Robust Acceleration Control Based on Acceleration Measurement Using Optical Encoder

Toshiaki Tsuji, *Member, IEEE,* and Hiroshi Kobayashi, *Member, IEEE*
Department of Mechanical Engineering Faculty of Engineering Division I
Tokyo University of Science
Tokyo, Japan
Email: tsuji@kobalab.com, hiroshi@kobalab.com

***Abstract*— The purpose of this study is to realize robust acceleration control using optical encoders. A quantization error of an optical encoder limits performance of the acceleration control. Hence, the study extends the premise of the S method, a high performance velocity measurement method, to acceleration measurement. The proposed method was shown to considerably improve the accuracy of acceleration measurements. High performance of acceleration control is realized due to the accurate measurement. Experimental results verify the validity of the proposed method.**

I. Introduction

Studies on motion control are becoming increasingly important as more robots and electric vehicles come into practical use. And industry is demanding even higher-performance motion control, which is characterized by rapid response and high robustness.

Many studies on velocity measurement have been conducted in an effort to improve motion control performance. Among them, the M method and the T method are the two most commonly used methods. The M method, also called the fixed-time method, counts the number of pulses from an optical encoder during a fixed time interval and calculates velocity by a finite-difference derivative. On the other hand, the T method, or fixed-position method, calculates velocity as the interpulse angle divided by the time between sequential pulses. The accuracy of the M method deteriorates in the low-speed range, but the T method is able to attain high accuracy in this region. The T method, however, is only applicable to the low-speed range. Ohmae, Matsuda, Kamiyama and Tachikawa [1] proposed the M/T method that works for all speed ranges and has a high accuracy in the low-speed range. This method has been applied in many studies[2], [3] since it is effective for practical use. Tsuji, Mizuochi and Ohnishi have proposed the S method, which measures velocity synchronous with the alteration of pulse numbers[4]. Compared with the other methods, the S method has the advantage of acquiring a high accuracy over all speed ranges.

There are some studies that derive acceleration for acceleration control based on velocity measurements using optical encoders. Acceleration control has some advantages: it achieves a high robustness; the controlled plant can be assumed to be a nominalized system; and both position/force control can be treated in a unified manner. Sensorless acceleration control using optical encoders is useful due to acceleration sensor costs and calibration requirements. Disturbance observer (DOB)[5] is a typical example of a sensorless acceleration control. Resolved acceleration control[6] and acceleration feedback control[7] are also good candidates for sensorless acceleration control.

These methods, however, have a common problem that the acceleration control performance is limited due to the noise on a measured acceleration value. The noise is caused by a quantization error of the optical encoder that is amplified through a second order derivative. Thus, this study attempts to improve acceleration measurements by introducing an advanced differentiation method. The S method, a differentiation method for velocity measurement, is then extended to acceleration measurement.

This paper is organized as follows. Section II describes the experimental setup. Section III shows the acceleration control mechanism and Section IV describes velocity measurement using S method and discusses the solution for sensor nonideality. The S method is extended for acceleration control in Section V. Section VI and Section VII detail the simulation and experimental results, respectively. Section VIII concludes the paper.

II. Experimental Setup

Fig. 1 shows an overview of the experimental system. The mechanical part of the system is a one degree-of-freedom flywheel driven by a DC motor. An optical encoder generates pulses in proportion to arm displacement, a counter board counts the pulses from the optical encoder, and a PC reads the pulse number from the board. Here, the pulse number denotes the number of pulse signals over every sampling period T_s. T_s is controlled to be constant with the real-time architecture of RT-Linux. The performance of the proposed method is compared with other methods on this system.

The parameters on the experimental setup are shown in Table I.

1-4244-0755-9/07/$20.00 ©2007 IEEE

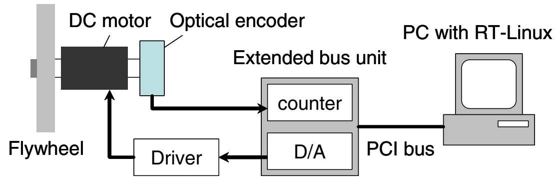


Fig. 1. Overview of experimental setup

TABLE I
EXPERIMENTAL PARAMETERS

Sampling period [ms]	1.0
Flywheel MOI [Kgm^2]	0.002
Type of motor	Maxon RE40
Stall torque [mNm]	2500
Torque constant [mNm/A]	60.3
Type of optical encoder	Maxon HEDS 5540
Resolution(quadruple) [PPR]	2000

III. ACCELERATION CONTROL

A. Acceleration control system with disturbance observer

This subsection describes the mechanism for acceleration control used in this study, which employs a disturbance observer as the basic technique for acceleration control. Fig. 2 is a block diagram of disturbance observer. In this figure, τ_l is the mechanical load, $\hat{\tau}_{dis}$ is the estimated disturbance torque, G_{dis} is the cut-off frequency of the disturbance observer, G_v is the cut-off frequency of the low-pass filter (LPF) for measured velocity, I_a is the input current, K_t is the torque constant, θ is the position response of the controlled object, ω is the velocity response, and J is the inertia. s denotes a Laplace operator, and a bar over a variable denotes a calculated value, a subscript n denotes a nominal value, a superscript ref denotes a reference value, and a superscript cmp denotes a compensation value.

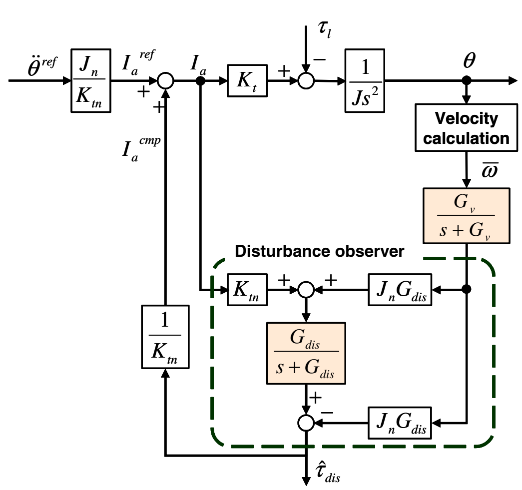


Fig. 2. Disturbance observer

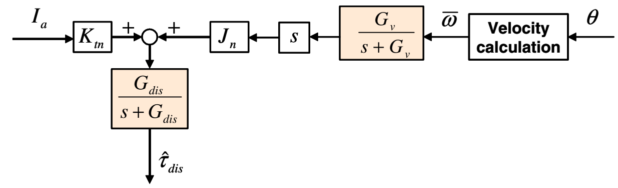


Fig. 3. Equivalent transformation form of disturbance observer

Disturbance torque τ_{dis} can be determined by (1).

$$\tau_{dis} = K_{tn} I_a^{ref} - J_n \omega s \tag{1}$$

Here, the Laplace operator s denotes a derivative calculation. The estimated disturbance value includes a considerable amount of noise amplified by the derivative calculation. In order to reduce this noise, the disturbance torque is estimated through the LPF, as shown in (2).

$$\hat{\tau}_{dis} = \frac{G_{dis}}{s + G_{dis}} \left(K_{tn} I_a^{ref} - \frac{G_v}{s + G_v} J_n \omega s \right) \tag{2}$$

The disturbance observer estimates the disturbance on the control system and compensates for it. This disturbance estimation is based on the acceleration ωs derived from the output of the optical encoder. In other words, the control system has a type of acceleration feedback. Consequently, the control system with the disturbance observer is essentially an acceleration control system.

B. Quantization error in acceleration control

This subsection describes the influence of a quantization error on optical encoders in order to show the importance of velocity measurement accuracy in acceleration control.

Fig. 3 shows the equivalent transformation form of the disturbance observer. In this figure, $\bar{\omega}$ includes a certain amount of noise due to the quantization error of the optical encoder, whose accuracy and delay depend on the calculation method. The finite-difference derivative is the simplest calculation method, although it amplifies the noise. Fig. 4 shows the velocity values for the simulation. It also displays the acceleration values derived from the velocity. The velocity values are derived using the finite-difference derivative of the position values. At the same time, the acceleration values are derived using the finite-difference derivative of the velocity values. The parameters in this simulation, such as the encoder resolution and the sampling period, are equal to the experimental parameters, although they do not exhibit the irregularities of the experimental system. The result shows that the noise on the measured acceleration is extremely large relative to the true value.

Two LPFs are introduced in practical acceleration control to reduce this noise, although they introduce a delay in disturbance estimation. The cutoff frequency of the LPFs should be high since this delay may deteriorate the performance of the control system. An accurate velocity and acceleration measurement is indispensable for increasing the cutoff frequency.

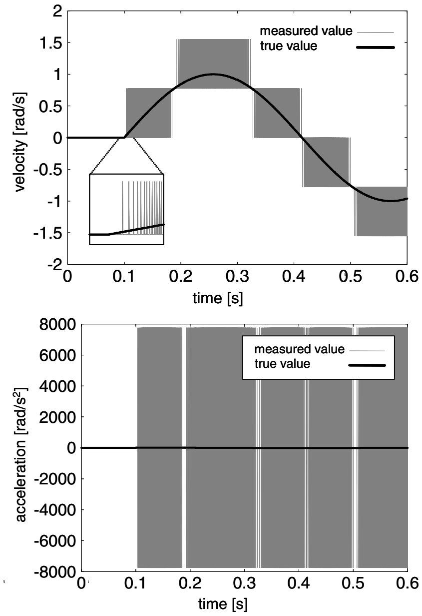


Fig. 4. Velocity and acceleration derived by finite-difference derivative

IV. Velocity measurement by S method

A. Principle of S method

This subsection briefly describes the principle of the S method, a velocity measurement method proposed in [4]. As previously shown in Fig. 4, a finite-difference derivative generates a certain amount of error. The measured value is accurate only if the velocity is a multiple of the unit velocity. Here, the unit velocity represents the velocity at which exactly one pulse occurs during one sampling period. The unit velocity ω_u is derived as follows:

$$\omega_u = \frac{2\pi}{PT_s} \tag{3}$$

where P is number of pulses per revolution. If velocity is other than a multiple of the unit velocity, the pulse number varies. Fig. 5 shows an example of pulse number patterns. When the time constant of the control system is much longer than the sampling period, the typical pulse number pattern alternates after a sequence of constant values. This alternation of the pulse number is called "pulse alternation". The S method calculates a velocity value synchronous with the pulse alternation. The procedure for the S method is as follows.

1) Count the pulse number $m_e(i)$ in the ith sampling period;
2) Do not update the velocity value when the pulse number is constant; and
3) Calculate the velocity value if the pulse number changes (i.e. if pulse alternation occurs).

The velocity value is derived from (4).

$$\bar{\omega}(i) = \frac{2\pi \sum_{j=0}^{m_s-1} m_e(i-j)}{m_s P T_s} \tag{4}$$

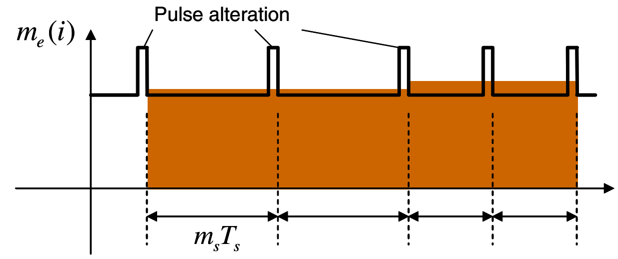


Fig. 5. Pulse number pattern in high-speed range

Here, m_s is the number of samples during a pulse alternation interval. This equation is quite similar to the M method with averaging equation. The difference, however, is that the averaging calculation for the S method is synchronized with the pulse alternation.

The method is named the "synchronous-measurement method (S method)" since its calculation is synchronous with pulse alternation.

B. S method resolution and measurement time

The S method calculates a much more accurate velocity than the velocity averaged at constant intervals. On the other hand, the S method has the disadvantage of fluctuant interval time due to the synchronization. When pulse alternation does not occur for a long time, the average velocity is compulsively derived since a long measurement time may degrade control performance. Maximum number of interval samples m_s^{max} is decided based on the method in [8]. The relationship between the measured velocity value and the measurement time is shown in Tables II and III. The measurement time is long when the velocity is nearly a multiple of the unit velocity. After m_s^{max} samples, the average velocity is compulsively derived. In this case, the maximum quantization error is ω_u/m_s^{max}. On the other hand, the quantization error is smaller when the interval time of pulse alternation is shorter than the maximum measurement time. Suppose the pulse number alternates from 0 to 1 after n samples and the velocity is measured as ω_u/n, while the true value is in the range between $\omega_u/(n+1)$ and $\omega_u/(n-1)$. As a result, the maximum quantization error Q_V of the S method with the interval of m_s samples is derived by (5). And the measurement time T_m is derived by (6).

$$\begin{aligned} Q_V &= \max\left(\frac{\omega_u}{m_s-1} - \frac{\omega_u}{m_s}, \frac{\omega_u}{m_s} - \frac{\omega_u}{m_s+1}\right) \\ &= \frac{\omega_u}{m_s(m_s-1)} \qquad (5) \\ T_m &= m_s T_s \qquad (6) \end{aligned}$$

Although Tables II and III cover most of the possible pulse patterns, other patterns may be generated in rare cases. This happens when the velocity variation is extremely large. In this case, the velocity is immediately calculated because pulse alteration always occurs in such a case. Here, the accuracy of the velocity value is equal to that of the M method.

TABLE II

MEASURED VELOCITY VALUE AND MEASUREMENT TIME (LOWER THAN UNIT VELOCITY)

Pattern of pulse number	$\underbrace{0,\cdots,0}_{m_s^{max}}$	$\underbrace{0,\cdots,0,1}_{m_s^{max}}$	$\cdots$	$\underbrace{0,\cdots,0,1}_{n+1}$	$\underbrace{0,\cdots,0,1}_{n}$	$\underbrace{0,\cdots,0,1}_{n-1}$	$\cdots$	$\cdots$	$\underbrace{0,0,0,1}_{4}$	$\underbrace{0,0,1}_{3}$	$\underbrace{0,1}_{2}$	$\underbrace{1,1,0}_{3}$	$\underbrace{1,1,1,0}_{4}$	$\cdots$	$\underbrace{1,\cdots,1,0}_{m_s^{max}}$	$\underbrace{1,\cdots,1}_{m_s^{max}}$
Measurement time	$m_s^{max}T_s$	$m_s^{max}T_s$	$\cdots$	$(n+1)T_s$	nT_s	$(n-1)T_s$	$\cdots$	$\cdots$	$4T_s$	$3T_s$	$2T_s$	$3T_s$	$4T_s$	$\cdots$	$m_s^{max}T_s$	$m_s^{max}T_s$
Measured velocity (ratio to unit vel.)	0	$\frac{1}{m_s^{max}}$	$\cdots$	$\frac{1}{n+1}$	$\frac{1}{n}$	$\frac{1}{n-1}$	$\cdots$	$\cdots$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\cdots$	$\frac{m_s^{max}-1}{m_s^{max}}$	1
Quantization error (ratio to unit vel.)	$\frac{1}{m_s^{max}}$			$\frac{1}{n(n+1)}$	$\frac{1}{n(n-1)}$				$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$			$\frac{1}{m_s^{max}}$	

TABLE III

MEASURED VELOCITY VALUE AND MEASUREMENT TIME (HIGHER THAN UNIT VELOCITY)

Pattern of pulse number	$\underbrace{1,\cdots,1}_{m_s^{max}}$	$\underbrace{1,\cdots,1,2}_{m_s^{max}}$	$\cdots$	$\underbrace{1,\cdots,1,2}_{n+1}$	$\underbrace{1,\cdots,1,2}_{n}$	$\underbrace{1,\cdots,1,2}_{n-1}$	$\cdots$	$\cdots$	$\underbrace{1,1,1,2}_{4}$	$\underbrace{1,1,2}_{3}$	$\underbrace{1,2}_{2}$	$\underbrace{2,2,1}_{3}$	$\underbrace{2,2,2,1}_{4}$	$\cdots$	$\underbrace{2,\cdots,2,1}_{m_s^{max}}$	$\underbrace{2,\cdots,2}_{m_s^{max}}$	$\underbrace{2,\cdots,2,3}_{m_s^{max}}$	$\cdots$
Measurement time	$m_s^{max}T_s$	$m_s^{max}T_s$	$\cdots$	$(n+1)T_s$	nT_s	$(n-1)T_s$	$\cdots$	$\cdots$	$4T_s$	$3T_s$	$2T_s$	$3T_s$	$4T_s$	$\cdots$	$m_s^{max}T_s$	$m_s^{max}T_s$	$m_s^{max}T_s$	$\cdots$
Measured velocity (ratio to unit vel.)	1	$\frac{m_s^{max}+1}{m_s^{max}}$	$\cdots$	$\frac{n+2}{n+1}$	$\frac{n+1}{n}$	$\frac{n}{n-1}$	$\cdots$	$\cdots$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{7}{4}$	$\cdots$	$\frac{2m_s^{max}-1}{m_s^{max}}$	2	$\frac{2m_s^{max}+1}{m_s^{max}}$	$\cdots$
Quantization error (ratio to unit vel.)	$\frac{1}{m_s^{max}}$			$\frac{1}{n(n+1)}$	$\frac{1}{n(n-1)}$				$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$			$\frac{1}{m_s^{max}}$	$\frac{1}{m_s^{max}}$		

C. Solution for nonideality of optical encoder

In practice, the acceleration measurement contains nonideal noise since the interpulse angle of an optical encoder is not necessarily even. It has already been mentioned in [9] that alternate pulse alternation degrades accuracy when the velocity is near a multiple of the unit velocity. The study also showed that this issue can be solved by canceling out the alternate pulse alternation. However, the cancellation procedure in the study was complicated. On the other hand, Equation (7) cancels out the alternate pulse alternation with a simple calculation.

$$\bar{\omega}(i) = \frac{2\pi(\frac{1}{2}m_e(i-m_s)+\sum_{j=1}^{m_s-1}m_e(i-j)+\frac{1}{2}m_e(i))}{Pm_sT_s} \tag{7}$$

V. ACCELERATION MEASUREMENT BY S METHOD

This section describes how to extend the S method to acceleration measurement. The S method can be easily extended to acceleration measurement by synchronizing the calculation of the acceleration to the pulse alternation. The procedure for the acceleration measurement is as follows:

1) Count pulse number $m_e(i)$ in each sampling period;
2) Do not update the velocity and acceleration values while the pulse number is constant; and
3) Calculate the velocity and the acceleration values if the pulse number changes (i.e., if pulse alternation occurs).

The acceleration value is derived from (8).

$$\bar{\alpha}(i) = \frac{\bar{\omega}(i)-\bar{\omega}(i-m_s)}{m_sT_s} \tag{8}$$

Here, $\bar{\alpha}$ is a measured acceleration value. Equation (9) shows the maximum error of the acceleration measurement.

$$Q_A = \frac{Q_V}{T_m} = \frac{\omega_u}{m_s^2(m_s-1)T_s} \tag{9}$$

The measurement time T_m is the same as for the velocity measurement using the conventional S method.

This proposed method is now compared with conventional methods. (10) and (11) show the maximum acceleration error and the measurement time based on M method, respectively.

$$Q_A = \frac{\omega_u}{n^2T_s} \tag{10}$$

$$T_m = nT_s \tag{11}$$

Here, n samples of acceleration values were averaged. This equation indicates that acceleration measurement synchronized with the S method has a higher accuracy when the measurement time T_m is the same. The method in [4] derives acceleration from velocity based on S method while the derivative calculation for acceleration is not synchronized with pulse alteration. Accuracy of the acceleration measurement further improves with the synchronization.

The S method procedure is carried out on a PC with RT-Linux. One of the advantages of this method is that it is applicable to a system with a relatively long sampling period since it works in all speed ranges. It should be noted that the T method often requires a short sampling period for the extension of its measurable speed range. Furthermore, this method can also be applied to a system with an auxiliary processor to acquire a shorter sampling period. The accuracy of the velocity measurement improves with a shorter sampling period.

VI. Simulation

The mean squared error (MSE) of the acceleration measurement is compared using simulation results since true acceleration cannot be determined in experiments. The parameters in the simulation are the same as the experimental parameters. The simulation modeled the nonideality of the optical encoder and the error model used in Kavanagh's study[2] was applied here. The position measurement error is uniformly distributed over $[-\varepsilon, \varepsilon]$, where ε is the ratio of the maximum error to the interpulse angle.

The trajectory of the flywheel was given by (12), so the acceleration should then be given by (13).

$$\theta = A \sin \lambda t \tag{12}$$

$$\ddot{\theta} = A\lambda^2 \sin \lambda t \tag{13}$$

$$A = 5.0, \quad \lambda = 1.0$$

The MSE values for a 10 second simulation are compared in Table IV. The cutoff frequency of the LPF was 50 rad/sec. The results show that the S method acquired acceleration values with a higher accuracy and that accuracy degradation due to encoder nonideality was reduced by using (7).

VII. Experiment

Several experiments were carried out to verify the performance of the proposed method.

The performance of the acceleration measurement is first examined. Fig. 6 shows an acceleration measurement result. The input torque $\tau_{inp} = 0.01 \sin t$ was given then without any position, velocity, or acceleration feedback. The majority of the noise is due to measurement error since it does not contain any noise produced by control feedback. The result shows that acceleration measurement by the extended S method significantly reduces the measurement noise compared with the acceleration measurement based on M method.

Next, the acceleration control performance of the proposed method is examined. The control system was a PD control with disturbance observer. The parameters are shown in Table. V. Fig. 7 shows the result when the velocity and acceleration feedback was derived based on the M method. The figure also shows the velocity and acceleration values measured by both methods, the M method and the S method. Sound noise occurred during the experiment since the acceleration control system generated vibrations. A large noise was also recognized on the measured acceleration values based on both the M method and the S method. Fig. 8 shows the result when velocity and acceleration feedback was derived based on the S method. Sound noise was not detected during this experiment. The measured acceleration value based on the S method shows that vibration was rarely generated.

In summary, the experimental results show that acceleration measurement based on the S method achieved high gain feedback control while the conventional method using the same feedback gain generated vibration.

VIII. Conclusion

This paper proposes an acceleration measurement method using an optical encoder. The method is an extension of the S method, which is a method that measures velocity synchronized with the alternation of the pulse number. The proposed method considerably reduces the noise on the acceleration values. Thus, control performance is improved because the method makes it possible to increase the feedback gain of the acceleration control systems. The validity of the proposed method was verified by experiments.

Acknowledgment

This research was supported by a grant from the Casio Science Promotion Foundation.

References

[1] T. Ohmae, T. Matsuda, K. Kamiyama and M. Tachikawa: "A Microprocessor-Controlled High-Accuracy Wide-Range Speed Regulator for Motor Drives," IEEE Trans. on Industrial Electronics, Vol. 29, No. 3, pp. 207–211, (1982)

[2] Richard C. Kavanagh: "Improved Digital Tachometer with Reduced Sensitivity to Sensor Nonideality," IEEE Trans. on Industrial Electronics, Vol. 47, No. 4, pp. 890–897, (2000)

[3] R. Bonert: "Design of a high performance digital tachometer with a microcontroller", *IEEE Trans. on Instrumentation and Measurement*, Vol. 38, No. 6, pp. 1104–1108, (1989).

[4] T. Tsuji, M. Mizuochi, H. Nishi, K. Ohnishi: "A Velocity Measurement Method for Acceleration Control," *Proc. of Int. Conf. on Industrial Electronics, Control and Instrumentation*, pp. 1943–1948, (2005)

[5] K. Ohnishi, M. Shibata, T. Murakami: "Motion Control for Advanced Mechatronics," IEEE/ASME Trans. on Mechatronics, Vol. 1, No. 1, pp. 56–67 (1996)

TABLE IV

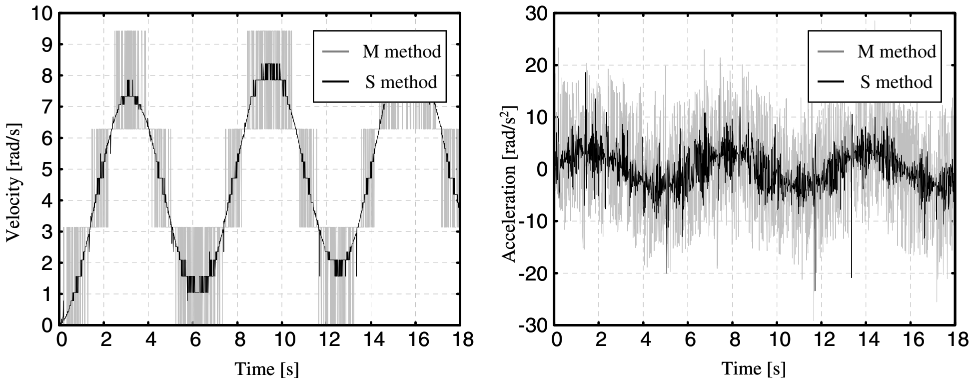
MSE of acceleration measurement [rad/sec^2]

	$\varepsilon = 0.0$	$\varepsilon = 0.03$	$\varepsilon = 0.1$
M method	3.9612	3.9742	4.2851
S method with (4)	3.2160	2.8424	3.9307
S method with (7)	1.5335	1.6815	2.6496

TABLE V

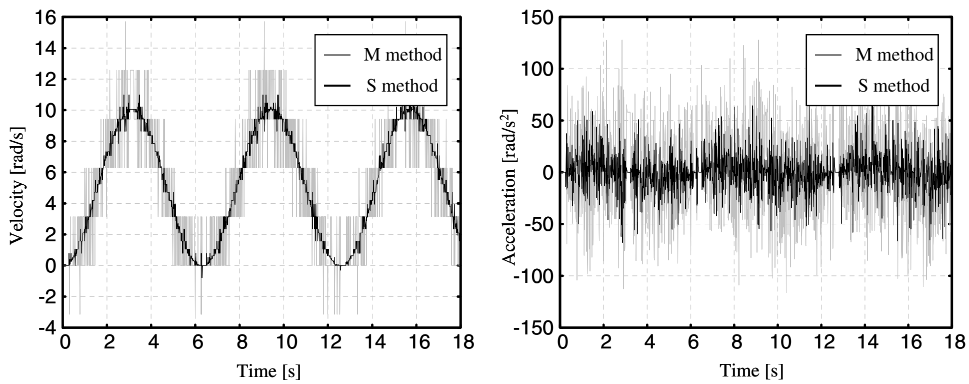
Control parameters in experiment

K_p	Position gain on PD controller	400.0
K_v	Velocity gain on PD controller	40.0
G_{dis}	Cutoff Freq. of DOB	70.0
G_v	Cutoff Freq. of LPF on measured Vel.	200.0



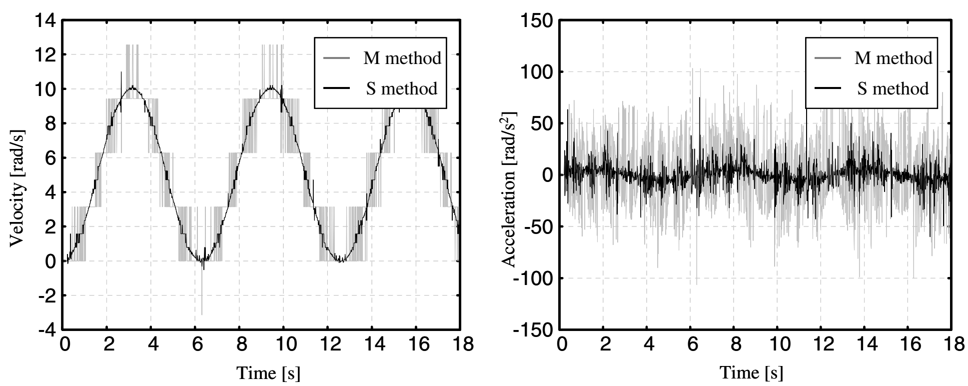
a) Measured velocity (w/o LPF) b) Measured acceleration (with LPF)

Fig. 6. Open loop control



a) Measured velocity (w/o LPF) b) Measured acceleration (with LPF)

Fig. 7. Acceleration control with feedback derived by M method



a) Measured velocity (w/o LPF) b) Measured acceleration (with LPF)

Fig. 8. Acceleration control with feedback derived by S method

[6] Luh J. Y. S., M. W. Walker, and R. P. C. Paul: "Resolved-acceleration Control of Mechanical Manipulators," IEEE Trans. on Automatic Control, Vol. 25, pp. 486–474, (1980)

[7] I. Godler, K. Hashimoto, T. Ninomiya: "Stability, gain and sampling time of discrete time acceleration control loop," Proc. IEEE Int. Symp. Industrial Electronics (ISIE 2001), pp. 1305–1308, (2001)

[8] T. Tsuji, M. Mizuochi, K. Ohnishi:"Modification of Measurement Time for Digital Tachometers," Japan Industry Applications Society Conference, (2005)

[9] T. Tsuji, M. Mizuochi and K. Ohnishi: "Technical issues on velocity measurement for motion control," *Proc. of Int. Power Electronics and Motion Control Conf. (EPE-PEMC '06)*, pp. 336–341 (2006).

[10] M. Mizuochi, T. Tsuji, K. Ohnishi:"Multirate Sampling Method for Acceleration Control System," IEEE Intr. Symp. on Industrial Electronics, (2005)

[11] S. H. Lee, J. B. Song: "Acceleration Estimator for Low-Velocity and Low-Acceleration Regions Based on Encoder Position Data, " IEEE/ASME Trans. on Mechatronics, Vol. 6, No. 1, pp. 58–64, (2001)

[12] R. H. Brown, S. C. Schneider, M. G. Mulligan: "Analysis of Algorithms for Velocity Estimation from Dicrete Position Versus Time Data," IEEE Trans. on Industrial Electronics, Vol. 39, No. 1, pp. 11–19, (1992)