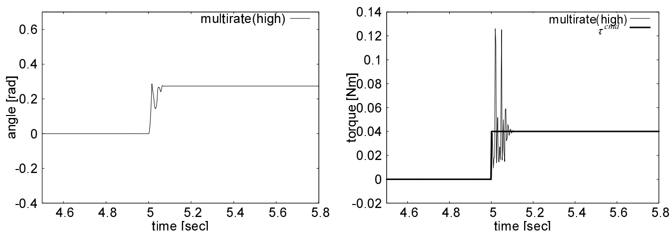
(a) Position Response (b) Force Response

Fig. 14. Experimental Results (Case 4 and Case 5)



(a) Position Response (b) Force Response

Fig. 15. Experimental Results (Case 6)



(a) Position Response (b) Force Response

Fig. 16. Experimental Results (Case 7)

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