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CP and T violation in long baseline experiments with low energy neutrino

Joe Sato
Research Center for Higher Education, Kyushu University,
Ropponmatsu, Chuo-ku, Fukuoka, 810-8560
E-mail: joe@rc.kyushu-u.ac.jp

Stimulated by the idea of PRISM, a very high intensity muon ring with rather low energy, we consider possibilities of observing CP-violation effects in neutrino oscillation experiments. Destructive sum of matter effect and CP-violation effect can be avoided with use of initial $\nu_{\rm e}$ beam. We finally show that the experiment with (a few) × 100 MeV of neutrino energy and (a few) × 100 km of baseline length, which is considered in this paper, is particularly suitable for a search of CP violation in view of 3-generation nature of CP violation.

1 Introduction

Many experiments and observations have shown evidences for neutrino oscillation one after another. The solar neutrino deficit has long been observed[1,2,3,4,5]. The atmospheric neutrino anomaly has been found[6,7,8,9] and recently almost confirmed by SuperKamiokande[10]. There is also another suggestion given by LSND[11]. All of them can be understood by neutrino oscillation and hence indicates that neutrinos are massive and there is a mixing in lepton sector[12].

Since there is a mixing in lepton sector, it is quite natural to imagine that there occurs CP violation in lepton sector. Several physicists have considered whether we may see CP-violation effect in lepton sector through long baseline neutrino oscillation experiments[13].

The use of neutrinos from muon beam has great advantages compared with those from pion beam. Neutrinos from $\mu^+(\mu^-)$ beam consist of pure $\nu_{\rm e}$ and $\bar\nu_\mu$ ($\bar\nu_{\rm e}$ and ν_μ) and will contain no contamination of other kinds of neutrinos. Also their energy distribution will be determined very well. In addition we can test T violation in long baseline experiments by using (anti-)electron neutrino[14,15].

What energy range is suitable for observing CP violation? Since CP-violation effect arise as three(or more)-generation phenomena[16,17,18], we should make an experiment with "not too high" and "not too low" energy to see "3-generation". In an oscillation experiment, there are two energy scales,

$$E \sim \left\{ \begin{array}{l} \delta m^2_{31} L \\ \delta m^2_{21} L \end{array} \right. . \qquad (1)$$

Then the above energy range is expected to be suitable for a neutrino oscillation experiment to see CP violation in lepton sector.

More to say, to avoid matter effect which gives a fake CP violation, the lower energy is expected to be more preferable. Though unfortunately neutrinos in neutrino factory seem to have very high energy[19], very luckily we will have very intense muon source with rather low energy, PRISM[20]. It will located at Tokai, Ibaraki Prefecture, about 50 km from KEK. Since the muons will have energy less than 1 GeV, we can expect that we will have very intense neutrino beam with energy less than 500 MeV. With baseline length of several hundreds km, it will be very suitable to explore CP violation in lepton sector with neutrino oscillation experiments. With such a low energy beam, we will be able to detect neutrinos experimentally with good energy resolution. Stimulated by the possibility that we will have a low energy neutrino source with very high intensity, we consider here how large CP-violation effect we will see with such neutrino beam.

In this paper we will consider three active neutrinos without any sterile one by attributing the solar neutrino deficit and atmospheric neutrino anomaly to the neutrino oscillation[21]. We will use the following notation for the mixing matrix U,

$$
\begin{aligned}
U &= \mathrm{e}^{\mathrm{i}\psi\lambda_7}\Gamma\mathrm{e}^{\mathrm{i}\phi\lambda_5}\mathrm{e}^{\mathrm{i}\omega\lambda_2} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\psi & s_\psi \\ 0 & -s_\psi & c_\psi \end{pmatrix}
\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mathrm{e}^{\mathrm{i}\delta} \end{pmatrix}
\begin{pmatrix} c_\phi & 0 & s_\phi \\ 0 & 1 & 0 \\ -s_\phi & 0 & c_\phi \end{pmatrix}
\begin{pmatrix} c_\omega & s_\omega & 0 \\ -s_\omega & c_\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_\phi c_\omega & c_\phi s_\omega & s_\phi \\ -c_\psi s_\omega - s_\psi s_\phi c_\omega \mathrm{e}^{\mathrm{i}\delta} & c_\psi c_\omega - s_\psi s_\phi s_\omega \mathrm{e}^{\mathrm{i}\delta} & s_\psi c_\phi \mathrm{e}^{\mathrm{i}\delta} \\ s_\psi s_\omega - c_\psi s_\phi c_\omega \mathrm{e}^{\mathrm{i}\delta} & -s_\psi c_\omega - c_\psi s_\phi s_\omega \mathrm{e}^{\mathrm{i}\delta} & c_\psi c_\phi \mathrm{e}^{\mathrm{i}\delta} \end{pmatrix}, \qquad (2)
\end{aligned}
$$

where $c_\psi = \cos\psi, s_\phi = \sin\phi$, etc, and matter effect[22,23] a,

$$
a \equiv 2\sqrt{2}G_{\mathrm{F}}n_{\mathrm{e}}E = 7.56 \times 10^{-5}\mathrm{eV}^2 \cdot \left(\frac{\rho}{\mathrm{g\,cm^{-3}}}\right)\left(\frac{E}{\mathrm{GeV}}\right). \qquad (3)
$$

We will assume above energy range $\sim$ several hundreds MeV and hence from (1) with baseline length $\sim$ several hundreds km. With such an experimental setting the oscillation probabilities are calculated, e.g., as[15]

$$
\begin{aligned}
&P(\nu_\mu \to \nu_{\mathrm{e}}; E, L) = 4\sin^2\frac{\delta m^2_{31}L}{4E}c^2_\phi s^2_\phi s^2_\psi \left\{1 + \frac{a}{\delta m^2_{31}} \cdot 2(1-2s^2_\phi)\right\} \\
&+ 2\frac{\delta m^2_{31}L}{2E}\sin\frac{\delta m^2_{31}L}{2E}c^2_\phi s_\phi s_\psi \left\{-\frac{a}{\delta m^2_{31}}s_\phi s_\psi(1-2s^2_\phi) + \frac{\delta m^2_{21}}{\delta m^2_{31}}s_\omega(-s_\phi s_\psi s_\omega + c_\delta c_\psi c_\omega)\right\} \\
&- 4\frac{\delta m^2_{21}L}{2E}\sin^2\frac{\delta m^2_{31}L}{4E}s_\delta c^2_\phi s_\phi c_\psi s_\psi c_\omega s_\omega. \qquad (4)
\end{aligned}
$$

2 CP violation search in long baseline experiments

2.1 Magnitude of CP violation and matter effect

The available neutrinos as an initial beam are ν_μ and $\bar{\nu}_\mu$ in the current long baseline experiments[24,25]. The "CP violation" gives the nonzero difference of the oscillation probabilities between, e.g., $P(\nu_\mu \to \nu_{\rm e})$ and $P(\bar{\nu}_\mu \to \bar{\nu}_{\rm e})$[15]. This gives

$$\begin{aligned} P(\nu_\mu \to \nu_{\rm e}; L) - P(\bar{\nu}_\mu \to \bar{\nu}_{\rm e}; L) = {} & 16\frac{a}{\delta m^2_{31}} \sin^2 \frac{\delta m^2_{31} L}{4E} c^2_\phi s^2_\phi s^2_\psi (1 - 2s^2_\phi) \\ & - 4\frac{aL}{2E} \sin \frac{\delta m^2_{31} L}{2E} c^2_\phi s^2_\phi s^2_\psi (1 - 2s^2_\phi) \\ & - 8\frac{\delta m^2_{21} L}{2E} \sin^2 \frac{\delta m^2_{31} L}{4E} s_\delta c^2_\phi s_\phi c_\psi s_\psi c_\omega s_\omega. \end{aligned} \tag{5}$$

The difference of these two, however, also includes matter effect, or the fake CP violation, proportional to a. We must somehow distinguish these two to conclude the existence of CP violation as discussed in ref.[15].

On the other hand, a muon ring enables to extract $\nu_{\rm e}$ and $\bar{\nu}_{\rm e}$ beam. It enables direct measurement of pure CP violation through "T violation", e.g., $P(\nu_\mu \to \nu_{\rm e}) - P(\nu_{\rm e} \to \nu_\mu)$ as

$$P(\nu_\mu \to \nu_{\rm e}) - P(\nu_{\rm e} \to \nu_\mu) = -8\frac{\delta m^2_{21} L}{2E} \sin^2 \frac{\delta m^2_{31} L}{4E} s_\delta c^2_\phi s_\phi c_\psi s_\psi c_\omega s_\omega. \tag{6}$$

Note that this difference gives pure CP violation.

By measuring "CPT violation", e.g. the difference between $P(\nu_\mu \to \nu_{\rm e})$ and $P(\bar{\nu}_{\rm e} \to \bar{\nu}_\mu)$, we can check the matter effect.

$$\begin{aligned} P(\nu_\mu \to \nu_{\rm e}; L) - P(\bar{\nu}_{\rm e} \to \bar{\nu}_\mu; L) = {} & 16\frac{a}{\delta m^2_{31}} \sin^2 \frac{\delta m^2_{31} L}{4E} c^2_\phi s^2_\phi s^2_\psi (1 - 2s^2_\phi) \\ & - 4\frac{aL}{2E} \sin \frac{\delta m^2_{31} L}{2E} c^2_\phi s^2_\phi s^2_\psi (1 - 2s^2_\phi) \end{aligned} \tag{7}$$

We present in Fig.1 "T-violation" part (6) and "CPT-violation" part (7) for some parameters allowed by the present experiments[26] with $\sin^2 \omega = 1/2$, $\sin^2 \psi = 1/2$, $\sin \delta = 1$ fixed[a]. The matter density is also fixed to the constant

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[a] Although the Chooz reactor experiment have almost excluded $\sin^2 \phi = 0.1$[27], there remains still small chance to take this value.

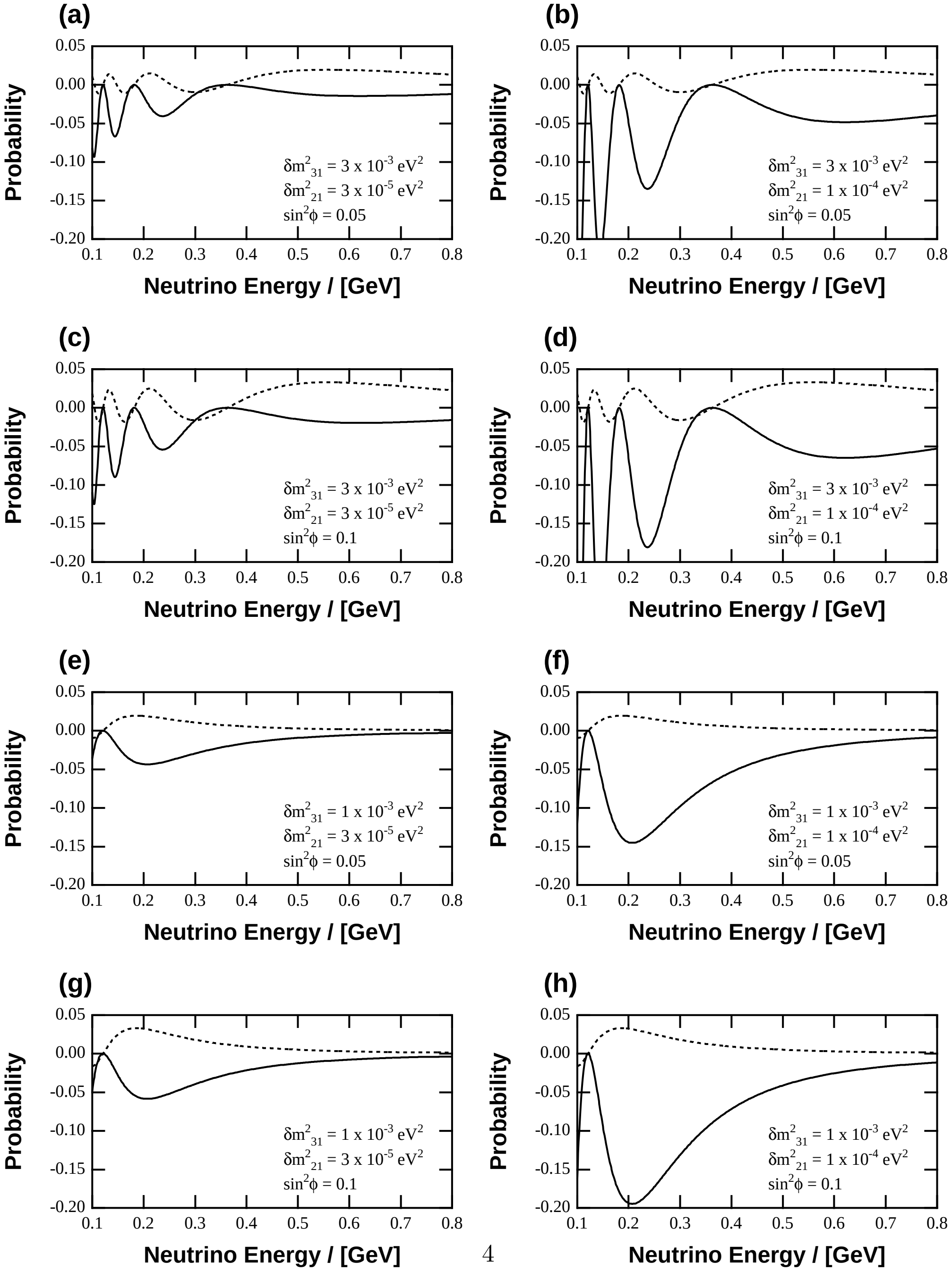

Figure 1: Graphs of $P(\nu_\mu \to \nu_\mathrm{e}) - P(\nu_\mathrm{e} \to \nu_\mu)$ (solid lines; pure CP-violation effects) and $P(\nu_\mu \to \nu_\mathrm{e}) - P(\bar{\nu}_\mathrm{e} \to \bar{\nu}_\mu)$ (dashed lines; matter effects) as functions of neutrino energy. Parameters not shown in the graphs are taken $\sin^2\omega = 1/2, \sin^2\psi = 1/2, \sin\delta = 1; \rho = $ g/cm^3 and $L = 300$km.

value[28] $\rho = 2.5\text{g/cm}^3$. Other parameters are taken as $\delta m^2_{31} = 3\times 10^{-3}\text{eV}^2$ and $1\times 10^{-3}\text{eV}^2$, $\delta m^2_{21} = 1\times 10^{-4}\text{eV}^2$ and $3\times 10^{-5}\text{eV}^2$.

"T-violation" effect is proportional to $\delta m^2_{21}/\delta m^2_{31}$ and, for $\phi \ll 1$, also to $\sin\phi$ as seen in eq.(6) and Fig.1. Recalling that the energy of neutrino beam is of several hundreds MeV, we see in Fig.1 that the "T-violation" effect amounts to at least about 5%, hopefully 10∼20%. This result gives hope to detect the pure leptonic CP violation directly with the neutrino oscillation experiments.

The "T violation" is, however, less than 10% in the case that δm^2_{21} is as small as $3\times 10^{-5}\text{eV}^2$ (see the left four graphs of Fig.1). In this case matter effect is as large in magnitude as "T violation" and has an opposite sign for $\sin\delta > 0$ as seen in Fig.1. In such a case the sum of the two, eq.(5), is destructive and has even more smaller magnitude than "T violation", thus the experiments will be more difficult. Thanks to $\nu_{\rm e}$ and $\bar{\nu}_{\rm e}$ available from low energy muon source, one can measure "T violation". This makes the measurement much easier.

2.2 Estimation of statistical error in CP-violation searches

Here we state that the energy range considered here is probably best in view of statistical errors in order to observe CP violation effect. To this end let us estimate how $\delta P/\Delta P$ scales with E and L, where δP be statistical error of transition probabilities such as $P(\nu_{\rm e} \to \nu_\mu)$ and $\Delta P = P(\nu_{\rm e} \to \nu_\mu) - P(\nu_\mu \to \nu_{\rm e})$. We denote in this section the transition probabilities $P(\nu_\alpha \to \nu_\beta)(\alpha \neq \beta)$ simply by P. Suppose that n neutrinos out of N detected neutrinos has changed its flavor. With a number of decaying muons fixed, the number of detected neutrinos N are roughly proportional to E^3, and hence $N \sim E^3 L^{-2}$. We estimate δP as

$$\begin{aligned}\delta P &= \delta\left(\frac{n}{N}\right)\\ &= \frac{|N\delta n| + |n\delta N|}{N^2}\\ &= \frac{|N\sqrt{NP}| + |NP\sqrt{N}|}{N^2}\\ &= \frac{\sqrt{P}+P}{\sqrt{N}},\end{aligned} \tag{8}$$

where we used $\delta n = \sqrt{n}$, $\delta N = \sqrt{N}$ and $n = NP$. From eqs.(4), (6) and (8), we can estimate how $\delta P/\Delta P$ scales for E with L fixed. We summarize the results in Table 1. There we see that $\delta P/\Delta P$ reaches minimum at the region

E		$\delta m^2_{21} L$		$\delta m^2_{31} L$	
P	"const."		$1/E$ or "const."		$1/E^2$
δP	$1/E^{1.5}$		$1/E^{1.5} \sim 1/E^2$		$1/E^{2.5}$
ΔP	"const"		$1/E$		$1/E^3$
$\delta P/\Delta P$	$1/E^{1.5}$		$1/E^{0.5} \sim 1/E$		$E^{0.5}$
	$\searrow$		$\searrow$	minimum	$\nearrow$

Table 1: The E-dependence of oscillation envelopes of some quantities with L fixed. Here "const." means that the oscillation envelope of the quantity is independent of E. $\delta P/\Delta P$ reaches minimum at the region $E \sim \delta m^2_{31} L$.

L		$E/\delta m^2_{31}$		$E/\delta m^2_{21}$	
P	L^2		L or "const."		"const."
δP	L^3		$L^{1.5} \sim L$		L
ΔP	L^3		L		"const."
$\delta P/\Delta P$	"const."		$L^{0.5} \sim$ "const."		L
	$\rightarrow$		$\nearrow$		$\nearrow$

Table 2: The L-dependence of oscillation envelopes of some quantities with E fixed.

$E \sim \delta m^2_{31} L$. Note that indeed the 3-generation nature (1) is satisfied here. With such a concrete example, we can see how important the 3-genaration nature is.

By a similar consideration one can obtain how $\delta P/\Delta P$ scales for L with E fixed. The result for this case is shown in Table 2. We can see there that we should keep not too large L so that the error $\delta P/\Delta P$ should not get large.

We need a few hundreds MeV of neutrino energy to reach the threshold energy of muon production reaction $\mathrm{N} + \nu_\mu \to \mathrm{N} + \mu$, where N is nucleon. We have also seen in Table 1 that the error comes to minimum at the region $E \sim \delta m^2_{31} L$. Considering these results, we conclude that $E \sim$ (a few) $\times$ 100 MeV and $L \sim$ (a few) $\times$ 100 km, which we have just considered in this paper, is the best configuration to search CP violation in view of statistical error.

3 Summary and conclusion

We considered how large CP/T violation effects can be observed making use of low-energy neutrino beam, inspired by PRISM. More than 10%, hopefully 20% of the pure CP-violation effects may be observed within the allowed region of present experiments.

We have also seen that in some case the pure CP-violation effects are as small as the matter effect but have opposite sign. In such a case the "CP violation" gets smaller through the destructive sum of the pure CP-violation effect and matter effect. We pointed out that we can avoid this difficulty by observing "T-violation" effect using initial $\nu_{\rm e}$ beam.

We finally discussed that the configuration we have considered here, $E \sim$ (a few) $\times$ 100 MeV and $L \sim$ (a few) $\times$ 100 km is best to search lepton CP violation in terms of statistical error. With such consideration we also found how important the 3-generation nature (1) is. It is thus worth making an effort to develop leptonic CP violation search using neutrinos from low energy muons.

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